Thai Journal of Mathematics : 293-301
Special Issue : Annual Meeting in Mathematics 2019

http://thaijmath.in.cmu.ac.th
Online ISSN 1686-0209

Common Fixed Point Theorems for Firmly Nonspreading Mappings and Quasi-Nonexpansive Mappings in CAT(0) Spaces

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Abstract : In this paper, we prove the existence of a common fixed point for firmly nonspreading mappings and quasi-nonexpansive mappings in CAT(0) spaces. Using the concept of Ishikawa iterative scheme, we define the sequence $\{x_n\}$ by

$$(A)\begin{cases} x_1 \in E, \\ x_{n+1} = \alpha_n x_n \oplus (1-\alpha_n)S(Tx_n), \end{cases}$$

for all $n \in N$, where E is a nonempty closed and convex subset of a complete CAT(0) space, S and T are mappings defined on E.

We prove that the sequence $\{x_n\}$ Δ - converges to a common fixed point for firmly nonspreading mappings and quasi-nonexpansive mappings in CAT(0) spaces.

This research was supported by Department of Mathematics, Faculty of Science, Chiang Mai University.

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Keywords : firmly nonspreading; quasi-nonexpansive; common fixed point; CAT(0) spaces.
2010 Mathematics Subject Classification : 47H05; 47H09; 47H10; 47J25.

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1 Introduction

Let (X, d) be a metric space and $T : X \to X$ be a mapping. A point $x \in X$ is called *fixed point of* T if $Tx = x$. We denote the set of fixed points of T by $F(T)$, i.e. $F(T) = \{x \in X : Tx = x\}$. A mapping $T : X \to X$ is said to be nonexpansive if $d(Tx, Ty) \leq d(x, y)$ for all $x, y \in X$. The generalization of nonexpansive mappings which we are interested in are quasi-nonexpansive mappings, i.e., $d(Tx, y) \leq d(x, y)$ for all $x \in X$ and for all $y \in F(T)$. In 2011, Lin et al. [1] introduced a generalized nonspreading mapping on CAT(0) spaces, call a generalized hybrid mapping : let E be a nonempty closed and convex subset of a CAT(0) space X, we say that $T : E \to X$ is a generalized hybrid mapping if there exist mappings $a_1 : E \to [0, 1]$, $a_2, a_3 : E \to [0, 1)$ such that

P(1) $d^2(Tx, Ty) \leq a_1(x) d^2(x, y) + a_2(x) d^2(Tx, y) + a_3(x) d^2(x, Ty) + k_1(x) d^2(Tx, x) + k_2(x) d^2(Ty, y)$ for all $x, y \in E$;

P(2) $a_1(x) + a_2(x) + a_3(x) \leq 1$ for all $x, y \in E$;

P(3) $2k_1(x) < 1 - a_2(x)$ and $k_2(x) < 1 - a_3(x)$ for all $x, y \in E$.

The authors also introduced the notion of nonspreading mappings on CAT(0) spaces as follows : let E be a nonempty closed and convex subset of a complete CAT(0) space X. A mapping $T : E \to E$ is said to be a nonspreading mapping if

$$2d^2(Tx, Ty) \leq d^2(Tx, y) + d^2(Ty, x)$$

for all $x, y \in E$. From the definition of a generalized nonspreading mapping, if we set $a_1(x) = k_1(x) = k_2(x) = 0$ and $a_2(x) = a_3(x) = \frac{1}{2}$ for all $x \in E$, then T is a nonspreading mapping. In 2018, Kimura and Kishi [2] introduced the concept of firmly nonspreading mappings on complete CAT(0) spaces : let E be a nonempty closed and convex subset of a complete CAT(0) space X. A mapping $T : E \to E$ is said to be a firmly nonspreading mapping if

$$2d^2(Tx, Ty) \leq d^2(Tx, y) + d^2(Ty, x) - d^2(x, Tx) - d^2(y, Ty)$$

for all $x, y \in E$. We see that firmly nonspreading mapping is a nonspreading mapping.

To study convergence theorems, we are interested in the iteration of a sequence defined by, in 1953, W. Robert Mann [3] as follows: let E be a compact and convex subset of a Banach space X and $T : E \to E$ a continuous mapping. Let $x \in E$ and $\{x_n\}$ be a sequence generated by

$$\begin{cases} x_1 = x \\ x_{n+1} = \alpha_n x_n + (1 - \alpha_n) T x_n, \quad n \geq 1. \end{cases}$$

In 1974, Ishikawa [4] defined a new iteration which is a generalization of Mann iterative scheme by

$$\begin{cases} y_n = \beta_n x_n + (1-\beta_n)Tx_n, \\ x_{n+1} = \alpha_n x_n + (1-\alpha_n)Ty_n, \quad n \geq 1, \end{cases}$$

In this paper, using the concept of Ishikawa iterative scheme, we define the sequence $\{x_n\}$ by

$$(A)\begin{cases} x_1 \in E, \\ x_{n+1} = \alpha_n x_n \oplus (1-\alpha_n)S(Tx_n),\ n \geq 1, \end{cases}$$

where E is a nonempty closed and convex subset of a complete CAT(0) space, S and T are mappings defined on E. Furthermore, we prove that the sequence $\{x_n\}$ Δ - converges to a common fixed point for firmly nonspreading mappings and quasi-nonexpansive mappings in CAT(0) spaces.

2 Preliminaries

In 1976, Lim [5] introduced a concept of convergence in a general metric space which is called Δ-convergence. In 2008, Kirk and Panyanak [6] specialized Lim's concept to CAT(0) spaces and showed that many Banach space results involving weak convergence have precise analogs in this setting. Since then the notion of Δ-convergence has been widely studied and a number of papers have appeared see for instance [7, 8, 9, 10, 11]. For more detail about CAT(0) spaces see [12].

Let X be a complete CAT(0) space, let $\{x_n\}$ be a bounded sequence in X and for $x \in X$ set

$$r\left(x, \{x_n\}\right) = \limsup_{n\to\infty} d\left(x, x_n\right).$$

The *asymptotic radius* $r\left(\{x_n\}\right)$ of $\{x_n\}$ is given by

$$r\left(\{x_n\}\right) = \inf\left\{r\left(x, \{x_n\}\right) : x \in X\right\}$$

and the *asymptotic center* $A\left(\{x_n\}\right)$ of $\{x_n\}$ is the set

$$A\left(\{x_n\}\right) = \left\{x \in X : r\left(x, \{x_n\}\right) = r\left(\{x_n\}\right)\right\}.$$

It is known from Proposition 7 of [13] that in a CAT(0) space, $A(\{x_n\})$ consists of exactly one point.

We now give the definition and collect some basic properties of $\Delta-$ convergence and recall the related concepts which will be used in our work.

Definition 2.1 ([6, 5])**.** A sequence $\{x_n\}$ in a complete CAT(0) space X is said to Δ-converge to $x \in X$ if x is the unique asymptotic center of $\{u_n\}$ for every subsequence $\{u_n\}$ of $\{x_n\}$. In this case, we write $\Delta-\lim_{n\to\infty} x_n = x$ and call x the Δ - limit of $\{x_n\}$.

Lemma 2.2 ([7])**.** *Let X be a CAT(0) space. Then for all $x, y, z \in X$ and $t \in [0,1]$.*
(i) $d\big((1-t)x \oplus ty, z\big) \leq (1-t)d(x,z) + td(y,z)$.
(ii) $d^2\big((1-t)x \oplus ty, z\big) \leq (1-t)d^2(x,z) + td^2(y,z) - t(1-t)d^2(x,y)$.

Lemma 2.3 ([6])**.** *Every bounded sequence in a complete CAT(0) space always has a $\Delta-$convergent subsequence.*

Lemma 2.4 ([14])**.** *If E is a closed and convex subset of a complete CAT(0) space and if $\{x_n\}$ is a bounded sequence in E, then the asymptotic center of $\{x_n\}$ is in E.*

Lemma 2.5 ([7])**.** *Let E be a nonempty closed and convex subset of a CAT(0) space (X,d). Let $\{x_n\}$ be a bounded sequence in X with $A(\{x_n\}) = \{x\}$, $\{u_n\}$ be a subsequence of $\{x_n\}$ with $A(\{u_n\}) = \{u\}$. If $\lim\limits_{n\to\infty} d(x_n, u)$ exists, then $x = u$.*

Lemma 2.6 ([1])**.** *Let X be a CAT(0) space. Let $\{x_n\}$ and $\{y_n\}$ be two bounded sequences in X with $\lim\limits_{n\to\infty} d(y_n, x_n) = 0$. If $\Delta-\lim\limits_{n\to\infty} x_n = x$, then $\Delta-\lim\limits_{n\to\infty} y_n = x$.*

We defined $\omega_w(\{x_n\}) := \cup A(\{u_n\})$ where the union is taken over any subsequence $\{u_n\}$ of $\{x_n\}$. In order to prove our main theorem, the following facts are needed.

Lemma 2.7 ([1])**.** *Let E be a nonempty closed and convex subset of a complete CAT(0) space X and $T : E \to X$ be a generalized hybrid mapping. If $\{x_n\}$ is a bounded sequence in E such that $\lim\limits_{n\to\infty} d(x_n, Tx_n) = 0$ and $\{d(x_n, v)\}$ converges for all $v \in F(T)$, then $\omega_w(\{x_n\}) \subset F(T)$. Furthermore, $\omega_w(\{x_n\})$ consists of exactly one point.*

Remark 2.8 ([1])**.** *The conclusion of Lemma 2.7 is still true if $T : E \to X$ is any one of nonexpansive mappings, firmly nonspreading mapping, nonspreading mapping, TJ-1 mapping, TJ-2 mapping, and hybrid mapping.(For other mapping, one can also refer [1].)*

Let X be a real Banach space and let E be a nonempty closed and convex subset of X. A mapping $T : E \to X$ is demiclosed (at y) if $T(x) = y$ whenever $\{x_n\}$ is a sequence in E, x_n converges weakly to x and Tx_n converges strongly to y.

In 1967, Browder [15] gave the following result called Browder's demiclosedness principle, which states that let E be a nonempty closed and convex subset of a uniformly convex Banach space X and $T : E \to X$ be a nonexpansive mapping, then $I - T$ is demiclosed where I is the identity mapping of X. In 2008, Kirk and Panyanak [6] extend Lim's concept [5] to CAT(0) spaces, they obtain the following result. Let E be a nonempty closed and convex subset of a complete CAT(0) space X and $T : E \to E$ be a nonexpansive mapping. If $\{x_n\}$ is a bounded sequence in E such that $\lim\limits_{n\to\infty} d(Tx_n, x_n) = 0$ and $\Delta-\lim\limits_{n\to\infty} x_n = z$ for some $z \in X$, then $z \in E$ and $z = Tz$.

3 Main Results

Firstly, we need the following lemmas for complete the proof of main results.

Lemma 3.1. *Let E be a nonempty closed and convex subset of a complete CAT(0) space X, and let $T : E \to X$ be a quasi-nonexpansive mapping having demiclosed principle such that $F(T) \neq \emptyset$. If $\{x_n\}$ is a bounded sequence in E such that $\lim_{n\to\infty} d(Tx_n, x_n) = 0$ and $\{d(x_n, v)\}$ converges for all $v \in F(T)$, then $\omega_w(\{x_n\}) \subset F(T)$. Furthermore, $\omega_w(\{x_n\})$ consists of exactly one point.*

Proof. By the assumption $\{x_n\}$ is a bounded sequence in E such that $\lim_{n\to\infty} d(Tx_n, x_n) = 0$. Let $u \in \omega_w(\{x_n\})$, then there exists a subsequence $\{u_n\}$ of $\{x_n\}$ such that $\mathrm{A}(\{u_n\}) = \{u\}$. By Lemma 2.3 and Lemma 2.4 there exists a subsequence $\{v_n\}$ of $\{u_n\}$ such that $\Delta - \lim_{n\to\infty} u_n = v \in E$. Since $\lim_{n\to\infty} d(Tv_n, v_n) = 0$, we can conclude that $v \in F(T)$. From Lemma 2.5 and $\{d(x_n, v)\}$ converges for all $v \in F(T)$, we have $u = v \in F(T)$. This implies that $\omega_w(\{x_n\}) \subset F(T)$. Finally, $\omega_w(\{x_n\})$ consists of exactly one point. Indeed, let $\mathrm{A}(\{x_n\}) = \{x\}$ and $\{u_n\}$ be a subsequence of $\{x_n\}$ with $\mathrm{A}(\{u_n\}) = \{u\}$. Since $u \in \omega_w(\{x_n\}) \subset F(T)$, we have $u = v \in F(T)$ and hence $\{d(x_n, u)\}$ converges. We can apply Lemma 2.5 again to conclude that $x = u$. □

Lemma 3.2. *Let E be a nonempty closed and convex subset of a complete CAT(0) space X, and let $T, S : E \to E$ are quasi-nonexpansive mappings having demiclosed principle such that $F(T) \cap F(S) \neq \emptyset$. If $\{x_n\}$ be a sequence defined by (A) then $\lim_{n\to\infty} d(x_n, w)$ exists for all $w \in F(T) \cap F(S)$.*

Proof. Let $\{x_n\}$ be a sequence defined by (A) and $w \in F(T) \cap F(S)$. Then $d(x, w) \leq d(x, w)$ and $d(Sy, w) \leq d(y, w)$ for all $x, y \in E$. By Lemma 2.2(ii), we have

$$\begin{aligned} d^2(x_{n+1}, w) &= d^2(\alpha_n x_n \oplus (1-\alpha_n)S(Tx_n), w) \\ &\leq \alpha_n d^2(x_n, w) + (1-\alpha_n) d^2(S(Tx_n), w) \\ &- \alpha_n(1-\alpha_n) d^2(x_n, S(Tx_n)) \\ &\leq \alpha_n d^2(x_n, w) + (1-\alpha_n) d^2(Tx_n, w) \\ &- \alpha_n(1-\alpha_n) d^2(x_n, S(Tx_n)) \qquad (3.1) \\ &\leq \alpha_n d^2(x_n, w) + (1-\alpha_n) d^2(x_n, w) \\ &- \alpha_n(1-\alpha_n) d^2(x_n, S(Tx_n)) \\ &\leq d^2(x_n, w) - \alpha_n(1-\alpha_n) d^2(x_n, S(Tx_n)) \qquad (3.2) \\ &\leq d^2(x_n, w). \end{aligned}$$

Therefore, $\{d(x_n, w)\}$ is bounded and decreasing sequence which imply that $\lim_{n\to\infty} d(x_n, w)$ exists. □

Lemma 3.3. *Let E be a nonempty closed and convex subset of a complete CAT(0) space X and $T : E \to E$ be a firmly nonspreading mapping. Suppose that $S : E \to E$ is a quasi-nonexpansive mapping having demiclosed principle such that $F(T) \cap F(S) \neq \emptyset$ and let $\{x_n\}$ be defined as (A). If $\{\alpha_n\}$ is a sequences in $(0,1)$ such that $\liminf_{n\to\infty} \alpha_n(1-\alpha_n) > 0$, then $\lim_{n\to\infty} d(x_n, Tx_n) = 0$ and $\lim_{n\to\infty} d(Tx_n, w)$ exists.*

Proof. Let $\{x_n\}$ be a sequence defined by (A) and $w \in F(T) \cap F(S)$. By Lemma 3.2, we have $\lim_{n\to\infty} d(x_n, w)$ exists. Since $d(Tx_n, w) \leq d(x_n, w) \leq d(x_1, w)$, $\{x_n\}$ and $\{Tx_n\}$ are bounded.
From (3.2), we have

$$d^2(x_{n+1}, w) \leq d^2(x_n, w) - \alpha_n(1-\alpha_n)d^2(x_n, S(Tx_n)).$$

Thus,

$$\alpha_n(1-\alpha_n)d^2(x_n, S(Tx_n)) \leq d^2(x_n, w) - d^2(x_{n+1}, w).$$

Since $\liminf_{n\to\infty} \alpha_n(1-\alpha_n) > 0$, there exist $k > 0$ and $N \in \mathbb{N}$ such that $\alpha_n(1-\alpha_n) \geq k$ for all $n \geq N$, and hence

$$\begin{aligned}
\limsup_{n\to\infty} kd^2(x_n, S(Tx_n)) &\leq \limsup_{n\to\infty} \alpha_n(1-\alpha_n)d^2(x_n, S(Tx_n)) \\
&\leq \limsup_{n\to\infty} \left(d^2(x_n, w) - d^2(x_{n+1}, w)\right) \\
&= 0.
\end{aligned}$$

Therefore, $0 \leq \liminf_{n\to\infty} d^2(x_n, S(Tx_n)) \leq \limsup_{n\to\infty} d^2(x_n, S(Tx_n)) \leq 0$, which implies that $\lim_{n\to\infty} d^2(x_n, S(Tx_n)) = 0$. Thus,

$$\lim_{n\to\infty} d(x_n, S(Tx_n)) = 0. \tag{3.3}$$

Furthermore, we have from (3.1) that

$$d^2(x_{n+1}, w) \leq \alpha_n d^2(x_n, w) + (1-\alpha_n)d^2(Tx_n, w) - \alpha_n(1-\alpha_n)d^2(x_n, S(Tx_n)).$$

Therefore,

$$d^2(x_{n+1}, w) \leq \alpha_n d^2(x_n, w) + d^2(Tx_n, w) - \alpha_n d^2(Tx_n, w),$$

and hence

$$\begin{aligned}
\alpha_n[d^2(Tx_n, w) - d^2(x_n, w)] &\leq d^2(Tx_n, w) - d^2(x_{n+1}, w) \\
&\leq d^2(x_n, w) - d^2(x_{n+1}, w).
\end{aligned}$$

Since $\alpha_n(1-\alpha_n) < \alpha_n$, $\liminf_{n\to\infty} \alpha_n > 0$.
Using the same argument we can conclude that

$$\lim_{n\to\infty}(d^2(Tx_n, w) - d^2(x_n, w)) = 0.$$

Since $w \in F(T)$,

$$\begin{aligned} d^2(Tx_n, w) &= d^2(Tx_n, Tw) \\ &\leq \frac{1}{2}(d^2(x_n, Tw) + d^2(Tx_n, w) - d^2(x_n, Tx_n) - d^2(w, Tw)) \\ &\leq d^2(x_n, Tw) - \frac{1}{2}d^2(x_n, Tx_n) \\ &= d^2(x_n, w) - \frac{1}{2}d^2(x_n, Tx_n). \end{aligned}$$

Thus,

$$d^2(Tx_n, w) \leq d^2(x_n, w) - \frac{1}{2}d^2(x_n, Tx_n). \tag{3.4}$$

Again by (3.1), we get

$$d^2(x_{n+1}, w) \leq \alpha_n d^2(x_n, w) + (1 - \alpha_n)d^2(Tx_n, w).$$

From (3.4), we get

$$\begin{aligned} d^2(x_{n+1}, w) &\leq \alpha_n d^2(x_n, w) + (1 - \alpha_n)[d^2(x_n, w) - \frac{1}{2}d^2(x_n, Tx_n)] \\ &= d^2(x_n, w) - \frac{1 - \alpha_n}{2}d^2(x_n, Tx_n). \end{aligned}$$

Therefore,

$$(1 - \alpha_n)d^2(x_n, Tx_n) \leq 2(d^2(x_n, w) - d^2(x_{n+1}, w)).$$

Since $\alpha_n(1 - \alpha_n) < (1 - \alpha_n)$, $\liminf_{n\to\infty}(1 - \alpha_n) > 0$.

Using the same argument, we have $\lim_{n\to\infty} d^2(x_n, Tx_n) = 0$.
This implies that

$$\lim_{n\to\infty} d(x_n, Tx_n) = 0. \tag{3.5}$$

Since $\lim_{n\to\infty}(d^2(Tx_n, w) - d^2(x_n, w)) = 0$ and $\lim_{n\to\infty} d(x_n, w)$ exists, we can conclude that $\lim_{n\to\infty} d(Tx_n, w)$ exists. □

Now we are ready to prove the $\Delta-$convergence theorem as follows.

Theorem 3.4. *Let E be a nonempty closed and convex subset of a complete CAT(0) space X, and $T : E \to E$ be a firmly nonspreading mapping. Suppose that $S : E \to E$ is a quasi-nonexpansive mapping having demiclosed principle such that $F(T) \cap F(S) \neq \emptyset$ and let $\{x_n\}$ be a sequence defined by (A). If $\{\alpha_n\}$ is a sequence in $(0, 1)$ such that $\liminf_{n\to\infty} \alpha_n(1 - \alpha_n) > 0$, then $\Delta - \lim_n x_n = w \in F(T) \cap F(S)$.*

Proof. Let $\{x_n\}$ be a sequence defined by (A) and $w \in F(T) \cap F(S)$. From Lemma 3.2, we have $\lim_{n\to\infty} d(x_n, w)$ exists. Then $\{x_n\}$ is bounded. From Lemma 3.3, we

have $\lim_{n\to\infty} d(x_n, Tx_n) = 0$ and $\lim_{n\to\infty} d(Tx_n, w)$ exists which implies that $\{Tx_n\}$ is also bounded. By (3.5) we have $\lim_{n\to\infty} d(x_n, Tx_n) = 0$ and by (3.3) we have $\lim_{n\to\infty} d(x_n, S(Tx_n)) = 0$. Since $d(S(Tx_n), Tx_n) \leq d(S(Tx_n), x_n) + d(x_n, Tx_n)$, $\lim_{n\to\infty} d(S(Tx_n), Tx_n) = 0$. By Lemma 3.1 and Remark 2.8, there exist $\bar{x}, \bar{y} \in E$ such that $\omega_w(\{x_n\}) = \{\bar{x}\} \subset F(T)$ and $\omega_w(\{Tx_n\}) = \{\bar{y}\} \subset F(S)$. Thus, $\Delta-\lim_n x_n = \bar{x}$ and
$\Delta- \lim_n Tx_n = \bar{y}$. By using Lemma 2.6, we have $\bar{x} = \bar{y}$. □

Acknowledgements : We would like to thank the referee(s) for comments and suggestions on the manuscript. This work was supported by Chiang Mai University.

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(Received 17 June 2019)
(Accepted 24 December 2019)