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AN FUZZY SCALED WEIGHTED VARIANCE S CONTROL CHART FOR SKEWED POPULATIONS

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Abstract A control chart is one of the most important tools in statistical process control (SPC) that leads to improving quality of processes and ensures the required quality levels. The usual assumption for designing a control chart is the data or measurement should have a normal distribution. However, this assumption may not be true for some processes, there are some factors that cause an uncertainty data, such as human, measurement device or environmental conditions. Therefore, the purposes of this paper are to study a new S control chart for monitoring process dispersion of skewed populations, which has a non-normal distribution as Weibull, gamma and lognormal. This control chart, called fuzzy scaled weighted variance S control chart (FSWV-S), that is an improvement of the scaled weighted variance S control chart (SWV-S) proposed Atta et al [17]. Finally, the efficiency of the FSWV S control charts is compared with the average run length (ARL) by Monte Carlo (MC) simulation technique.

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1. Introduction

We use statistical process control (SPC), is an industry standard methodology for measuring and controlling quality during the manufacturing process. The SPC is often referred, to flowcharts, check sheet, histogram, Pareto chart, scatter diagram, control charts and cause-and-effect diagram. Generally, the most important tool in SPC is control chart invented by Dr.Walter Andrew Shewhart in 1924. It is a time-ordered plot of representative sample such as sample obtained from an ongoing process. The main objectives of using control chart are to improve and examine the analysis in order to reduce random variation in the manufacturing process. There are two types of control charts used in the SPC to detect changes in large data, which are variable and attribute control charts. The first type of control chart is used to monitor measurable characteristics on numerical data. The variable control charts and attribute control charts could monitor more than one quality characteristic in the same time; however, the inspection uses a lot

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of expenses and consumes time. There are many causes of problems on the control chart. One of that is the uncertainty of data which can possibly occur from human measurement devices or environmental conditions. Therefore, applying fuzzy theorem to the control chart can reduce this problem.

The fuzzy sets theory suggested by Zadeh [1]. In the literature, different attempts were done to develop control charts consistent with fuzzy logic and fuzzy sets theory. Pongpullponsak et al.[2] compared fuzzy control charts by using nonnormal distribution data, such as Weibull, lognormal and Burrs then Rungsarit [3] developed and compared the fuzzy weighted variance (FWV) control charts using the non-normal distribution data as Weibull, gamma and chi-squared. Sogandi et al. [4] developed fuzzy P control chart to monitor attribute quality characteristics based, on α -level fuzzy midrange. Senturk and Erginel [5], Senturk [6] and Senturk et al. [7] proposed the fuzzy $\tilde{\bar{X}}$ -$\tilde{R}$ and $\tilde{\bar{X}}$ -$\tilde{S}$ control charts with α - cuts, fuzzy regression control charts and studied exponential weighted moving average (EWMA) control chart for univariate data are developed under fuzzy theory respectively. Sorooshian [8] preferred monitoring attribute quality characteristics by consideration of ambiguity and uncertainty data. Kaya et al. [9] proposed fuzzy control charts used in determining the variability of the process. Kaya et al.[10] and Kaya et al.[11], studied fuzzy control chart to analyze the variability in the process. Wang and Raz [12] proposed approaches by the combination of fuzzy set with each linguistic data using rules of fuzzy arithmetic. Gullbay et al. [13] advised α -cut fuzzy control charts for linguistic data that are mainly based on probability and membership function. Gullbay and Kahraman [14] developed fuzzy, c control charts to check the unnatural sample. Roland and Wang [15] proposed fuzzy control chart to create and design a fuzzy SPC evaluation and control method based on the application of fuzzy logic to SPC zone rules.

In this paper, we consider the use of FSWV S control chart to compute the limits of S chart, when underlying distributions are skewed. is used for an effective criteria and calculated by the MC simulation method under mean process shift 0σ, 0.5σ, 1σ, 1.5σ, 2.0σ, 2.5σ and 3σ .

Section 2 review fuzzy numbers and transformation techniques. In section 3, firstly we review SWV-S control chart; then, α - level fuzzy is developed for observations in these charts. Finally, α - level fuzzy midrange is presented for SWV-S control chart. In section 4, the performance and advantage of the proposed fuzzy control char are measured in terms of ARL with respect to traditional control chart via MC simulation. The conclusion and the future research are presented in the final section.

2. Fuzzy numbers and fuzzy transformation techniques

Fortune, vagueness, lack of knowledge naturally cause indecision and ambiguity. Vagueness in humanistic system is exemplified by mathematical way provided by the fuzzy sets [16]. α- cuts approximation is always applied when the formulation of the FSWV S control chart takes place. Moreover, the triangular membership statistics represent the process, samples in fuzzy set approach as illustrated Figure 1.

The representative numbers for additional calculations represent the fuzzy sets and sampled linguistic data whenever the fuzzy data are used. Fuzzy measurement used in descriptive statistic is as follows [7]:

The fuzzy mode, f_{mode} : The fuzzy mode of a fuzzy set F is the value of the base variable where the membership function equals 1. This is stated as:

$f_{mode} = \{x|\mu_F(x) = 1\}, \forall x \in F$

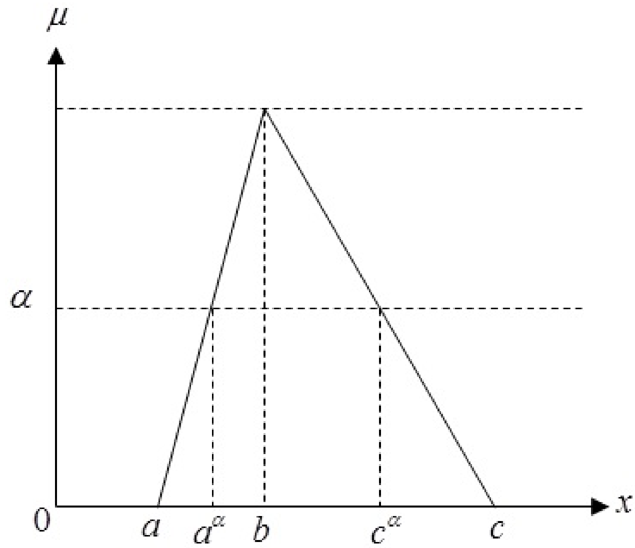

FIGURE 1. Represents of a sample of triangular fuzzy number.

If the membership function is unimodal, it is special. The α- level fuzzy midrange, f^{α}_{mr}: The average of end points of an α- cuts that is denoted by F_α is a non- fuzzy subset of the base variable x (comprising all values with membership function values greater than or equal to α). Therefore, $f_\alpha = \{x|\mu_F(x) \geq x\}$ .Its the end points of α- cuts F_α are a_α and c_α such that $a_\alpha = Min\{F_\alpha\}$ and $c_\alpha = Max\{F_\alpha\}$, then

$f^{\alpha}_{mr} = \frac{1}{2}(a_\alpha + c_\alpha)$

The curve under the membership function of a fuzzy set is screened by the fuzzy median, f_{med} into two equal regions in the equation of

$$\int_a^{f_{med}} \mu_F(x)\,dx = \int_{f_{med}}^{c} \mu_F(x)\,dx = \frac{1}{2}\int_a^{c} \mu_F(x)\,dx \tag{2.1}$$

where a and c are the end points in the base variable of the fuzzy set F such that $a < c$. The fuzzy average, f_{avg} : is

$f_{avg} = Av(x;F) = \frac{\int_0^1 x\mu_F(x)dx}{\int_0^1 \mu_F(x)dx}$

It is indicated that there is no theoretical basis to encourage any particular one or selection among them. Commonly, because of simplicity of practical calculation, the fuzzy mode and α level fuzzy midrange transformation method are mostly applied. Then Fig 1 illustrates the transformation techniques on triangular membership function. In the next section, the proposal of FSWV-S control charts are similarly presented as other fuzzy control charts.

3. Fuzzy Scaled Weighted Variance S Control Charts

Control charts are used for monitoring the SPC, but fuzzy control charts are used when considering the uncertainty data, to provide flexibility on control limits so that it can prevent false alarms. FSWV S is developed from SWV S control charts introduced by Atta et.al.[17]. The SWV S control charts adjust the control limits based on skewness population.

Castagliola [18] suggested a new method, called SWV method to adjust the performance of weighted variance (WV) method presented by Choobineh and Ballard [19]. The function $f_L(x)$ and $f_U(x)$ are not certainly separated by two normal probability density distributions $\phi(x,\mu,\sigma_L)$ and $\phi(x,\mu,\sigma_U)$ but are replaced by bell-shape functions $\phi(x,\mu,\sigma'_L,2P_x)$, and $\phi(x,\mu,\sigma'_U,2(1-P_x))$ centered on μ, having σ'_L and σ'_U for second central moments and to $2P_x$ and $2(1-P_x)$ for areas. Castagliola [19] defined the function $\phi(x,\mu,t,k)$ as

$\phi(x,\mu,t,k) = \frac{k^{3/2}}{t}\varphi(\frac{(x-\mu)\sqrt{k})}{t})$

This function has the following required properties presenting more details about the derivations (Castagliola [19]):

$$\int_{-\infty}^{\infty} \phi(x,\mu,t,k)dx = k \tag{3.1}$$

$$\int_{-\infty}^{\infty} (x-\mu)^2\phi(x,\mu,t,k)dx = t^2 \tag{3.2}$$

Using $\phi(x,\mu,t,k)$ instead of probability density function $\phi(x,\mu,t)$ gives new limits for the WV- S control chart proposed by Khoo et al.[20].

If the process parameters are known, the control charts of proposed SWV- S control chart is computed as follows:

$$\begin{aligned} UCL_{SWV-S} &= \mu_s + \Phi^{-1}(1-\frac{\alpha}{4(1-P_x)})\sqrt{\frac{P_x}{(1-P_x)}}\sigma \\ LCL_{SWV-S} &= \mu_s - \Phi^{-1}(1-\frac{\alpha}{4(P_x)})\sqrt{\frac{(1-P_x)}{(P_x)}}\sigma \end{aligned} \tag{3.3}$$

where, μ_s and σ_s are mean and standard deviation of the S respectively, and α is a Type I error rate (False alarm).

When $P_X = \frac{1}{2}$, the SWV-S control chart reduces to standard S control chart. If the process parameters are unknown, control charts of the proposed SWV-S control chart is computed as follows:

$$\begin{aligned} CL_{SWV-S} &= \overline{S} \\ UCL_{SWV-S} &= \overline{S}(1+\Phi^{-1}(1-\frac{\alpha}{4(1-\hat{P}_X)})\frac{\sqrt{1-(C'_4)^2}}{C'_4}\sqrt{\frac{\hat{P}_x}{(1-\hat{P}_x)}}) \\ LCL_{SWV-S} &= \overline{S}(1-\Phi^{-1}(1-\frac{\alpha}{4(1-\hat{P}_x)})\frac{\sqrt{1-(C'_4)^2}}{C'_4}\sqrt{\frac{(1-\hat{P}_x)}{\hat{P}_x}}) \end{aligned} \tag{3.4}$$

Here, $C_4' = \frac{E(S)}{\sigma_X}$ is a constant for a given skewed population and $\overline{S} = \frac{\sum_{i=1}^{r} S_i}{n}$ is the average of the sample standard deviations estimated from n preliminary subgroups. $\overline{S}$ for each sample plots on the control limits. If some sample plot outside the control limits, it shows the process that should be stopped to check assignable causes. In this study, we consider the number of sample standard deviations as triangular fuzzy numbers that represented by $(\overline{S}_a, \overline{S}_b, \overline{S}_c)$ for each fuzzy sample. CL, UCL and LCL represent the center line, upper control limit and lower control limit of the FSWV S control chart, respectively and they are triangular fuzzy sets; then we determined by the following equations:

$$\widetilde{CL}_{FSWV-S} = (CL_a, CL_b, CL_c) = (\overline{S}_a, \overline{S}_b, \overline{S}_c) \tag{3.5}$$

$$(\overline{S}_a, \overline{S}_b, \overline{S}_c) = (\frac{\sum S_a}{n}, \frac{\sum S_b}{n}, \frac{\sum S_c}{n}) \tag{3.6}$$

$$\begin{aligned} U\widetilde{CL}_{FSWV-S} &= (UCL_a, UCL_b, UCL_c) \\ &= (\overline{S}_a B_{4a}, \overline{S}_b B_{4b}, \overline{S}_c B_{4c}) \end{aligned} \tag{3.7}$$

$$\begin{aligned} L\widetilde{CL}_{FSWV-S} &= (LCL_a, LCL_b, LCL_c) \\ &= (\overline{S}_a B_{3a}, \overline{S}_b B_{3b}, \overline{S}_c B_{3c}) \end{aligned} \tag{3.8}$$

where n is the fuzzy sample size

$B_{4a} = (1 + \Phi^{-1}(1 - \frac{\alpha}{4(1-\hat{P}_{xa})})\frac{\sqrt{1-(C'_{4a})^2}}{C'_{4a}}\sqrt{\frac{\hat{P}_{xa}}{(1-\hat{P}_{xa})}})$

$B_{4b} = (1 + \Phi^{-1}(1 - \frac{\alpha}{4(1-\hat{P}_{xb})})\frac{\sqrt{1-(C'_{4b})^2}}{C'_{4b}}\sqrt{\frac{\hat{P}_{xb}}{(1-\hat{P}_{xb})}})$

$B_{4c} = (1 + \Phi^{-1}(1 - \frac{\alpha}{4(1-\hat{P}_{xc})})\frac{\sqrt{1-(C'_{4c})^2}}{C'_{4c}}\sqrt{\frac{\hat{P}_{xc}}{(1-\hat{P}_{xc})}})$

$B_{3a} = (1 - \Phi^{-1}(1 - \frac{\alpha}{4-\hat{P}_{xa})})\frac{\sqrt{1-(C'_{4a})^2}}{C'_{4a}}\sqrt{\frac{(1-\hat{P}_{xa})}{\hat{P}_{xa}}})$

$B_{3b} = (1 - \Phi^{-1}(1 - \frac{\alpha}{4-\hat{P}_{xb})})\frac{\sqrt{1-(C'_{4b})^2}}{C'_{4b}}\sqrt{\frac{(1-\hat{P}_{xb})}{\hat{P}_{xb}}})$

$B_{3c} = (1 - \Phi^{-1}(1 - \frac{\alpha}{4-\hat{P}_{xc})})\frac{\sqrt{1-(C'_{4c})^2}}{C'_{4c}}\sqrt{\frac{(1-\hat{P}_{xc})}{\hat{P}_{xc}}})$

3.1α - cut FSWV S Control Chart

In this paper, we construct a FSWV S control chart with α - cut theory. The interpretation of these charts is the same as mentioned in the section 3. By applying α - cut on fuzzy sets, the values of center line are determined as follows:

$$\widetilde{CL}_{FSWV-S} = (CL_a^{\alpha}, CL_b, CL_c^{\alpha}) = (\overline{S}_a^{\alpha}, \overline{S}_b, \overline{S}_c^{\alpha}) \tag{3.9}$$

Similarly, α- cut FSWV S control chart based on ranges can be stated as follows:

$$(\overline{S}_a^{\alpha}, \overline{S}_b, \overline{S}_c^{\alpha}) = (\frac{\sum S_a^{\alpha}}{n}, \frac{\sum S_b}{n}, \frac{\sum S_c^{\alpha}}{n}) \tag{3.10}$$

where in $(\overline{S}_a^\alpha)$ and $(\overline{S}_c^\alpha)$ as follows:

$$S_a^\alpha = \overline{S}_a + \alpha(\overline{S}_b - \overline{S}_a) \tag{3.11}$$

$$S_c^\alpha = \overline{S}_c + \alpha(\overline{S}_c - \overline{S}_b) \tag{3.12}$$

3.2 α - Level Fuzzy Midrange for FSWV S Control Chart

As stated α- level fuzzy midrange is one of four transformation techniques used to design the FSWV S control charts. These control limits are used to decide whether the process is in control or out of control. In this study, α-level fuzzy midrange is used as the fuzzy transformation techniques while calculating α -level fuzzy midrange for α-cut FSWV S control chart is:

$$C\widetilde{L}_{mr-FSWV-S}^\alpha = \frac{1}{2}(\overline{S}_a^\alpha + \overline{S}_c^\alpha) \tag{3.13}$$

$$U\widetilde{CL}_{mr-FSWV-S}^\alpha = (C\widetilde{L}_{mr-FSWV-S}^\alpha)\frac{1}{2}(B_{4a} + B_{4c}) \tag{3.14}$$

$$L\widetilde{CL}_{mr-FSWV-S}^\alpha = (C\widetilde{L}_{mr-FSWV-S}^\alpha)\frac{1}{2}(B_{3a} + B_{3c}) \tag{3.15}$$

As mentioned in section 2, a definition of α -level fuzzy midrange for sample j for FSWV S control charts is:

$$S_{mr-FSWV-S,j}^\alpha = \frac{(S_{a_j} + S_{c_j}) + \alpha[(S_{b_j} - S_{a_j}) - [(S_{c_j} - S_{b_j})]}{2} \tag{3.16}$$

Then, the condition of process control for each sample can be defined as

$$Processcontrol = \begin{cases} in - control & \text{for } LCL_{mr-FSWV-S}^\alpha \\ & \quad \le S_{mr-FSWV-S,j}^\alpha \\ & \quad \le UCL_{mr-FSWV-S}^\alpha \\ out - of control & \text{for } otherwise \end{cases} \tag{3.17}$$

4. Results and Discussions

The aim of this research is to compare the efficiency of FSWV S control charts in each distribution are Weibull, lognormal and gamma distributions, in order to show that the proposed approach has better performance for detection shifts. Before reaching the process of comparing the effectiveness of control charts, the construction of the FSWV S control chart should be considered. The researcher generated data from a Weibull distribution as shown in Table 1. The data consist of 750 skewed observations that were grouped into 50 subgroups, n=5 each. The shape parameter and scale were 1 and 1 respectively. As a result, it presented $(\hat{P}_{xa}, \hat{P}_{xb}, \hat{P}_{xc}) = (0.616, 0.620, 0.632), (C'_{4a}, C'_{4b}, C'_{4c}) = (0.013, 0.012, 0.028), \alpha = 0.0027$ and $(\overline{S}_a^\alpha, \overline{S}_b, \overline{S}_c^\alpha) = (0.013, 0.013, 0.049)$; therefore, the FSWV S control chart computed using equations (3.14) and (3.15) are equal to

$$U\widetilde{CL}_{mr-FSWV-S}^\alpha = 6.462$$

and

$$\widetilde{LCL}^{\alpha}_{mr-FSWV-S} = -4.429$$

After obtaining a control chart, then, we evaluate ARL in order to compare the control chart efficiency. ARL is an average of samples numbers, which should occur before a sample that is out of control condition. If the process observations are not auto correlated, the ARL could be calculated for every type of control charts to appraise their ability [21]. Hence, in this paper, the ARL is applied as an evaluation criterion to compare the performance of proposed FSWV S control charts. It is pointed out that there are two different ARLs as follows:

1. In the control state, ARL is shown by ARL_0 and is the number of points that, on average, will be plotted on a control chart before an out of control condition is indicated.

2. An ARL is shown by ARL_1 when the process is out of control. ARL_1 is calculated from $ARL_1 = \frac{1}{1-\beta}$ where β is the probability of a Type II error, that is a probability to have no signal when there is real change.

In this research, $ARL_0 = 300$, the fuzzy sample size (n=5,7,10) , the number of fuzzy samples (m=300) are randomly generated from Weibull, Lognormal and Gamma distributions with parameters $\theta = 1, \beta = 1, \mu = 1, \sigma = 1$ and $\theta = 1, k = 1$, respectively. The, ARL_0 and ARL_1 of FSWV S control charts are calculated by using MATLAB, 7.6.0(R2009a) via Monte Carlo (MC) simulation method, which is calculated repeatedly up to 10,000 times for shift sizes of $0.5\sigma, 1.0\sigma, 2.0\sigma, 2.5\sigma$ and 3.0σ .We use ARL_1 to determine the efficiency of FSWV S control charts. When input data in the process are out of control, we stop and count the number of samples that are in control. We use the ARL_1 to decide which is the most effective. The results of ARL_1 of FSWV S control charts, which have Weibull, lognormal and gamma distributions are shown in Table 2. That can be concluded that the presented FSWV S control chart, has gamma distribution. It has smaller ARL_1 than other distributions under different shift. Hence, FSWV S control chart, data in case of gamma distribution has been significantly improved to detect assignable causes with high performance and detect shifts in the process faster than Weibull and lognormal distributions. On the other hand, it can be concluded that the magnitude shifts increases and the power of control chart is then augmented and when the sample size is increased, the ARL_1 is decreased.

Table 1. An example of illustration using simulated data from the skewed population (Weibull distribution)

Sample	X_a	X_b	X_c	$\overline{X}$	S	$\overline{S}$
S1-1	0.002	0.008	0.014	$\overline{X}_{a1} = 0.006$	$S_{a1} = 0.003$	$\overline{S}_a = 0.012$
S1-2	0.005	0.010	0.021	$\overline{X}_{b1} = 0.011$	$S_{b1} = 0.002$	$\overline{S}_b = 0.013$
S1-3	0.005	0.010	0.022	$\overline{X}_{c1} = 0.018$	$S_{c1} = 0.004$	$\overline{S}_c = 0.035$
S1-4	0.008	0.012	0.023	...	...	...
S1-5	0.008	0.013	0.023	...	...	...
S2-1	0.023	0.027	0.033	$\overline{X}_{a2} = 0.024$	$S_{a2} = 0.001$	
S2-2	0.023	0.028	0.035	$\overline{X}_{b2} = 0.028$	$S_{a2} = 0.001$	
S2-3	0.025	0.029	0.037	$\overline{X}_{c2} = 0.037$	$S_{c2} = 0.003$	
S2-4	0.025	0.029	0.039	...	...	...
S2-5	0.026	0.029	0.040	...	...	...
...	...	...	...	...	...	...
S50-1	4.484	4.940	5.786	$\overline{X}_{a50} = 4.763$	$S_{a50} = 0.158$	
S50-2	4.799	4.953	6.327	$\overline{X}_{b50} = 5.177$	$S_{b50} = 0.226$	
S50-3	4.826	5.207	6.987	$\overline{X}_{c50} = 7.181$	$S_{c50} = 1.193$	
S50-4	4.829	5.342	8.394	...	...	...
S50-5	4.875	5.441	8.410	...	...	...

Table 2. Comparison of ARL_1 of FSWV S Control Chart

Shift	Weibull			Lognormal			Gamma		
	n=5	n=7	n=10	n=5	n=7	n=10	n=5	n=7	n=10
0.5	40.3206	40.3205	1.4501	40.2306	40.2210	0.7521	40.1515	40.1222	0.0514
1.0	38.2321	38.2320	0.7258	38.2214	38.1148	0.4147	38.1447	37.2528	0.0414
1.5	36.2514	36.2513	0.3236	36.2212	36.2103	0.2589	36.1658	35.2650	0.0213
2.0	34.2130	34.2129	0.1369	34.0714	34.0202	0.0896	33.2569	31.2148	0.0207
2.5	33.2525	33.2523	0.1362	33.2222	33.1963	0.0782	32.1420	30.6696	0.0204
3.0	32.2144	32.2140	0.0789	32.1496	32.1236	0.0651	30.0125	28.0589	0.0112

5. Conclusions

Control charts have extensive applications to find shifts in the process and indicate abnormal process condition. One of the most important SPC tools is control chart that monitors quality characteristics. Some causes as human, incomplete data or environmental conditions in quality characteristic leading to exist some level of uncertainty in attribute control chart. In these situations it is better to use fuzzy set theory applied in control charts. Thus, in this paper, we developed an FSWV S control chart to monitor attribute quality characteristic. Results of comparative study using ARL criterion showed that FSWV S control chart with gamma distribution, has a high performance and could detect shifts in the process faster than Weibull and lognormal distributions. For future research, considering trapezoidal fuzzy numbers and applying other control charts such as EWMA or cumulative sum (CUSUM) can be interesting topics for future research.

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