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NON-DiFFERENTIABLE DELAY-INTERVAL -DEPENDENT EXPONENTIALLY PASSIVE CONDITIONS FOR NEUTRAL INTEGRO -DiFFERENTIAL EQUATIONS WITH TIME-VARYING DELAYS

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Abstract The problem of non-differentiable delay-interval-dependent exponentially passive conditions for certain neutral integro-differential equation with time-varying delays is studied. The time-varying delays, being continuous functions, are under the allowed interval delays, therefore the lower and upper bounds of time-varying delays are accessible. But the condition on the derivatives of the distributed and discrete interval time-varying delays are eliminated. These conditions are interested on distributed and discrete delays for the equation. By applying decomposition technique of coefficient constant, descriptor model transformation, new class of augmented Lyapunov-Krasovskii functional, Leibniz-Newton formula, improved integral inequalities, utilization of zero equation and Peng-Park's integral inequality, new delay-interval-dependent exponentially passive conditions are in form of linear matrix inequalities (LMIs). Furthermore, improved delay-dependent stability conditions for certain neutral differential equation with time-varying delays are presented. Numerical examples represent the improvement and effectiveness of results over another research.

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1. Introduction

During the last few decades, the delay-interval-dependent stability conditions for differential neutral systems with time-varying delays were considered by several researchers since they describe heartbeat, memorization, locomotion, mastication and respiration,

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see [18, 21, 30]. The time-varying delays have the lower and upper bounds as well as the delay functions are required to be differentiable. Delay-dependent asymptotic stability criteria for certain neutral differential equations (CNDE) with constant delays have been discussed in [8, 17, 23] by applying Lyapunov-Krasovskii functional and several model transformation. In [5, 6, 15], the researchers considered the exponential stability problem for CNDE with time-varying delays by several methods. In [5], the results were established without the use of the bounding technique and the model transformation method, while researchers have studied it by using radially unboundedness, Lyapunov-Krasovskii functional approach and model transformation method in [15]. Furthermore, the researchers have shown stability criteria for uncertain neutral systems with distributed and discrete delays such as [33]. The robust stability of uncertain linear neutral systems with distributed and discrete delays has been considered in [14]. However, upper bounds of derivative for neutral and discrete time-varying delays are necessary in existing methods.

On the one hand, the importance of concept for passivity has been popularly applied in various fields, such as complexity [7], fuzzy control [4], and signal processing [32], since 1970s. Passivity presents more than just stability of systems, which relates the input and output of the systems to the storage function. Passive properties of systems can maintain the systems internally stable. In the present, the passivity analysis have been studied several researchers [20, 29, 34, 36]. The exponentially passivity condition for delayed neural networks was obtained in [36]. In [34], the issue of robust passivity conditions for neural networks with distributed and discrete delays has been extensively studied. The researchers have presented the passivity condition for neural network with norm-bounded uncertainties and time-varying delays in [20]. The researchers have investigated the issue of passivity of neutral-type neural networks with mixed and leakage delays in [29]. However, no result has been obtained for exponentially passive condition of certain neutral integro-differential equations with non-differentiable interval time-varying delays.

The point of this paper is the delay-interval-dependent stability conditions for differential systems with time-varying delays. The restrictions on the derivatives of the discrete and distributed time-varying delays for the equation are eliminated. Mixed integral inequalities, the combination of mixed model transformation, application of zero equation, Lyapunov-Krasovskii functional and separation of coefficient constant are applied to establish conditions for exponential stability and passivity. Then, improved delay-dependent exponential stability criteria for certain neutral differential equation with time-varying delays [6, 15] are presented. Lastly, numerical examples have shown that the presented conditions are improvement and effective over another researches.

2. Preliminaries

We introduce the following certain neutral integro-differential equation with interval time-varying delays

$$\begin{aligned} \frac{d}{dt}[x(t)+px(t-\tau(t))] &= -ax(t)+b\tanh x(t-\sigma(t))+c\int_{t-\rho(t)}^{t}x(s)ds \\ &\quad +du(t), \qquad (2.1) \\ z(t) &= \tilde{a}x(t)+\tilde{b}\tanh x(t-\sigma(t))+\tilde{d}u(t), \qquad (2.2) \end{aligned}$$

for the state vector $x(t)\in\mathbb{R}$. $z(t)\in\mathbb{R}$ is the output of the equation. $u(t)\in\mathbb{R}$ stands for the external inputs. $\tilde{a}$, $\tilde{b}$, c, d, $\tilde{d}$, p are real numbers with $|p|<1$ and a, b are positive real numbers. The variables $\rho(t)$, $\sigma(t)$ and $\tau(t)$ are distributed, discrete and neutral interval time-varying delays,serially,

$$0 \le \rho_1 \le \rho(t) \le \rho_2, \tag{2.3}$$
$$0 \le \sigma_1 \le \sigma(t) \le \sigma_2, \tag{2.4}$$
$$0 \le \tau_1 \le \tau(t) \le \tau_2, \quad \dot{\tau}(t) \le \tau_d, \tag{2.5}$$

where τ_1, τ_2, τ_d, σ_1, σ_2, ρ_1 and ρ_2 are given positive real numbers. The initial condition for equation (2.1) are

$$x_0(t) = \varsigma(t), \quad t \in [-\kappa, 0],$$

for $\varsigma \in C([-\kappa, 0]; \mathbb{R})$ such that $\kappa = \max\{\tau_2, \sigma_2, \rho_2\}$. We denote the state trajectory of (2.1) as $x(t, \varsigma)$. The Leibniz-Newton equations are considered

$$0 = x(t) - x(t-\tau(t)) - \int_{t-\tau(t)}^{t} \dot{x}(s)ds, \tag{2.6}$$
$$0 = x(t) - x(t-\sigma(t)) - \int_{t-\sigma(t)}^{t} \dot{x}(s)ds. \tag{2.7}$$

For $\epsilon_1, \epsilon_2 \in \mathbb{R}$, we utilize the previous Leibniz-Newton equations

$$0 = \epsilon_1 x(t) - \epsilon_1 x(t-\tau(t)) - \epsilon_1 \int_{t-\tau(t)}^{t} \dot{x}(s)ds, \tag{2.8}$$
$$0 = \epsilon_2 x(t) - \epsilon_2 x(t-\sigma(t)) - \epsilon_2 \int_{t-\sigma(t)}^{t} \dot{x}(s)ds. \tag{2.9}$$

The equation (2.1) is tranformed by descriptor model transformation and (2.6)-(2.9),

$$\begin{aligned} \dot{x}(t) &= y(t) + \epsilon_1 x(t) - \epsilon_1 x(t-\tau(t)) - \epsilon_1 \int_{t-\tau(t)}^{t} y(s)ds \\ &\quad + \epsilon_2 x(t) - \epsilon_2 x(t-\sigma(t)) - \epsilon_2 \int_{t-\sigma(t)}^{t} y(s)ds, \end{aligned} \tag{2.10}$$
$$\begin{aligned} y(t) &= -ax(t) + b \tanh x(t-\sigma(t)) + c \int_{t-\rho(t)}^{t} x(s)ds \\ &\quad - py(t-\tau(t)) + du(t). \end{aligned} \tag{2.11}$$

For $a = a_1 + a_2 + a_3$ with $a_1, a_2, a_3 \in \mathbb{R}$, we rewrite the (2.10)-(2.11)

$$\begin{aligned} \dot{x}(t) &= y(t) + \epsilon_1 x(t) - \epsilon_1 x(t-\tau(t)) - \epsilon_1 \int_{t-\tau(t)}^{t} y(s)ds \\ &\quad + \epsilon_2 x(t) - \epsilon_2 x(t-\sigma(t)) - \epsilon_2 \int_{t-\sigma(t)}^{t} y(s)ds, \end{aligned} \tag{2.12}$$
$$\begin{aligned} y(t) &= -a_1 x(t) - a_2 x(t-\tau(t)) - a_2 \int_{t-\tau(t)}^{t} y(s)ds - a_3 x(t-\sigma(t)) \\ &\quad - a_3 \int_{t-\sigma(t)}^{t} y(s)ds + b \tanh x(t-\sigma(t)) + c \int_{t-\rho(t)}^{t} x(s)ds \\ &\quad - py(t-\tau(t)) + du(t). \end{aligned} \tag{2.13}$$

Definition 2.1. [9] The equation (2.1)-(2.2) is exponentially passive from input $u(t)$ to output $z(t)$, if there is a Lyapunov function $V(t)$ and a positive real number k satisfy :

$$\dot{V}(t) + kV(t) \leq 2z(t)u(t), \quad t \geq t_0,$$

for all $u(t)$, all initial conditions $x(t_0)$.

Lemma 2.2. [31] *For constant symmetric positive definite matrix $\Lambda \in \mathbb{R}^{n\times n}$, $0 \leq \sigma_1 \leq \sigma(t) \leq \sigma_2$ and vector function $x : [-\sigma_2, -\sigma_1] \to \mathbb{R}^n$ is well defined, then*

$$\begin{aligned} &-[\sigma_2 - \sigma_1] \int_{-\sigma_2}^{-\sigma_1} x^T(s)\Lambda x(s)ds \\ \leq\ & -\int_{-\sigma(t)}^{-\sigma_1} x^T(s)ds\Lambda \int_{-\sigma(t)}^{-\sigma_1} x(s)ds - \int_{-\sigma_2}^{-\sigma(t)} x^T(s)ds\Lambda \int_{-\sigma_2}^{-\sigma(t)} x(s)ds. \end{aligned}$$

Lemma 2.3. [31] *For $M_1, M_2, M_3 \in \mathbb{R}^{n\times n}$ are constant matrices such that $M_1 \geq 0, M_3 > 0$, $\begin{bmatrix} M_1 & M_2 \\ * & M_3 \end{bmatrix} \geq 0$, $0 \leq \sigma_1 \leq \sigma(t) \leq \sigma_2$ and vector function $\dot{x} : [-\sigma_2, -\sigma_1] \to \mathbb{R}^n$ is well defined, then*

$$\begin{aligned} &-[\sigma_2 - \sigma_1] \int_{t-\sigma_2}^{t-\sigma_1} \begin{bmatrix} x(s) \\ \dot{x}(s) \end{bmatrix}^T \begin{bmatrix} M_1 & M_2 \\ * & M_3 \end{bmatrix} \begin{bmatrix} x(s) \\ \dot{x}(s) \end{bmatrix} ds \\ &\leq A^T(t) \begin{bmatrix} -M_3 & M_3 & 0 & -M_2^T & 0 \\ * & -M_3 - M_3 & M_3 & M_2^T & -M_2^T \\ * & * & -M_3 & 0 & M_2^T \\ * & * & * & -M_1 & 0 \\ * & * & * & * & -M_1 \end{bmatrix} A(t), \end{aligned}$$

where

$$A(t) := \begin{bmatrix} x(t-\sigma_1) \\ x(t-\sigma(t)) \\ x(t-\sigma_2) \\ \int_{t-\sigma(t)}^{t-\sigma_1} x(s)ds \\ \int_{t-\sigma_2}^{t-\sigma(t)} x(s)ds \end{bmatrix}.$$

Lemma 2.4. [31] *For vector function $\dot{x} : [-\sigma_2, -\sigma_1] \to \mathbb{R}^n$ is well defined, $0 \leq \sigma_1 \leq \sigma(t) \leq \sigma_2$ and $X, M_i (i = 1, 2, \ldots, 5) \in \mathbb{R}^{n\times n}$ are constant matrices, then*

$$\begin{aligned} &-\int_{t-\sigma_2}^{t-\sigma_1} \dot{x}^T(s)X\dot{x}(s)ds \\ \leq\ & B^T(t) \begin{bmatrix} M_1 + M_1^T & -M_1^T + M_2 & 0 \\ * & M_1 + M_1^T - M_2 - M_2^T & -M_1^T + M_2 \\ * & * & -M_2 - M_2^T \end{bmatrix} B(t) \\ &+[\sigma_2 - \sigma_1]B^T(t) \begin{bmatrix} M_3 & M_4 & 0 \\ * & M_3 + M_5 & M_4 \\ * & * & M_5 \end{bmatrix} B(t), \end{aligned}$$

where $\begin{bmatrix} X & M_1 & M_2 \\ * & M_3 & M_4 \\ * & * & M_5 \end{bmatrix} \geq 0$ *and* $B(t) := \begin{bmatrix} x(t-\sigma_1) \\ x(t-\sigma(t)) \\ x(t-\sigma_2) \end{bmatrix}$.

Lemma 2.5. (Peng-Park's integral inequality) [26] *Let* m *and* s *be real constants such that* $\begin{bmatrix} m & s \\ * & m \end{bmatrix} \geq 0$, σ *and* $\sigma(t)$ *be positive scalars satisfying* $0 < \sigma(t) < \sigma$, *vector function* $\dot{x} : [-\sigma, 0] \to \mathbb{R}^n$ *be well defined. Then*

$$-\sigma \int_{t-\sigma}^{t} \dot{x}^T(s) m \dot{x}(s) ds \leq C^T(t) \begin{bmatrix} -m & m-s & s \\ * & -2m+s+s & m-s \\ * & * & -m \end{bmatrix} C(t),$$

where $C(t) := \begin{bmatrix} x(t) \\ x(t-\sigma(t)) \\ x(t-\sigma) \end{bmatrix}$.

3. Main Results

We define a new parameter

$$\sum = \left[\Delta_{(i,j)}\right]_{27\times 27}, \tag{3.1}$$

for $\Delta_{(i,j)} = \Delta^T_{(j,i)}$,

$$\Delta_{(1,1)} = 2\alpha k_1 + n_9 + k_7 + n_{10} + k_8 + 2z_1 + 2z_4 + 2k_1\epsilon_1 + 2k_1\epsilon_2 - 2q_2a_1 + k_2\tau_2^2 + k_3\sigma_2^2 + n_1\tau_2^2 - 2n_1\tau_2\tau_1 + n_1\tau_1^2 + n_2\sigma_2^2 - 2n_2\sigma_2\sigma_1 + n_2\sigma_1^2 - k_4e^{-2\alpha\tau_2} - k_5e^{-2\alpha\sigma_2} + \tau_1^2 f_1 + \tau_2^2 r_1 - r_3e^{-2\alpha\tau_2} + f_4\tau_2^2 - 2\tau_1\tau_2 + \tau_1^2 f_4 + \sigma_1^2 f_7 + \sigma_2^2 r_4 + \sigma_2^2 f_{10} - 2\sigma_1\sigma_2 f_{10} + \sigma_1^2 f_{10} - r_6e^{-2\alpha\sigma_2} + n_7\sigma_1^2 + k_6\sigma_2^2 + n_8\sigma_2^2 - 2n_8\sigma_1\sigma_2 + n_8\sigma_1^2 + n_{11}\rho_2^2 - 2n_{11}\rho_1\rho_2 + n_{11}\rho_1^2, \quad \Delta_{(1,2)} = q_1\epsilon_1 + q_1\epsilon_2 + \tau_1^2 f_2 + \tau_2^2 r_2 + \tau_2^2 f_5 - 2\tau_1\tau_2 f_5 + \tau_1^2 f_5 + \sigma_1^2 f_8 + \sigma_2^2 r_5 + \sigma_2^2 f_{11} - 2\sigma_1\sigma_2 f_{11} + \sigma_1^2 f_{11} + k_1 - q_2 - q_3a_1, \quad \Delta_{(1,3)} = -z_1 + z_2 - k_1\epsilon_1 - q_2a_2 - q_4a_1 + k_4e^{-2\alpha\tau_2} - s_1 + r_3e^{-2\alpha\tau_2}, \quad \Delta_{(1,4)} = -z_1 + z_3 - k_1\epsilon_1 - q_2a_2 - q_5a_1, \quad \Delta_{(1,5)} = -z_4 + z_5 - k_1\epsilon_2 - q_2a_3 - q_6a_1 + k_5e^{-2\alpha\sigma_2} - s_2 + r_6e^{-2\alpha\sigma_2}, \quad \Delta_{(1,6)} = -z_4 + z_6 - k_1\epsilon_2 - q_2a_3 - q_7a_1, \quad \Delta_{(1,7)} = q_2b - q_8a_1, \quad \Delta_{(1,8)} = q_2c - q_{10}a_1, \quad \Delta_{(1,9)} = -q_2p - q_9a_1, \quad \Delta_{(1,10)} = -r_2e^{-2\alpha\tau_2}, \quad \Delta_{(1,12)} = -r_5e^{-2\alpha\sigma_2}, \quad \Delta_{(1,17)} = s_1, \quad \Delta_{(1,19)} = s_2, \quad \Delta_{(1,27)} = q_2d - \tilde{a}, \quad \Delta_{(2,2)} = k_9 + n_3\tau_1^2 + k_4\tau_2^2 + n_4\sigma_1^2 + k_5\sigma_2^2 + n_5\tau_2^2 - 2n_5\tau_1\tau_2 + n_5\tau_1^2 + n_6\sigma_2^2 - 2n_6\sigma_1\sigma_2 + n_6\sigma_1^2 + \tau_1^2 f_3 + \tau_2^2 r_3 + \tau_2^2 f_6 - 2\tau_1\tau_2 f_6 + \tau_1^2 f_6 + \sigma_1^2 f_9 + \sigma_2^2 f_{12} - 2\sigma_1\sigma_2 f_{12} + \sigma_1^2 f_{12} - 2q_3 + \sigma_2^2 r_6, \quad \Delta_{(2,3)} = -q_1\epsilon_1 - q_3a_2 - q_4, \quad \Delta_{(2,4)} = -q_1\epsilon_1 - q_3a_2 - q_5, \quad \Delta_{(2,5)} = -q_1\epsilon_2 - q_3a_3 - q_6, \quad \Delta_{(2,6)} = -q_1\epsilon_2 - q_3a_3 - q_7, \quad \Delta_{(2,7)} = q_3b - q_8, \quad \Delta_{(2,8)} = q_3c - q_{10}, \quad \Delta_{(2,9)} = -q_3p - q_9, \quad \Delta_{(2,27)} = q_3d, \quad \Delta_{(3,3)} = -2z_2 - 2q_4a_2 - 2k_4e^{-2\alpha\tau_2} + 2s_1 + 2m_1 - 2m_2 + \tau_2m_3 - \tau_1m_3 + \tau_2m_5 - \tau_1m_5 - 2r_3e^{-2\alpha\tau_2} - 2f_6e^{-2\alpha\tau_2}, \quad \Delta_{(3,4)} = -z_2 - z_3 - q_4a_2 - q_5a_2, \quad \Delta_{(3,5)} = -q_4a_3 - q_6a_2,$$

$\Delta_{(3,6)} = -q_4a_3 - q_7a_2, \quad \Delta_{(3,7)} = q_4b - q_8a_2, \quad \Delta_{(3,8)} = q_4c - q_{10}a_2,$
$\Delta_{(3,9)} = -q_4p - q_9a_2, \quad \Delta_{(3,10)} = r_2e^{-2\alpha\tau_2}, \quad \Delta_{(3,11)} = -r_2e^{-2\alpha\tau_2}$
$-f_5e^{-2\alpha\tau_2}, \quad \Delta_{(3,14)} = f_5e^{-2\alpha\tau_2}, \quad \Delta_{(3,16)} = -m_1 + m_2 + \tau_2m_4 - \tau_1m_4$
$+f_6e^{-2\alpha\tau_2}, \quad \Delta_{(3,17)} = k_4e^{-2\alpha\tau_2} - s_1 - m_1 + m_2 + \tau_2m_4 - \tau_1m_4 + r_3e^{-2\alpha\tau_2}$
$+f_6e^{-2\alpha\tau_2}, \quad \Delta_{(3,27)} = q_4d, \quad \Delta_{(4,4)} = -2z_3 - 2q_5a_2, \quad \Delta_{(4,5)} = -q_5a_3$
$-q_6a_2, \quad \Delta_{(4,6)} = -q_5a_3 - q_7a_2, \quad \Delta_{(4,7)} = q_5b - q_8a_2, \quad \Delta_{(4,8)} = q_5c$
$-q_{10}a_2, \quad \Delta_{(4,9)} = -q_5p - q_9a_2, \quad \Delta_{(4,27)} = q_5d, \quad \Delta_{(5,5)} = w - 2z_5 - 2q_6a_3$
$-2k_5e^{-2\alpha\sigma_2} + 2s_2 + 2m_6 - 2m_7 + \sigma_2m_8 - \sigma_1m_8 + \sigma_2m_{10} - \sigma_1m_{10}$
$-2r_6e^{-2\alpha\sigma_2} - 2f_{12}e^{-2\alpha\sigma_2}, \quad \Delta_{(5,6)} = -z_5 - z_6 - q_6a_3 - q_7a_3, \quad \Delta_{(5,7)} = q_6b$
$-q_8a_3, \quad \Delta_{(5,8)} = q_6c - q_{10}a_3, \quad \Delta_{(5,9)} = -q_6p - q_9a_3, \quad \Delta_{(5,12)} = r_5e^{-2\alpha\sigma_2},$
$\Delta_{(5,13)} = -r_5e^{-2\alpha\sigma_2} - f_{11}e^{-2\alpha\sigma_2}, \quad \Delta_{(5,15)} = f_{11}e^{-2\alpha\sigma_2}, \quad \Delta_{(5,18)} = -m_6$
$+m_7 + \sigma_2m_9 - \sigma_1m_9 + f_{12}e^{-2\alpha\sigma_2}, \quad \Delta_{(5,19)} = k_5e^{-2\alpha\sigma_2} - s_2 - m_6 + m_7$
$+\sigma_2m_9 - \sigma_1m_9 + r_6e^{-2\alpha\sigma_2} + f_{12}e^{-2\alpha\sigma_2}, \quad \Delta_{(5,27)} = q_6d, \quad \Delta_{(6,6)} = -2z_6$
$-2q_7a_3, \quad \Delta_{(6,7)} = q_7b - q_8a_3, \quad \Delta_{(6,8)} = q_7c - q_{10}a_3, \quad \Delta_{(6,9)} = -q_7p$
$-q_9a_3, \quad \Delta_{(6,27)} = q_7d, \quad \Delta_{(7,7)} = -w + 2q_8b, \quad \Delta_{(7,8)} = q_8c + q_{10}b,$
$\Delta_{(7,9)} = -q_8p + q_9b, \quad \Delta_{(7,27)} = q_8d - \tilde{b}, \quad \Delta_{(8,8)} = 2q_{10}c - n_{11}e^{-2\alpha\rho_2},$
$\Delta_{(8,9)} = -q_{10}p + q_9c, \quad \Delta_{(8,27)} = q_{10}d, \quad \Delta_{(9,9)} = -k_9e^{-2\alpha\tau_2} + k_9\tau_d - 2q_9p,$
$\Delta_{(9,27)} = q_9d, \quad \Delta_{(10,10)} = -k_2e^{-2\alpha\tau_2} - r_1e^{-2\alpha\tau_2}, \quad \Delta_{(11,11)} = -k_2e^{-2\alpha\tau_2}$
$-n_1e^{-2\alpha\tau_2} - r_1e^{-2\alpha\tau_2} - f_4e^{-2\alpha\tau_2}, \quad \Delta_{(11,17)} = r_2e^{-2\alpha\tau_2} + f_5e^{-2\alpha\tau_2},$
$\Delta_{(12,12)} = -k_3e^{-2\alpha\sigma_2} - r_4e^{-2\alpha\sigma_2}, \quad \Delta_{(13,13)} = -k_3e^{-2\alpha\sigma_2} - n_2e^{-2\alpha\sigma_2}$
$-r_4e^{-2\alpha\sigma_2} - f_{10}e^{-2\alpha\sigma_2}, \quad \Delta_{(13,19)} = r_5e^{-2\alpha\sigma_2} + f_{11}e^{-2\alpha\sigma_2}, \quad \Delta_{(14,14)} =$
$-n_1e^{-2\alpha\tau_2} - f_4e^{-2\alpha\tau_2}, \quad \Delta_{(14,16)} = -f_5e^{-2\alpha\tau_2}, \quad \Delta_{(15,15)} = -n_2e^{-2\alpha\sigma_2}$
$-f_{10}e^{-2\alpha\sigma_2}, \quad \Delta_{(15,18)} = -f_{11}e^{-2\alpha\sigma_2}, \quad \Delta_{(16,16)} = -n_9e^{-2\alpha\tau_1} + 2m_1 + \tau_2m_3$
$-\tau_1m_3 - f_6e^{-2\alpha\tau_2}, \quad \Delta_{(17,17)} = -k_7e^{-2\alpha\tau_2} - k_4e^{-2\alpha\tau_2} - 2m_2 + \tau_2m_5 - \tau_1m_5$
$-r_3e^{-2\alpha\tau_2} - f_6e^{-2\alpha\tau_2}, \quad \Delta_{(18,18)} = -n_{10}e^{-2\alpha\sigma_1} + 2m_6 + \sigma_2m_8 - \sigma_1m_8$
$-f_{12}e^{-2\alpha\sigma_2}, \quad \Delta_{(19,19)} = -k_8e^{-2\alpha\sigma_2} - k_5e^{-2\alpha\sigma_2} - 2m_7 + \sigma_2m_{10} - \sigma_1m_{10}$
$-r_6e^{-2\alpha\sigma_2} - f_{12}e^{-2\alpha\sigma_2}, \quad \Delta_{(20,20)} = -n_3e^{-2\alpha\tau_1} - f_3e^{-2\alpha\tau_1}, \quad \Delta_{(20,22)} =$
$-f_2e^{-2\alpha\tau_1}, \quad \Delta_{(21,21)} = -n_4e^{-2\alpha\sigma_1} - f_9e^{-2\alpha\sigma_1}, \quad \Delta_{(21,23)} = -f_8e^{-2\alpha\sigma_1},$
$\Delta_{(22,22)} = -f_1e^{-2\alpha\tau_1}, \quad \Delta_{(23,23)} = -f_7e^{-2\alpha\sigma_1}, \quad \Delta_{(24,24)} = -n_7e^{-2\alpha\sigma_1},$
$\Delta_{(25,25)} = -k_6e^{-2\alpha\sigma_2}, \quad \Delta_{(26,26)} = -n_8e^{-2\alpha\sigma_2}, \quad \Delta_{(27,27)} = -2\tilde{d},$
and other terms are 0.

Theorem 3.1. *Let* $\alpha, r_1, r_3, r_4, r_6, f_4, f_6, f_{10}, f_{12}, w, k_i (i = 1, 2, \ldots, 9), n_j (j = 1, 2, \ldots, 11)$ *be positive real numbers and* $s_1, s_2, r_2, r_5, f_1, f_2, f_3, f_5, f_7, f_8, f_9, f_{11}, q_k, m_k (k = 1, 2, \ldots, 10)$ *be real numbers such that satisfy the following symmetric linear matrix inequalities*

$$\sum < 0, \tag{3.2}$$

$$\begin{bmatrix} k_4 e^{-2\alpha\tau_2} & s_1 \\ * & k_4 e^{-2\alpha\tau_2} \end{bmatrix} \geq 0, \tag{3.3}$$

$$\begin{bmatrix} k_5 e^{-2\alpha\sigma_2} & s_2 \\ * & k_5 e^{-2\alpha\sigma_2} \end{bmatrix} \geq 0, \tag{3.4}$$

$$\begin{bmatrix} n_5(\tau_2 - \tau_1) e^{-2\alpha\tau_2} & m_1 & m_2 \\ * & m_3 & m_4 \\ * & * & m_5 \end{bmatrix} \geq 0, \tag{3.5}$$

$$\begin{bmatrix} n_6(\sigma_2 - \sigma_1) e^{-2\alpha\sigma_2} & m_6 & m_7 \\ * & m_8 & m_9 \\ * & * & m_{10} \end{bmatrix} \geq 0, \tag{3.6}$$

$$\begin{bmatrix} r_1 & r_2 \\ * & r_3 \end{bmatrix} \geq 0, \tag{3.7}$$

$$\begin{bmatrix} r_4 & r_5 \\ * & r_6 \end{bmatrix} \geq 0, \tag{3.8}$$

$$\begin{bmatrix} f_4 & f_5 \\ * & f_6 \end{bmatrix} \geq 0, \tag{3.9}$$

$$\begin{bmatrix} f_{10} & f_{11} \\ * & f_{12} \end{bmatrix} \geq 0. \tag{3.10}$$

Then equations (2.1)-(2.2) *are exponentially passive.*

Proof. Let $\alpha, r_1, r_3, r_4, r_6, f_4, f_6, f_{10}, f_{12}, w, k_i (i = 1, 2, \ldots, 9), n_j (j = 1, 2, \ldots, 11)$ be positive real numbers and $s_1, s_2, r_2, r_5, f_1, f_2, f_3, f_5, f_7, f_8, f_9, f_{11}, q_k, m_k (k = 1, 2, \ldots, 10)$ be real numbers. The Lyapunov-Krasovskii functional is defined by (2.12)-(2.13)

$$V(t) = \sum_{i=1}^{7} V_i(t), \tag{3.11}$$

where

$$\begin{aligned}
V_1(t) &= k_1 x^2(t), \\
V_2(t) &= k_2 \tau_2 \int_{-\tau_2}^{0} \int_{t+s}^{t} e^{2\alpha(\theta - t)} x^2(\theta) d\theta ds + k_3 \sigma_2 \int_{-\sigma_2}^{0} \int_{t+s}^{t} e^{2\alpha(\theta - t)} x^2(\theta) d\theta ds \\
&\quad + n_1(\tau_2 - \tau_1) \int_{-\tau_2}^{-\tau_1} \int_{t+s}^{t} e^{2\alpha(\theta - t)} x^2(\theta) d\theta ds \\
&\quad + n_2(\sigma_2 - \sigma_1) \int_{-\sigma_2}^{-\sigma_1} \int_{t+s}^{t} e^{2\alpha(\theta - t)} x^2(\theta) d\theta ds,
\end{aligned}$$

$$
\begin{aligned}
V_3(t) &= n_3\tau_1 \int_{-\tau_1}^{0}\int_{t+s}^{t} e^{2\alpha(\theta-t)} y^2(\theta)d\theta ds + k_4\tau_2 \int_{-\tau_2}^{0}\int_{t+s}^{t} e^{2\alpha(\theta-t)} y^2(\theta)d\theta ds \\
&\quad + n_4\sigma_1 \int_{-\sigma_1}^{0}\int_{t+s}^{t} e^{2\alpha(\theta-t)} y^2(\theta)d\theta ds + k_5\sigma_2 \int_{-\sigma_2}^{0}\int_{t+s}^{t} e^{2\alpha(\theta-t)} y^2(\theta)d\theta ds \\
&\quad + n_5(\tau_2-\tau_1) \int_{-\tau_2}^{-\tau_1}\int_{t+s}^{t} e^{2\alpha(\theta-t)} y^2(\theta)d\theta ds \\
&\quad + n_6(\sigma_2-\sigma_1) \int_{-\sigma_2}^{-\sigma_1}\int_{t+s}^{t} e^{2\alpha(\theta-t)} y^2(\theta)d\theta ds, \\
V_4(t) &= \tau_1 \int_{-\tau_1}^{0}\int_{t+s}^{t} e^{2\alpha(\theta-t)} \begin{bmatrix} x(\theta) \\ y(\theta) \end{bmatrix}^T \begin{bmatrix} f_1 & f_2 \\ * & f_3 \end{bmatrix} \begin{bmatrix} x(\theta) \\ y(\theta) \end{bmatrix} d\theta ds \\
&\quad + \tau_2 \int_{-\tau_2}^{0}\int_{t+s}^{t} e^{2\alpha(\theta-t)} \begin{bmatrix} x(\theta) \\ y(\theta) \end{bmatrix}^T \begin{bmatrix} r_1 & r_2 \\ * & r_3 \end{bmatrix} \begin{bmatrix} x(\theta) \\ y(\theta) \end{bmatrix} d\theta ds \\
&\quad + (\tau_2-\tau_1) \int_{-\tau_2}^{-\tau_1}\int_{t+s}^{t} e^{2\alpha(\theta-t)} \begin{bmatrix} x(\theta) \\ y(\theta) \end{bmatrix}^T \begin{bmatrix} f_4 & f_5 \\ * & f_6 \end{bmatrix} \begin{bmatrix} x(\theta) \\ y(\theta) \end{bmatrix} d\theta ds \\
&\quad + \sigma_1 \int_{-\sigma_1}^{0}\int_{t+s}^{t} e^{2\alpha(\theta-t)} \begin{bmatrix} x(\theta) \\ y(\theta) \end{bmatrix}^T \begin{bmatrix} f_7 & f_8 \\ * & f_9 \end{bmatrix} \begin{bmatrix} x(\theta) \\ y(\theta) \end{bmatrix} d\theta ds \\
&\quad + \sigma_2 \int_{-\sigma_2}^{0}\int_{t+s}^{t} e^{2\alpha(\theta-t)} \begin{bmatrix} x(\theta) \\ y(\theta) \end{bmatrix}^T \begin{bmatrix} r_4 & r_5 \\ * & r_6 \end{bmatrix} \begin{bmatrix} x(\theta) \\ y(\theta) \end{bmatrix} d\theta ds \\
&\quad + (\sigma_2-\sigma_1) \int_{-\sigma_2}^{-\sigma_1}\int_{t+s}^{t} e^{2\alpha(\theta-t)} \begin{bmatrix} x(\theta) \\ y(\theta) \end{bmatrix}^T \begin{bmatrix} f_{10} & f_{11} \\ * & f_{12} \end{bmatrix} \begin{bmatrix} x(\theta) \\ y(\theta) \end{bmatrix} d\theta ds, \\
V_5(t) &= n_7\sigma_1 \int_{-\sigma_1}^{0}\int_{t+s}^{t} e^{2\alpha(\theta-t)} \tanh^2 x(\theta)d\theta ds \\
&\quad + k_6\sigma_2 \int_{-\sigma_2}^{0}\int_{t+s}^{t} e^{2\alpha(\theta-t)} \tanh^2 x(\theta)d\theta ds \\
&\quad + n_8(\sigma_2-\sigma_1) \int_{-\sigma_2}^{-\sigma_1}\int_{t+s}^{t} e^{2\alpha(\theta-t)} \tanh^2 x(\theta)d\theta ds, \\
V_6(t) &= n_9 \int_{t-\tau_1}^{t} e^{2\alpha(s-t)} x^2(s)ds + k_7 \int_{t-\tau_2}^{t} e^{2\alpha(s-t)} x^2(s)ds \\
&\quad n_{10} \int_{t-\sigma_1}^{t} e^{2\alpha(s-t)} x^2(s)ds + k_8 \int_{t-\sigma_2}^{t} e^{2\alpha(s-t)} x^2(s)ds \\
&\quad + k_9 \int_{t-\tau(t)}^{t} e^{2\alpha(s-t)} y^2(s)ds, \\
V_7(t) &= n_{11}(\rho_2-\rho_1) \int_{-\rho_2}^{-\rho_1}\int_{t+s}^{t} e^{2\alpha(\theta-t)} x^2(\theta)d\theta ds.
\end{aligned}
$$

Computing the differential of $V(t)$ on the solution of (2.12)-(2.13)

$$\dot{V}(t) = \sum_{i=1}^{7} \dot{V}_i(t). \tag{3.12}$$

The differential of $V_1(t)$ is computed

$$\begin{aligned}\dot{V}_1(t) &= 2k_1x(t)\dot{x}(t)\\ &= 2k_1x(t)\Big[y(t)+\epsilon_1x(t)-\epsilon_1x(t-\tau(t))-\epsilon_1\int_{t-\tau(t)}^{t}y(s)ds+\epsilon_2x(t)\\ &\quad -\epsilon_2x(t-\sigma(t))-\epsilon_2\int_{t-\sigma(t)}^{t}y(s)ds\Big]+2q_1y(t)\Big[\epsilon_1x(t)-\epsilon_1x(t-\tau(t))\\ &\quad -\epsilon_1\int_{t-\tau(t)}^{t}y(s)ds+\epsilon_2x(t)-\epsilon_2x(t-\sigma(t))-\epsilon_2\int_{t-\sigma(t)}^{t}y(s)ds\Big]\\ &\quad +2\Big[q_2x(t)+q_3y(t)+q_4x(t-\tau(t))+q_5\int_{t-\tau(t)}^{t}y(s)ds+q_6x(t-\sigma(t))\\ &\quad +q_7\int_{t-\sigma(t)}^{t}y(s)ds+q_8\tanh x(t-\sigma(t))+q_9y(t-\tau(t))\\ &\quad +q_{10}\int_{t-\rho(t)}^{t}x(s)ds\Big]M_1(t)+2\alpha k_1x^2(t)-2\alpha V_1(t),\end{aligned}$$

where $M_1(t)=\Big[-y(t)-a_1x(t)-a_2x(t-\tau(t))-a_2\int_{t-\tau(t)}^{t}y(s)ds-a_3x(t-\sigma(t))$ $-a_3\int_{t-\sigma(t)}^{t}y(s)ds+b\tanh x(t-\sigma(t))+c\int_{t-\rho(t)}^{t}x(s)ds-py(t-\tau(t))+du(t)\Big]$.
Since $s\in[t-\tau_2,t]$ and $s\in[t-\sigma_2,t]$, we have $e^{-2\alpha\tau_2}\le e^{2\alpha(s-t)}\le 1$ and $e^{-2\alpha\sigma_2}\le e^{2\alpha(s-t)}\le 1$. Computing the differential of $V_2(t)$ and Lemma 2.2, we obtain

$$\begin{aligned}\dot{V}_2(t) &\le k_2(\tau_2)^2x^2(t)+k_3(\sigma_2)^2x^2(t)+n_1(\tau_2-\tau_1)^2x^2(t)+n_2(\sigma_2-\sigma_1)^2x^2(t)\\ &\quad -k_2e^{-2\alpha\tau_2}\Big(\int_{t-\tau(t)}^{t}x(s)ds\Big)^2-k_2e^{-2\alpha\tau_2}\Big(\int_{t-\tau_2}^{t-\tau(t)}x(s)ds\Big)^2\\ &\quad -k_3e^{-2\alpha\sigma_2}\Big(\int_{t-\sigma(t)}^{t}x(s)ds\Big)^2-k_3e^{-2\alpha\sigma_2}\Big(\int_{t-\sigma_2}^{t-\sigma(t)}x(s)ds\Big)^2\\ &\quad -n_1e^{-2\alpha\tau_2}\Big(\int_{t-\tau(t)}^{t-\tau_1}x(s)ds\Big)^2-n_1e^{-2\alpha\tau_2}\Big(\int_{t-\tau_2}^{t-\tau(t)}x(s)ds\Big)^2\\ &\quad -n_2e^{-2\alpha\sigma_2}\Big(\int_{t-\sigma(t)}^{t-\sigma_1}x(s)ds\Big)^2-n_2e^{-2\alpha\sigma_2}\Big(\int_{t-\sigma_2}^{t-\sigma(t)}x(s)ds\Big)^2\\ &\quad -2\alpha V_2(t).\end{aligned}$$

We use Lemma 2.2, 2.4 and 2.5 to compute the differential of $V_3(t)$

$$\begin{aligned}\dot{V}_3(t) &\le n_3(\tau_1)^2y^2(t)+k_4(\tau_2)^2y^2(t)+n_4(\sigma_1)^2y^2(t)+k_5(\sigma_2)^2y^2(t)\\ &\quad +n_5(\tau_2-\tau_1)^2y^2(t)+n_6(\sigma_2-\sigma_1)^2y^2(t)\\ &\quad -n_3e^{-2\alpha\tau_1}\Big(\int_{t-\tau_1}^{t}y(s)ds\Big)^2-n_4e^{-2\alpha\sigma_1}\Big(\int_{t-\sigma_1}^{t}y(s)ds\Big)^2\end{aligned}$$

$$
\begin{aligned}
&+M_2^T(t)\begin{bmatrix}-k_4e^{-2\alpha\tau_2} & k_4e^{-2\alpha\tau_2}-s_1 & s_1\\ * & -2k_4e^{-2\alpha\tau_2}+2s_1 & k_4e^{-2\alpha\tau_2}-s_1\\ * & * & -k_4e^{-2\alpha\tau_2}\end{bmatrix}M_2(t)\\
&+M_3^T(t)\begin{bmatrix}-k_5e^{-2\alpha\sigma_2} & k_5e^{-2\alpha\sigma_2}-s_2 & s_2\\ * & -2k_5e^{-2\alpha\sigma_2}+2s_2 & k_5e^{-2\alpha\sigma_2}-s_2\\ * & * & -k_5e^{-2\alpha\sigma_2}\end{bmatrix}M_3(t)\\
&+\begin{bmatrix}x(t-\tau_1))\\ x(t-\tau(t))\\ x(t-\tau_2)\end{bmatrix}^T\begin{bmatrix}2m_1 & -m_1+m_2 & 0\\ * & 2m_1-2m_2 & -m_1+m_2\\ * & * & -2m_2\end{bmatrix}\begin{bmatrix}x(t-\tau_1))\\ x(t-\tau(t))\\ x(t-\tau_2)\end{bmatrix}\\
&+[\tau_2-\tau_1]\begin{bmatrix}x(t-\tau_1))\\ x(t-\tau(t))\\ x(t-\tau_2)\end{bmatrix}^T\begin{bmatrix}m_3 & m_4 & 0\\ * & m_3+m_5 & m_4\\ * & * & m_5\end{bmatrix}\begin{bmatrix}x(t-\tau_1))\\ x(t-\tau(t))\\ x(t-\tau_2)\end{bmatrix}\\
&+\begin{bmatrix}x(t-\sigma_1)\\ x(t-\sigma(t))\\ x(t-\sigma_2)\end{bmatrix}^T\begin{bmatrix}2m_6 & -m_6+m_7 & 0\\ * & 2m_6-2m_7 & -m_6+m_7\\ * & * & -2m_7\end{bmatrix}\begin{bmatrix}x(t-\sigma_1)\\ x(t-\sigma(t))\\ x(t-\sigma_2)\end{bmatrix}\\
&+[\sigma_2-\sigma_1]\begin{bmatrix}x(t-\sigma_1)\\ x(t-\sigma(t))\\ x(t-\sigma_2)\end{bmatrix}^T\begin{bmatrix}m_8 & m_9 & 0\\ * & m_8+m_{10} & m_9\\ * & * & m_{10}\end{bmatrix}\begin{bmatrix}x(t-\sigma_1)\\ x(t-\sigma(t))\\ x(t-\sigma_2)\end{bmatrix}\\
&-2\alpha V_3(t),
\end{aligned}
$$

where $M_2(t)=\begin{bmatrix}x(t)\\ x(t-\tau(t))\\ x(t-\tau_2)\end{bmatrix}, M_3(t)=\begin{bmatrix}x(t)\\ x(t-\sigma(t))\\ x(t-\sigma_2)\end{bmatrix}$.

We compute the differential of $V_4(t)$ with Lemma 2.2 and 2.3

$$
\begin{aligned}
\dot{V}_4(t,x_t) \le\ & +e^{-2\alpha\tau_2}M_4^T(t)\begin{bmatrix}-r_3 & r_3 & 0 & -r_2 & 0\\ * & -r_3-r_3 & r_3 & r_2 & -r_2\\ * & * & -r_3 & 0 & r_2\\ * & * & * & -r_1 & 0\\ * & * & * & * & -r_1\end{bmatrix}M_4(t)\\
& +e^{-2\alpha\tau_2}M_5^T(t)\begin{bmatrix}-f_6 & f_6 & 0 & -f_5 & 0\\ * & -f_6-f_6 & f_6 & f_5 & -f_5\\ * & * & -f_6 & 0 & f_5\\ * & * & * & -f_4 & 0\\ * & * & * & * & -f_4\end{bmatrix}M_5(t)\\
& \tau_1^2\begin{bmatrix}x(t)\\ y(t)\end{bmatrix}^T\begin{bmatrix}f_1 & f_2\\ * & f_3\end{bmatrix}\begin{bmatrix}x(t)\\ y(t)\end{bmatrix}ds+\tau_2^2\begin{bmatrix}x(t)\\ y(t)\end{bmatrix}^T\begin{bmatrix}r_1 & r_2\\ * & r_3\end{bmatrix}\begin{bmatrix}x(t)\\ y(t)\end{bmatrix}ds\\
& -e^{-2\alpha\tau_1}\Big(\int_{t-\tau_1}^{t}\begin{bmatrix}x(s)\\ y(s)\end{bmatrix}ds\Big)^T\begin{bmatrix}f_1 & f_2\\ * & f_3\end{bmatrix}\Big(\int_{t-\tau_1}^{t}\begin{bmatrix}x(s)\\ y(s)\end{bmatrix}ds\Big)\\
& +(\tau_2-\tau_1)^2\begin{bmatrix}x(t)\\ y(t)\end{bmatrix}^T\begin{bmatrix}f_4 & f_5\\ * & f_6\end{bmatrix}\begin{bmatrix}x(t)\\ y(t)\end{bmatrix}ds
\end{aligned}
$$

$$+e^{-2\alpha\sigma_2}M_6^T(t)\begin{bmatrix}-r_6 & r_6 & 0 & -r_5 & 0\\ * & -r_6-r_6 & r_6 & r_5 & -r_5\\ * & * & -r_6 & 0 & r_5\\ * & * & * & -r_4 & 0\\ * & * & * & * & -r_4\end{bmatrix}M_6(t)$$
$$+e^{-2\alpha\sigma_2}M_7^T(t)\begin{bmatrix}-f_{12} & f_{12} & 0 & -f_{11} & 0\\ * & -f_{12}-f_{12} & f_{12} & f_{11} & -f_{11}\\ * & * & -f_{12} & 0 & f_{11}\\ * & * & * & -f_{10} & 0\\ * & * & * & * & -f_{10}\end{bmatrix}M_7(t)$$
$$+\sigma_1^2\begin{bmatrix}x(t)\\ y(t)\end{bmatrix}^T\begin{bmatrix}f_7 & f_8\\ * & f_9\end{bmatrix}\begin{bmatrix}x(t)\\ y(t)\end{bmatrix}ds+\sigma_2^2\begin{bmatrix}x(t)\\ y(t)\end{bmatrix}^T\begin{bmatrix}r_4 & r_5\\ * & r_6\end{bmatrix}\begin{bmatrix}x(t)\\ y(t)\end{bmatrix}ds$$
$$-e^{-2\alpha\sigma_1}\Big(\int_{t-\sigma_1}^{t}\begin{bmatrix}x(s)\\ y(s)\end{bmatrix}ds\Big)^T\begin{bmatrix}f_7 & f_8\\ * & f_9\end{bmatrix}\Big(\int_{t-\sigma_1}^{t}\begin{bmatrix}x(s)\\ y(s)\end{bmatrix}ds\Big)$$
$$+(\sigma_2-\sigma_1)^2\begin{bmatrix}x(t)\\ y(t)\end{bmatrix}^T\begin{bmatrix}f_{10} & f_{11}\\ * & f_{12}\end{bmatrix}\begin{bmatrix}x(t)\\ y(t)\end{bmatrix}ds-2\alpha V_4(t),$$

where

$$M_4(t)=\begin{bmatrix}x(t)\\ x(t-\tau(t))\\ x(t-\tau_2)\\ \int_{t-\tau(t)}^{t}x(s)ds\\ \int_{t-\tau_2}^{t-\tau(t)}x(s)ds\end{bmatrix},\ M_5(t)=\begin{bmatrix}x(t-\tau_1)\\ x(t-\tau(t))\\ x(t-\tau_2)\\ \int_{t-\tau(t)}^{t-\tau_1}x(s)ds\\ \int_{t-\tau_2}^{t-\tau(t)}x(s)ds\end{bmatrix},$$
$$M_6(t)=\begin{bmatrix}x(t)\\ x(t-\sigma(t))\\ x(t-\sigma_2)\\ \int_{t-\sigma(t)}^{t}x(s)ds\\ \int_{t-\sigma_2}^{t-\sigma(t)}x(s)ds\end{bmatrix},\ M_7(t)=\begin{bmatrix}x(t-\sigma_1)\\ x(t-\sigma(t))\\ x(t-\sigma_2)\\ \int_{t-\sigma(t)}^{t-\sigma_1}x(s)ds\\ \int_{t-\sigma_2}^{t-\sigma(t)}x(s)ds\end{bmatrix}.$$

By $\tanh^2 x(t)\le x^2(t)$ and Lemma 2.2, we get the differential of $V_5(t)$

$$\begin{aligned}\dot V_5(t) \le\ & n_7\sigma_1^2x^2(t)-n_7e^{-2\alpha\sigma_1}\Big(\int_{t-\sigma_1}^{t}\tanh x(s)ds\Big)^2\\ &+k_6\sigma_2^2x^2(t)-k_6e^{-2\alpha\sigma_2}\Big(\int_{t-\sigma_2}^{t}\tanh x(s)ds\Big)^2\\ &+n_8(\sigma_2-\sigma_1)^2x^2(t)-n_8e^{-2\alpha\sigma_2}\Big(\int_{t-\sigma_2}^{t-\sigma_1}\tanh x(s)ds\Big)^2-2\alpha V_5(t).\end{aligned}$$

The differential of $V_6(t)$ and $V_7(t)$ are computed

$$\begin{aligned}\dot V_6(t) =\ & (n_9+k_7+n_{10}+k_8)x^2(t)-n_9e^{-2\alpha\tau_1}x^2(t-\tau_1)\\ &-k_7e^{-2\alpha\tau_2}x^2(t-\tau_2)-n_{10}e^{-2\alpha\sigma_1}x^2(t-\sigma_1)\\ &-k_8e^{-2\alpha\sigma_2}x^2(t-\sigma_2)+k_9y^2(t)\\ &-k_9e^{-2\alpha\tau_2}y^2(t-\tau(t))+k_9\tau_dy^2(t-\tau(t))-2\alpha V_6(t),\end{aligned}$$

and

$$\dot{V}_7(t) \leq n_{11}(\rho_2-\rho_1)^2x^2(t) - n_{11}e^{-2\alpha\rho_2}\Big(\int_{t-\rho(t)}^{t} x(s)ds\Big)^2 \\ -2\alpha V_7(t).$$

By the utilization of zero equations, for real constants $z_m(m=1,2,...,6)$

$$2\Big[z_1x(t)+z_2x(t-\tau(t))+z_3\int_{t-\tau(t)}^{t} y(s)ds\Big] \\ \times\Big[x(t)-x(t-\tau(t)))-\int_{t-\tau(t)}^{t} y(s)ds\Big]=0, \tag{3.13}$$

$$2\Big[z_4x(t)+z_5x(t-\sigma(t))+z_6\int_{t-\sigma(t)}^{t} y(s)ds\Big] \\ \times\Big[x(t)-x(t-\sigma(t)))-\int_{t-\sigma(t)}^{t} y(s)ds\Big]=0. \tag{3.14}$$

Since $\tanh^2 x(s) \leq x^2(s)$ for all $s\in\mathbb{R}$, we get

$$0 \leq wx^2(t-\sigma(t)) - w\tanh^2 x(t-\sigma(t)), \quad w\geq 0. \tag{3.15}$$

We can conclude the following inequality by (3.12)-(3.15) and (2.2)

$$\dot{V}(t)+2\alpha V(t)-2z(t)u(t) \leq \xi^T(t)\sum\xi(t), \tag{3.16}$$

for

$$\xi(t) = col\{x(t),\ y(t),\ x(t-\tau(t)),\ \int_{t-\tau(t)}^{t} y(s)ds,\ x(t-\sigma(t)),\ \int_{t-\sigma(t)}^{t} y(s)ds,$$
$$\tanh x(t-\sigma(t)),\ \int_{t-\rho(t)}^{t} x(s)ds,\ y(t-\tau(t)),\ \int_{t-\tau(t)}^{t} x(s)ds,\ \int_{t-\tau_2}^{t-\tau(t)} x(s)ds,$$
$$\int_{t-\sigma(t)}^{t} x(s)ds,\ \int_{t-\sigma_2}^{t-\sigma(t)} x(s)ds,\ \int_{t-\tau(t)}^{t-\tau_1} x(s)ds,\ \int_{t-\sigma(t)}^{t-\sigma_1} x(s)ds,\ x(t-\tau_1),$$
$$x(t-\tau_2),\ x(t-\sigma_1),\ x(t-\sigma_2),\ \int_{t-\tau_1}^{t} y(s)ds,\ \int_{t-\sigma_1}^{t} y(s)ds,\ \int_{t-\tau_1}^{t} x(s)ds,$$
$$\int_{t-\sigma_1}^{t} x(s)ds,\ \int_{t-\sigma_1}^{t} \tanh x(s)ds,\ \int_{t-\sigma_2}^{t} \tanh x(s)ds,\ \int_{t-\sigma_2}^{t-\sigma_1} \tanh x(s)ds,\ u(t)\}.$$

Since $\sum$ is negative definite, then

$$\dot{V}(t)+2\alpha V(t) \leq 2z(t)u(t), \quad t\geq 0. \tag{3.17}$$

Therefore, the equations (2.1)-(2.2) are guaranteed to be exponentially passive from Definition 2.1. The proof is completed. ∎

Now the equation (2.1) when $d=0$ is demonstrated. We define a new parameter

$$\hat{\sum} = [\hat{\Delta}_{(i,j)}]_{26\times 26}, \tag{3.18}$$

where $\hat{\Delta}_{(i,j)} = \Delta_{(i,j)}$.

Corollary 3.2. *Let $r_1, r_3, r_4, r_6, f_4, f_6, f_{10}, f_{12}, w, k_i (i = 1, 2, \ldots, 9), n_j (j = 1, 2, \ldots, 11)$ be positive real numbers and $s_1, s_2, r_2, r_5, f_1, f_2, f_3, f_5, f_7, f_8, f_9, f_{11}, q_k, m_k (k = 1, 2, \ldots, 10)$ be real numbers satisfying the following symmetric linear matrix inequalities*

$$\hat{\sum} < 0, \tag{3.19}$$

$$\begin{bmatrix} k_4 e^{-2\alpha\tau_2} & s_1 \\ * & k_4 e^{-2\alpha\tau_2} \end{bmatrix} \geq 0, \tag{3.20}$$

$$\begin{bmatrix} k_5 e^{-2\alpha\sigma_2} & s_2 \\ * & k_5 e^{-2\alpha\sigma_2} \end{bmatrix} \geq 0, \tag{3.21}$$

$$\begin{bmatrix} n_5(\tau_2 - \tau_1) e^{-2\alpha\tau_2} & m_1 & m_2 \\ * & m_3 & m_4 \\ * & * & m_5 \end{bmatrix} \geq 0, \tag{3.22}$$

$$\begin{bmatrix} n_6(\sigma_2 - \sigma_1) e^{-2\alpha\sigma_2} & m_6 & m_7 \\ * & m_8 & m_9 \\ * & * & m_{10} \end{bmatrix} \geq 0, \tag{3.23}$$

$$\begin{bmatrix} r_1 & r_2 \\ * & r_3 \end{bmatrix} \geq 0, \tag{3.24}$$

$$\begin{bmatrix} r_4 & r_5 \\ * & r_6 \end{bmatrix} \geq 0, \tag{3.25}$$

$$\begin{bmatrix} f_4 & f_5 \\ * & f_6 \end{bmatrix} \geq 0, \tag{3.26}$$

$$\begin{bmatrix} f_{10} & f_{11} \\ * & f_{12} \end{bmatrix} \geq 0. \tag{3.27}$$

Then the equation (2.1) when $d = 0$ is exponential stability.

Now the equation (2.1) when $\tau_1 = \sigma_1 = c = d = 0$ is presented. We define a new parameter

$$\tilde{\sum} = \left[\tilde{\Delta}_{(i,j)}\right]_{15\times 15}, \tag{3.28}$$

where $\tilde{\Delta}(i,j) = \Delta(i,j)$, except

$\tilde{\Delta}(1,1) = 2\alpha k_1 + k_7 + k_8 + 2k_1\epsilon_1 + 2k_1\epsilon_2 - 2q_2 a_1 + k_2\tau_2^2 + k_3\sigma_2^2 - k_4 e^{-2\alpha\tau_2}$ $-k_5 e^{-2\alpha\sigma_2} + r_1\tau_2^2 - r_3 e^{-2\alpha\tau_2} + r_4\sigma_2^2 - r_6 e^{-2\alpha\sigma_2} + k_6\sigma_2^2 + 2z_1 + 2z_4$, $\tilde{\Delta}(1,2) = q_1\epsilon_1 + q_1\epsilon_2 + r_2\tau_2^2 + r_5\sigma_2^2 + k_1 - q_2 - q_3 a_1$, $\tilde{\Delta}(1,8) = -q_2 p$ $-q_9 a_1$, $\tilde{\Delta}(1,9) = -r_2 e^{-2\alpha\tau_2}$, $\tilde{\Delta}(1,10) = 0$, $\tilde{\Delta}(1,11) = -r_5 e^{-2\alpha\sigma_2}$, $\tilde{\Delta}(1,12) = 0$, $\tilde{\Delta}(1,13) = s_1$, $\tilde{\Delta}(1,14) = s_2$, $\tilde{\Delta}(2,2) = k_9 - 2q_3 + k_4\tau_2^2$ $+k_5\sigma_2^2 + r_3\tau_2^2 + r_6\sigma_2^2$, $\tilde{\Delta}(2,8) = -q_3 p - q_9$, $\tilde{\Delta}(2,9) = 0$, $\tilde{\Delta}(3,3) =$ $-2q_4 a_2 - 2k_4 e^{-2\alpha\tau_2} + 2s_1 - 2r_3 e^{-2\alpha\tau_2} - 2z_2$, $\tilde{\Delta}(3,8) = -q_4 p - q_9 a_2$, $\tilde{\Delta}(3,9) = r_2 e^{-2\alpha\tau_2}$, $\tilde{\Delta}(3,10) = -r_2 e^{-2\alpha\tau_2}$, $\tilde{\Delta}(3,11) = 0$, $\tilde{\Delta}(3,13) =$ $k_4 e^{-2\alpha\tau_2} - s_1 + r_3 e^{-2\alpha\tau_2}$, $\tilde{\Delta}(3,14) = 0$, $\tilde{\Delta}(4,8) = -q_5 p - q_9 a_2$, $\tilde{\Delta}(4,9)$

$= 0, \quad \tilde{\Delta}(5,5) = w - 2q_6a_3 - 2k_5e^{-2\alpha\sigma_2} + 2s_2 - 2r_6e^{-2\alpha\sigma_2} - 2z_5, \quad \tilde{\Delta}(5,8) = -q_6p - q_9a_3, \quad \tilde{\Delta}(5,9) = 0, \quad \tilde{\Delta}(5,11) = r_5e^{-2\alpha\sigma_2}, \quad \tilde{\Delta}(5,12) = -r_5e^{-2\alpha\sigma_2}, \quad \tilde{\Delta}(5,13) = 0, \quad \tilde{\Delta}(5,14) = k_5e^{-2\alpha\sigma_2} - s_2 + r_6e^{-2\alpha\sigma_2}, \quad \tilde{\Delta}(5,15) = 0, \quad \tilde{\Delta}(6,8) = -q_7p - q_9a_3, \quad \tilde{\Delta}(6,9) = 0, \quad \tilde{\Delta}(7,8) = -q_8p + q_9b, \quad \tilde{\Delta}(7,9) = 0, \quad \tilde{\Delta}(8,8) = -k_9e^{-2\alpha\tau_2} + k_9d - 2q_9p, \quad \tilde{\Delta}(8,9) = 0, \quad \tilde{\Delta}(9,9) = -k_2e^{-2\alpha\tau_2} - r_1e^{-2\alpha\tau_2}, \quad \tilde{\Delta}(10,10) = -k_2e^{-2\alpha\tau_2} - r_1e^{-2\alpha\tau_2}, \quad \tilde{\Delta}(10,13) = r_2e^{-2\alpha\tau_2}, \quad \tilde{\Delta}(11,11) = -k_3e^{-2\alpha\sigma_2} - r_4e^{-2\alpha\sigma_2}, \quad \tilde{\Delta}(12,12) = -k_3e^{-2\alpha\sigma_2} - r_4e^{-2\alpha\sigma_2}, \quad \tilde{\Delta}(12,14) = r_5e^{-2\alpha\sigma_2}, \tilde{\Delta}(13,13) = -k_7e^{-2\alpha\tau_2} - k_4e^{-2\alpha\tau_2} - r_3e^{-2\alpha\tau_2}, \quad \tilde{\Delta}(14,14) = -k_8e^{-2\alpha\sigma_2} - k_5e^{-2\alpha\sigma_2} - r_6e^{-2\alpha\sigma_2}, \quad \tilde{\Delta}(15,15) = -k_6e^{-2\alpha\sigma_2}.$

Corollary 3.3. *Let $r_1, r_3, r_4, r_6, w, k_i (i = 1, 2, \dots, 9)$ be positive real numbers and $r_2, r_5, s_1, s_2, q_j (j = 1, 2, \dots, 9)$ be real numbers satisfying the following symmetric linear matrix inequalities*

$$\tilde{\sum} < 0, \tag{3.29}$$

$$\begin{bmatrix} k_4e^{-2\alpha\tau_2} & s_1 \\ * & k_4e^{-2\alpha\tau_2} \end{bmatrix} \geq 0, \tag{3.30}$$

$$\begin{bmatrix} k_5e^{-2\alpha\sigma_2} & s_2 \\ * & k_5e^{-2\alpha\sigma_2} \end{bmatrix} \geq 0, \tag{3.31}$$

$$\begin{bmatrix} r_1 & r_2 \\ * & r_3 \end{bmatrix} \geq 0, \tag{3.32}$$

$$\begin{bmatrix} r_4 & r_5 \\ * & r_6 \end{bmatrix} \geq 0. \tag{3.33}$$

Then the equation (2.1) when $\tau_1 = \sigma_1 = c = d = 0$ is exponential stability.

4. Numerical examples

Example 4.1. The certain neutral integro-differential equations (4.1)-(4.2) are considered

$$\frac{d}{dt}[x(t) + 0.1x(t - \tau(t))] = -1.5x(t) + b\tanh x(t - \sigma(t)) + 0.5\int_{t-\rho(t)}^{t} x(s)ds + du(t), \tag{4.1}$$

$$z(t) = 0.5x(t) + 0.5\tanh x(t - \sigma(t)) + 0.1u(t), \tag{4.2}$$

for $\sigma(t) = 0.2 + \dfrac{|\sin(t)|}{2}$ and $\rho(t) = 0.2 + \dfrac{|\cos(t)|}{5}$.
Decompose constants a as $a = a_1 + a_2 + a_3$, where $a_i = 0.5$ for all $i \in \{1, 2, 3\}$.
Solving the LMIs (3.2)-(3.10) when $d = 0.1$, $\alpha = 0.4$, and $\tau(t) = 0.1 + \dfrac{\sin^2(t)}{2}$, we have the least upper bound of b which ensures the exponentially passive is 0.6117. Furthermore, Table 1 presents the least upper bound of b that guarantees the exponentially passive of this example with $d = 0.1$, $\tau_1 = 0.1$ and $\tau_2 = 0.6$ for various values of α, τ_d. Also the

least upper bound of b for exponentially passive of Example 4.1 with $\tau_d = 0.5,\ \tau_1 = 0.1$ and $\tau_2 = 0.6$ are represented in Table 2 for various values of $\alpha,\ d$.

TABLE 1. The least upper bounds of b for Example 4.1.

τ_d	$\alpha = 0$	$\alpha = 0.1$	$\alpha = 0.2$	$\alpha = 0.3$	$\alpha = 0.4$
0.3	1.1627	1.0386	0.9155	0.7930	0.6666
0.4	1.1556	1.0310	0.9060	0.7785	0.6498
0.5	1.1463	1.0190	0.8889	0.7559	0.6117

TABLE 2. The least upper bounds of b for Example 4.1.

d	$\alpha = 0$	$\alpha = 0.1$	$\alpha = 0.2$	$\alpha = 0.3$	$\alpha = 0.4$
0.05	1.1518	1.0231	0.8941	0.7608	0.6170
0.1	1.1463	1.0190	0.8889	0.7559	0.6117
0.2	1.1318	1.0031	0.8733	0.7391	0.5907

Example 4.2. The certain neutral integro-differential equation (4.3) is considered

$$\begin{aligned} \frac{d}{dt}[x(t) + 0.1x(t-\tau(t))] &= -1.5x(t) + b\tanh x(t-\sigma(t)) \\ &\quad + c\int_{t-\rho(t)}^{t} x(s)ds. \end{aligned} \tag{4.3}$$

for $\sigma(t) = 0.2 + \dfrac{|\sin(t)|}{2}$ and $\rho(t) = 0.2 + \dfrac{|\cos(t)|}{5}$.
Decompose constants a as $a = a_1 + a_2 + a_3$, where $a_i = 0.5$ for all $i \in \{1, 2, 3\}$.
Solving the LMIs (3.19)-(3.27) when $c = 0.5,\ \alpha = 0.4$ and $\tau(t) = 0.1 + \dfrac{\sin^2(t)}{2}$, we have the least upper bound of b that ensures the asymptotic and exponential stabilities are 2.4310 and 1.5220, respectively. Moreover, the least upper bound of b that ensures the asymptotic and exponential stabilities for this example with $c = 0.5,\ \tau_1 = 0.1$ and $\tau_2 = 0.6$ are presented in Table 3 for various values of $\alpha,\ \tau_d$. Also the least upper bound of b for ensures the asymptotic and exponential stabilities of Example 4.2 with $\tau_d = 0.5,\ \tau_1 = 0.1$ and $\tau_2 = 0.6$ are represented in Table 4 for various values of $\alpha,\ c$.

TABLE 3. The least upper bounds of b for Example 4.2.

τ_d	$\alpha = 0$	$\alpha = 0.1$	$\alpha = 0.2$	$\alpha = 0.3$	$\alpha = 0.4$
0.3	2.4807	2.2920	2.1060	1.9354	1.7611
0.4	2.4640	2.2529	2.0690	1.8760	1.6860
0.5	2.4310	2.2112	2.0002	1.7749	1.5220

Table 4. The least upper bounds of b for Example 4.2.

c	$\alpha = 0$	$\alpha = 0.1$	$\alpha = 0.2$	$\alpha = 0.3$	$\alpha = 0.4$
0.5	2.4310	2.2112	2.0002	1.7749	1.5220
1	2.4243	2.2310	2.0110	1.7719	1.5170
2	2.4256	2.2210	2.0004	1.7786	1.5151

Example 4.3. The certain neutral differential equation (4.4), investigated in [6, 15], is considered

$$\frac{d}{dt}[x(t) + 0.2x(t - \tau(t))] = -0.6x(t) + 0.5 \tanh x(t - \sigma(t)), \tag{4.4}$$

for $\tau(t) = \dfrac{\sin^2(t)}{10}$.
We separate constant a as $a = a_1 + a_2 + a_3$, where $a_i = 0.2$ for all $i \in \{1, 2, 3\}$.
By using the linear matrix inequalities (3.29)-(3.33), the least upper bound of $\sigma(t)$ which ensures exponential stability of this example are presented in the comparison in Table 5 for various values of α.

Table 5. The least upper bounds of time delay $\sigma(t)$ for Example 4.3

Methods	$\alpha = 0.02$	$\alpha = 0.028$
Chen and Meng (2011) [6]	infeasible	infeasible
Keadnarmol and Rojsiraphisal (2014)[15] ($\dot{\sigma}(t) < 0.2$)	0.5234	0.0321
Corollary 3.3 (no $\dot{\sigma}$(t))	1.0883	0.9723

5. Conclusions

In this paper, we propose the non-differentiable delay-interval-dependent exponentially passive conditions for certain neutral integro-differential equation with time-varying delays by using decomposition technique of coefficient constant, descriptor model transformation, new class of augmented Lyapunov-Krasovskii functional, Leibniz-Newton formula, improved integral inequalities, utilization of zero equation and Peng-Park's integral inequality. Then, we represented the delay-dependent exponential stability criteria for certain neutral differential equation with time-varying delays. Finally, three numerical examples represent that the proposed conditions are less conservative than another stability criteria.

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