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Exact Solutions of the Conformable Space-Time Chiral Nonlinear Schrödinger's Equations

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Abstract : In this paper, the modified simple equation method is used to obtain exact solutions of the space-time (1+1) and (2+1)-dimensional chiral nonlinear Schrödinger's equations in the sense of the conformable derivative. As a consequence, these obtained solutions with their constraint conditions can be useful to explain some physical phenomena such as dark or singular soliton solutions. Graphical representations of selected solutions are illustrated using a range of fractional orders. The performance of the method is concise, effective and reliable for solving nonlinear partial differential equations (NPDEs) including the NPDEs with conformable derivatives.

Keywords : exact solutions; conformable space-time (1+1) and (2+1)-dimensional chiral nonlinear Schrödinger's equations; modified simple equation method; conformable derivative.

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1 Introduction

Nonlinear evolution equations (NLEEs) play a vital role in nonlinear physical science and engineering, since they may well describe various natural phenomena, such as solitons, plasma physics, fluid dynamics, optical fiber, vibrations, plane wave and propagation with a finite speed. Many NLEEs can be written as nonlinear partial differential equations (NPDEs) for which their solutions assist us to analyze and understand such phenomena modeled by NLEEs. One kind of the important solutions of NPDEs is exact traveling wave solutions, which are closed form solutions. They can provide a complete explanation of the physical behaviors for the studied problems. Hence, searching such exact or closed form solutions of NPDEs is of great importance in several fields of nonlinear sciences. Recently, many efficient methods have been proposed and used to solve NPDEs for exact traveling wave solutions such as the (G'/G)-expansion method [1–3], the (G'/G^2)-expansion method [4, 5], the extended tanh–coth method [6], the exp-function method [7,8], the sine-cosine method [9,10], the first integral method [11,12], the generalized Kudryashov method [13], the generalized Riccati equation mapping method [14,15], the extended auxiliary equation method [16], the inverse scattering method [17] and so on.

Study of solitons is one of the most fascinating areas of research in theoretical physics since solitons appear in many aspects of our daily life. For example, solitons can be found in solitary waves observed on lake shore and beaches where shallow water occurs, biological sciences in the context of neurosciences [18], Langmuir waves and Alfven waves in plasma physics [19], nonlinear fiber optics [20] and nonperturbative developments in the quantum field theory [21]. The most important application of solitons is highlighted in quantum Hall effect where the governing equation is the chiral nonlinear Schrödinger's equation (NLSE) providing both bright and dark solitons.

The investigations of finding exact solutions of the chiral nonlinear Schrödinger's equation are as follows. In 1998, Nishino et al. [22] obtained the progressive wave solutions of the (1+1)-dimensional chiral nonlinear Schrödinger's equation. The obtained solutions include bright and dark soliton solutions depending upon the sign of the product of the velocity of envelop and the nonlinear coupling constant. In 2009, Biswas [23] studied the chiral nonlinear Schrödinger's equation in (2+1) dimensions to carry out the bright and dark soliton solutions using the soliton ansatz method. In 2010, Biswas [24] employed the ansatz method to solve the chiral nonlinear Schrödinger's equation with time-dependent coefficients. In 2011, Biswas and Milovic [25] used He's variational principle to solve the (1+1)-dimensional the chiral nonlinear Schrödinger's equation with Bohm potential to obtain the 1-soliton solution. Biswas et al. [26] solved the generalized version of (1+1)-dimensional chiral nonlinear Schrödinger's equation via conservation laws and the aid of He's semi-inverse variational principle. In the following year, Ebadi

et al. [27] utilized the (G'/G)-expansion method and the exponential function method to obtain traveling wave solutions of the (1+1)-dimensional chiral nonlinear Schrödinger's equation with Bohm potential. In 2014, Biswas [28] used the first integral method to obtain the 1-soliton solution of the generalized (1+1)-dimensional chiral nonlinear Schrödinger's equation with time-dependent coefficients. In 2016, Younis et al [29] investigated the (1+1)-dimensional chiral nonlinear Schrödinger's equation with Bohm potential via extended Fan sub-equation method to obtain soliton-like solutions, triangular type solutions, single and combined non-degenerate Jacobi elliptic wave function like-solutions. Eslami [30] applied the trial solution technique to solve the (2+1)-dimensional chiral nonlinear Schrödinger's equation for its exact solutions. In 2018, Bulut et al. [31] solved the (1+1) and (2+1)-dimensional chiral nonlinear Schrödinger's equation using the sine-Gordon expansion method (SGEM) for their exact solutions including dark soliton solutions and bright soliton solutions. Raza and Javid [32] obtained exact solutions of the (2+1)-dimensional chiral nonlinear Schrödinger's equation using the extended direct algebraic method and extended trial equation method. These methods provided many kinds of solutions of the equation including rational function solutions, hyperbolic function solutions, Jacobi elliptic function solutions. Here the (1+1)- and (2+1)-dimensional chiral nonlinear Schrödinger's equations are shown as follows.

The (1+1)-dimensional chiral nonlinear Schrödinger's equation given by [22,31] is written as

$$i\Psi_t + \Psi_{xx} - i\sigma\left(\Psi^*\Psi_x - \Psi\Psi_x^*\right)\Psi = 0, \tag{1.1}$$

where Ψ is a complex function of x and t, $i = \sqrt{-1}$, σ is a nonlinear coupling constant and the notation $*$ indicates the complex conjugate.

The (2+1)-dimensional chiral nonlinear Schrödinger's equation given by [30–32] is expressed as

$$i\Psi_t + a\left(\Psi_{xx} + \Psi_{yy}\right) + i\left(b_1\left(\Psi\Psi_x^* - \Psi^*\Psi_x\right) + b_2\left(\Psi\Psi_y^* - \Psi^*\Psi_y\right)\right)\Psi = 0, \tag{1.2}$$

where Ψ is the complex function of x and t, a is the coefficient of the dispersion terms and b_1, b_2 are nonlinear coupling constants and the notation $*$ represents the complex conjugate.

In this paper, we are interested in finding exact solutions, by means of the modified simple equation method, of the space-time chiral nonlinear Schrödinger's equations in the sense of the conformable derivative, which are modified from the original versions in Eqs. (1.1) and (1.2). The remaining paper is classified as follows. In Section 2, the conformable derivative and its properties are given. In Section 3, the description of the modified simple equation method is compactly provided. The applications of the method to the conformable space-time chiral nonlinear Schrödinger's equations are demonstrated in Section 4. In Section 5, the paper is concluded.

2 Conformable derivative and its properties

In this section, the definition of the conformable derivative and its important properties are shortly given as follows.

Definition 2.1. Given a function $f : [0, \infty) \to \mathbb{R}$. Then the conformable derivative of f of order α is defined by [33, 34]

$$D_t^\alpha f(t) = \lim_{\varepsilon \to 0} \frac{f(t + \varepsilon t^{1-\alpha}) - f(t)}{\varepsilon}, \quad \text{for all } t > 0,\ 0 < \alpha \leq 1. \tag{2.1}$$

If the limit in Eq. (2.1) exists, then we say that f is α-conformable differentiable at a point $t > 0$.

Remark 2.2. *Since the derivative in Eq. (2.1) was initially defined, it was originally called the conformable fractional derivative and had been used in many applications of fractional differential equations (FDEs) [35–37]. Until 2018, Tarasov [38] demonstrated that the conformable fractional derivative in Eq. (2.1) does not give anything new in the spaces of differentiable functions and is not a fractional derivative of non-integer order. Throughout this paper, we hence call the derivative in Eq. (2.1) that the conformable derivative.*

Theorem 2.3. *Let $\alpha \in (0, 1]$, and $f(t)$, $g(t)$ be α-conformable differentiable at a point $t > 0$, then [33, 34]*

$$\begin{aligned}
D_t^\alpha(\lambda) &= 0, \text{ where } \lambda = \text{constant}, \\
D_t^\alpha(t^\mu) &= \mu t^{\mu-\alpha}, \text{ for all } \mu \in \mathbb{R}, \\
D_t^\alpha(af(t) + bg(t)) &= aD_t^\alpha f(t) + bD_t^\alpha g(t), \text{ for all } a, b \in \mathbb{R}, \\
D_t^\alpha(f(t)g(t)) &= f(t)D_t^\alpha g(t) + g(t)D_t^\alpha f(t), \\
D_t^\alpha\left(\frac{f(t)}{g(t)}\right) &= \frac{g(t)D_t^\alpha f(t) - f(t)D_t^\alpha g(t)}{g(t)^2}.
\end{aligned}$$

Remark 2.4. *Conformable derivative of some interesting functions are as follows [33].*

(1) $D_t^\alpha(e^{ct}) = ct^{1-\alpha}e^{ct}$, $c \in \mathbb{R}$.

(2) $D_t^\alpha(\sin bt) = bt^{1-\alpha}\cos bt, b \in \mathbb{R}$.

(3) $D_t^\alpha(\cos bt) = -bt^{1-\alpha}\sin bt, b \in \mathbb{R}$.

(4) $D_t^\alpha(\frac{1}{\alpha}t^\alpha) = 1$.

(5) $D_t^\alpha(f(t)) = t^{1-\alpha}\dfrac{df(t)}{dt}$, *provided that $f(t)$ is differentiable.*

Theorem 2.5. *Let $f : (0, \infty) \to \mathbb{R}$ be a function such that f is differentiable and α-conformable differentiable. Also, let g be a differentiable function defined in the range of f. Then*

$$D_t^\alpha(f \circ g)(t) = t^{1-\alpha}f'(g(t))g'(t),$$

where the prime notation ($'$) denotes the classical derivative.

3 Algorithm of the modified simple equation method

The essential procedure of the modified simple equation method is as follows [39–44].

Step 1: We suppose that the given conformable nonlinear partial differential equation in $u(x,t)$ is of the following form

$$F(u, D_x^{\gamma_1}u, D_t^{\gamma_2}u, D_{2x}^{\gamma_1}u, D_t^{\gamma_2}D_x^{\gamma_1}u, ...) = 0, \tag{3.1}$$

where D_τ^α is the conformable derivatives of a dependent variable u with respect to independent variables τ. Eq. (3.1) can be converted into an ODE

$$G(u, u', u'', u''', ...) = 0, \tag{3.2}$$

by using the traveling wave transformation $\xi = \frac{x^{\gamma_1}}{\gamma_1} + \frac{t^{\gamma_2}}{\gamma_2}$.

Step 2: Suppose that the solution of Eq. (3.2) can be expressed by a finite series of the form

$$u(\xi) = \sum_{n=0}^{m} \alpha_n \left(\frac{\phi'(\xi)}{\phi(\xi)}\right)^n, \tag{3.3}$$

where $\alpha_n\,(n = 0, 1, 2, 3, ..., m)$ are arbitrary constants to be determined later, such that $\alpha_n \neq 0$, and $\phi(\xi)$ is an unidentified function to be determined subsequently. The positive integer m will be determined by balancing the highest order derivative term with the highest order nonlinear term of Eq. (3.2).

Step 3: We substitute Eq. (3.3) into Eq. (3.2). As a result of this substitution, we obtain a polynomial of $\phi^{-j}(\xi)$ in terms of the derivatives of $\phi(\xi)$. We equate all of the coefficients of $\phi^{-j}(\xi)$ for $j \geq 0$ to be zero. This process yields a system of algebraic equations which can be solved for the coefficients $\alpha_n\,(n = 0, 1, 2, 3, ..., m)$ and the function $\phi(\xi)$ with the aid of a symbolic software package.

Step 4: We substitute the values of $\alpha_n (n = 0, 1, 2, 3, ..., m)$, $\phi(\xi)$, $\phi'(\xi)$ and ξ into Eq. (3.3) to complete the determination of exact solutions of Eq. (3.1).

4 Applications

In this section, we will apply the modified simple equation method to the conformable space-time (1+1)- and (2+1)-dimensional chiral nonlinear Schrödinger's equations to obtain their exact solutions.

4.1 The conformable space-time (1+1)-dimensional chiral nonlinear Schrödinger's equation

The conformable space-time (1+1)-dimensional chiral nonlinear Schrödinger's equation is

$$iD_t^{\gamma_2}\Psi + D_x^{2\gamma_1}\Psi - i\sigma(\Psi^* D_x^{\gamma_1}\Psi - \Psi D_x^{\gamma_1}\Psi^*)\Psi = 0, \tag{4.1}$$

where $0 < \gamma_1, \gamma_2 \leq 1$, Ψ is a complex function of x and t, σ is a real constant and the notation $*$ indicates the complex conjugate. Assume that the solution form of Eq. (4.1) is

$$\Psi(x,t) = u(\xi)\exp(i\theta), \tag{4.2}$$

where the complex wave transformations ξ and θ are, respectively,

$$\xi = c\left(\frac{x^{\gamma_1}}{\gamma_1} + v\frac{t^{\gamma_2}}{\gamma_2}\right),\ \theta = k\frac{x^{\gamma_1}}{\gamma_1} + \omega\frac{t^{\gamma_2}}{\gamma_2} + \varphi, \tag{4.3}$$

where c, v, k, ω and φ are real constants. Substituting Ψ in Eq. (4.2) into Eq. (4.1), and then decomposing the resulting equation into real and imaginary parts, we obtain

$$\text{Re}: \quad c^2u'' + 2k\sigma u^3 - (\omega + k^2)u = 0, \tag{4.4}$$
$$\text{Im}: \quad (2k + v)cu' = 0. \tag{4.5}$$

From Eq. (4.5), one can get the relation

$$v = -2k. \tag{4.6}$$

Using the solution form in Eq. (3.3) and balancing the terms u'' and u^3 in Eq. (4.4) yields $m = 1$. Consequently, we have

$$u(\xi) = \alpha_0 + \alpha_1\left(\frac{\phi'}{\phi}\right), \tag{4.7}$$

where ϕ is the function of ξ. Substituting Eq. (4.7) into Eq. (4.4) and then setting the coefficients of $\phi^{-j}, j = 0, 1, 2, 3$ to be zero, we obtain a set of algebraic equations as follows:

$$\phi^{-3}: 2\,\sigma\,k\alpha_1^3(\phi')^3 + 2\,c^2\alpha_1(\phi')^3 = 0, \tag{4.8}$$
$$\phi^{-2}: 6\,\sigma\,k\alpha_0\alpha_1^2(\phi')^2 - 3\,c^2\alpha_1\phi''\phi' = 0, \tag{4.9}$$
$$\phi^{-1}: 6\,\sigma\,k\alpha_0^2\alpha_1\phi' - k^2\alpha_1\phi' + c^2\alpha_1\phi''' - \omega\,\alpha_1\phi' = 0, \tag{4.10}$$
$$\phi^{0}: 2\,k\sigma\alpha_0^3 - k^2\alpha_0 - \omega\alpha_0 = 0. \tag{4.11}$$

Solving Eqs. (4.8) and (4.11), we obtain

$$\alpha_0 = \pm\frac{1}{2}\sqrt{\frac{2\,k^2 + 2\,\omega}{\sigma\,k}},\ \alpha_1 = \pm\sqrt{-\frac{1}{\sigma\,k}}c. \tag{4.12}$$

Eqs. (4.9) and (4.10) can be reduced to

$$\phi' = \frac{c^2}{2\alpha_1\sigma\,k\alpha_0}\phi'', \tag{4.13}$$

and

$$\phi' = \frac{c^2}{-6\,k\sigma\,\alpha_0^2 + k^2 + \omega}\phi''', \tag{4.14}$$

respectively. Using Eq. (4.13) and Eq. (4.14), we have the following ODE

$$\frac{\phi'''}{\phi''} = \frac{-6\,k\sigma\,\alpha_0^2 + k^2 + \omega}{2\alpha_1\sigma\,k\alpha_0}. \tag{4.15}$$

Integrating Eq. (4.15) with respect to ξ, it yields

$$\phi'' = \varsigma_1 \exp\left(\frac{\left(-6k\sigma\alpha_0^2 + k^2 + \omega\right)\xi}{2\alpha_1\sigma k\alpha_0}\right), \tag{4.16}$$

where ς_1 is a constant of integration. Replacing ϕ'' in Eq. (4.16) into Eq. (4.13), we have

$$\phi' = \frac{c^2\varsigma_1}{2\alpha_1\sigma k\alpha_0}\exp\left(\frac{\left(-6k\sigma\alpha_0^2 + k^2 + \omega\right)\xi}{2\alpha_1\sigma k\alpha_0}\right). \tag{4.17}$$

Integrating Eq. (4.17) once, we obtain

$$\phi = \frac{c^2\varsigma_1}{-6k\sigma\alpha_0^2 + k^2 + \omega}\exp\left(\frac{\left(-6k\sigma\alpha_0^2 + k^2 + \omega\right)\xi}{2\alpha_1\sigma k\alpha_0}\right) + \varsigma_2, \tag{4.18}$$

where ς_2 is a constant of integration.

Next, we substitute $\phi(\xi)$ in Eq. (4.18) and $\phi'(\xi)$ in Eq. (4.17) into Eq. (4.7), then we obtain

$$u(\xi) = \alpha_0 + \frac{c^2\varsigma_1 \exp\left(\frac{\left(-6\,k\sigma\alpha_0^2 + k^2 + \omega\right)\xi}{2\alpha_1\sigma k\alpha_0}\right)}{2k\sigma\left(\frac{c^2\varsigma_1 \exp\left(\frac{\left(-6k\sigma\alpha_0^2 + k^2 + \omega\right)\xi}{2\alpha_1\sigma\,k\alpha_0}\right)}{-6k\sigma\alpha_0^2 + k^2 + \omega} + \varsigma_2\right)}. \tag{4.19}$$

Substituting the values of α_0, α_1 from Eq. (4.12) into Eq. (4.19) and then using Eq. (4.3), we obtain the exact solution of Eq. (4.1) as follows

$$\begin{aligned}\Psi_1(x,t) = \pm\,\frac{1}{2}\Bigg\{&\sqrt{\frac{2(k^2+\omega)}{k\sigma}} \\ &+ \frac{\sqrt{2}c^2\varsigma_1 \exp\left(\pm\sqrt{-2(k^2+\omega)}\left(\frac{x^{\gamma_1}}{\gamma_1} - 2k\frac{t^{\gamma_2}}{\gamma_2}\right)\right)}{\sqrt{(k^2+\omega)\,\sigma k}\left(\frac{c^2\varsigma_1 \exp\left(\pm\sqrt{-2(k^2+\omega)}\left(\frac{x^{\gamma_1}}{\gamma_1} - 2k\frac{t^{\gamma_2}}{\gamma_2}\right)\right)}{-2(k^2+\omega)} + \varsigma_2\right)}\Bigg\} \\ &\times \exp\left(i\left(k\frac{x^{\gamma_1}}{\gamma_1} + \omega\,\frac{t^{\gamma_2}}{\gamma_2} + \varphi\right)\right).\end{aligned} \tag{4.20}$$

In particular, if we first choose $\varsigma_2 = \frac{c^2\varsigma_1}{-6\,k\sigma\,\alpha_0^2+k^2+\omega}$ for Eq. (4.20), then the solitary wave solution (4.20) can be reduced in terms of the hyperbolic function solution as follows

$$\begin{aligned}\Psi_2(x,t) = &\pm \frac{1}{2}\sqrt{\frac{2\,(k^2+\omega)}{k\sigma}}\tanh\left(\frac{\sqrt{-2(k^2+\omega)}}{2}\left(\frac{x^{\gamma_1}}{\gamma_1}-2k\frac{t^{\gamma_2}}{\gamma_2}\right)\right)\\ &\times\exp\left(i\left(k\frac{x^{\gamma_1}}{\gamma_1}+\omega\frac{t^{\gamma_2}}{\gamma_2}+\varphi\right)\right),\end{aligned} \tag{4.21}$$

in which the dark soliton solution exists when $k^2+\omega<0$ and $k\sigma<0$. From Eq. (4.21), we alternatively obtain the trigonometric function solution of Eq. (4.21) as

$$\begin{aligned}\Psi_3(x,t) = &\pm \frac{1}{2}\sqrt{-\frac{2\,(k^2+\omega)}{k\sigma}}\tan\left(\frac{\sqrt{2(k^2+\omega)}}{2}\left(\frac{x^{\gamma_1}}{\gamma_1}-2k\frac{t^{\gamma_2}}{\gamma_2}\right)\right)\\ &\times\exp\left(i\left(k\frac{x^{\gamma_1}}{\gamma_1}+\omega\frac{t^{\gamma_2}}{\gamma_2}+\varphi\right)\right),\end{aligned} \tag{4.22}$$

in which the singular-periodic solution occurs when $k^2+\omega>0$ and $k\sigma<0$.

In addition, if we set $\varsigma_2 = \frac{\varsigma_1 c^2}{6\alpha_0^2 k\sigma-k^2-\omega}$, then the solitary wave solution (4.20) can be expressed in terms of the hyperbolic function solution as follows

$$\begin{aligned}\Psi_4(x,t) = &\pm \frac{1}{2}\sqrt{\frac{2\,(k^2+\omega)}{k\sigma}}\coth\left(\frac{\sqrt{-2(k^2+\omega)}}{2}\left(\frac{x^{\gamma_1}}{\gamma_1}-2k\frac{t^{\gamma_2}}{\gamma_2}\right)\right)\\ &\times\exp\left(i\left(k\frac{x^{\gamma_1}}{\gamma_1}+\omega\frac{t^{\gamma_2}}{\gamma_2}+\varphi\right)\right),\end{aligned} \tag{4.23}$$

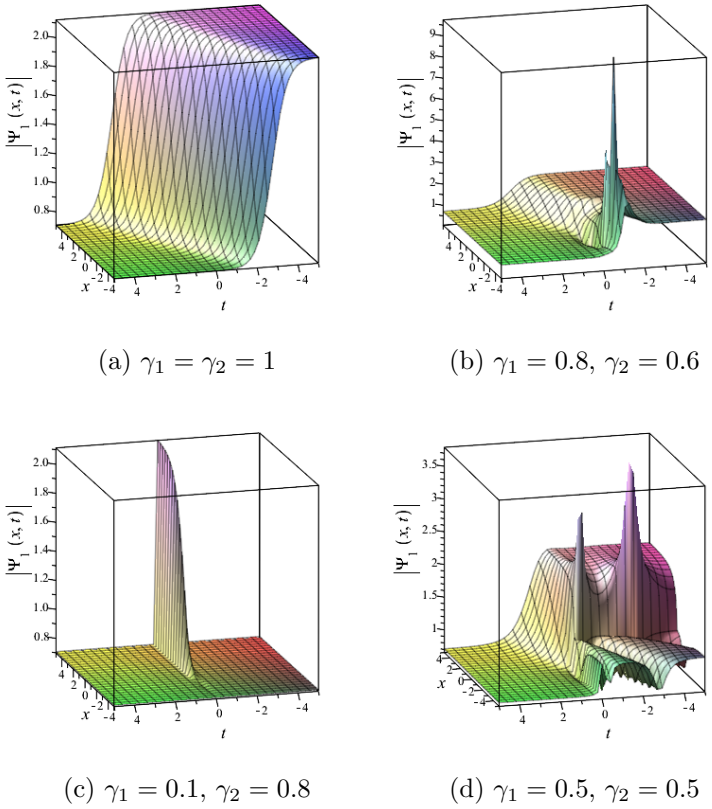
in which the singular soliton solution exists and is valid for $k^2+\omega<0$ and $k\sigma<0$. From Eq. (4.23), we alternatively get the trigonometric function solution of Eq. (4.23) as

$$\begin{aligned}\Psi_5(x,t) = &\pm \frac{1}{2}\sqrt{-\frac{2\,(k^2+\omega)}{k\sigma}}\cot\left(\frac{\sqrt{2(k^2+\omega)}}{2}\left(\frac{x^{\gamma_1}}{\gamma_1}-2k\frac{t^{\gamma_2}}{\gamma_2}\right)\right)\\ &\times\exp\left(i\left(k\frac{x^{\gamma_1}}{\gamma_1}+\omega\frac{t^{\gamma_2}}{\gamma_2}+\varphi\right)\right),\end{aligned} \tag{4.24}$$

in which the singular-periodic solution occurs and is valid when $k^2+\omega>0$ and $k\sigma<0$.

Next, we will give some graphical representations of the exact solutions of the conformable space-time (1+1)-dimensional chiral nonlinear Schrödinger's equation (4.1). We use the Maple package program to depict our results on the chosen domain is $-5\le x\le 5$ and $-5\le t\le 5$. The variations of the selected fractional orders γ_1 and γ_2 are as the following sets: $\{\gamma_1=1,\,\gamma_2=1\}$, $\{\gamma_1=0.8,\,\gamma_2=$

0.6}, $\{\gamma_1 = 0.1, \gamma_2 = 0.8\}$ and $\{\gamma_1 = 0.5, \gamma_2 = 0.5\}$. The absolute values of the exact solution (4.20) are plotted in Figure 1 using the mentioned variation sets of the fractional orders and the following parameter values $k = 1$, $\omega = -2$, $\sigma = -1$, $c = 1$, $\varsigma_1 = 1$, $\varsigma_2 = 1$. Figure 2 shows the absolute values of the exact solution

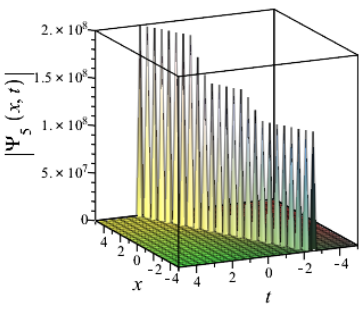


(a) $\gamma_1 = \gamma_2 = 1$ (b) $\gamma_1 = 0.8$, $\gamma_2 = 0.6$

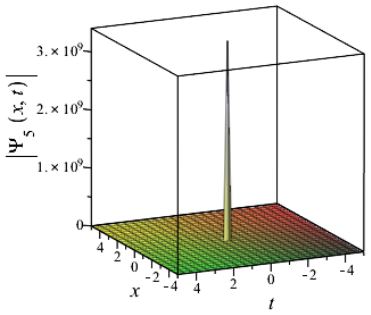
(c) $\gamma_1 = 0.1$, $\gamma_2 = 0.8$ (d) $\gamma_1 = 0.5$, $\gamma_2 = 0.5$

Figure 1: Graphs of the absolute value of the solution $\Psi_1(x,t)$ in Eq. (4.20) using $k = 1$, $\omega = -2$, $\sigma = -1$, $c = 1$, $\varsigma_1 = 1$, $\varsigma_2 = 1$ on $-5 \leq x \leq 5$ and $-5 \leq t \leq 5$.

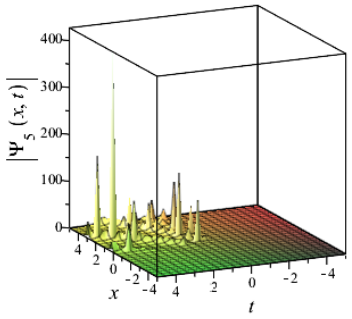
(4.24) using the mentioned variation sets of the fractional orders and the following parameter values $k = 1$, $\omega = 2$, $\sigma = -1$, $c = 1$, $\varsigma_1 = 1$, $\varsigma_2 = 1$.



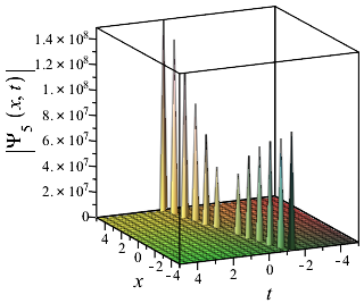
(a) $\gamma_1 = \gamma_2 = 1$



(b) $\gamma_1 = 0.8$, $\gamma_2 = 0.6$



(c) $\gamma_1 = 0.1$, $\gamma_2 = 0.8$



(d) $\gamma_1 = 0.5$, $\gamma_2 = 0.5$

Figure 2: Graphs of the absolute value of the solution $\Psi_5(x,t)$ in Eq. (4.24) using $k = 1$, $\omega = 2$, $\sigma = -1$, $c = 1$, $\varsigma_1 = 1$, $\varsigma_2 = 1$ on $-5 \leq x \leq 5$ and $-5 \leq t \leq 5$.

4.2 The conformable space-time (2+1)-dimensional chiral nonlinear Schrödinger's equation

The conformable space-time (2+1)-dimensional chiral nonlinear Schrödinger's equation is expressed as

$$\begin{aligned} &iD_t^{\gamma_2}\Psi + a\left(D_x^{2\gamma_1}\Psi + D_y^{2\gamma_3}\Psi\right) \\ &+ i\left(b_1\left(\Psi D_x^{\gamma_1}\Psi^* - \Psi^* D_x^{\gamma_1}\Psi\right) + b_2\left(\Psi D_y^{\gamma_3}\Psi^* - \Psi^* D_y^{\gamma_3}\Psi\right)\right)\Psi = 0, \end{aligned} \tag{4.25}$$

where $0 < \gamma_1, \gamma_2, \gamma_3 \leq 1$, Ψ is a complex function of x, y and t, the parameter a, b_1 and b_2 are real constants and the notation $*$ indicates the complex conjugate.

Assume that the solution form of Eq. (4.25) is

$$\Psi = u(\xi)\exp(i\Omega), \tag{4.26}$$

where the complex wave transformations ξ and Ω are, respectively,

$$\xi = c\frac{x^{\gamma_1}}{\gamma_1} - v\frac{t^{\gamma_2}}{\gamma_2} + k\frac{y^{\gamma_3}}{\gamma_3},\ \Omega = p\frac{x^{\gamma_1}}{\gamma_1} + \omega\frac{t^{\gamma_2}}{\gamma_2} + q\frac{y^{\gamma_3}}{\gamma_3} + \varphi, \tag{4.27}$$

where c, v, k, p, ω, q and φ are real constants. Substituting Ψ in Eq. (4.26) into Eq. (4.25) and then decomposing the resulting equation into real and imaginary parts, we get

$$\text{Re} : a\left(c^2 + k^2\right)u'' + 2\left(pb_1 + qb_2\right)u^3 - \left(a\left(p^2 + q^2\right) + \omega\right)u = 0, \tag{4.28}$$

$$\text{Im} : \left(2\,a(cp + kq) - v\right)u' = 0. \tag{4.29}$$

From Eq. (4.29), one can obtain the relation

$$v = 2\,a\left(cp + kq\right). \tag{4.30}$$

Using the solution in Eq. (3.3) and balancing the terms u'' and u^3 in Eq. (4.28) yields $m = 1$. In consequence, we have

$$u(\xi) = \alpha_0 + \alpha_1\left(\frac{\phi'}{\phi}\right), \tag{4.31}$$

where ϕ is the function of ξ. Substituting Eq. (4.31) into Eq. (4.28) and then equating the coefficients of ϕ^{-j}, $j = 0, 1, 2, 3$ to be zero, we obtain a set of algebraic equations as follows:

$$\phi^{-3} : 2\alpha_1\left(\left(pb_1 + qb_2\right)\alpha_1^2 + a\left(c^2 + k^2\right)\right)\left(\phi'\right)^3 = 0, \tag{4.32}$$

$$\phi^{-2} : 6^2\alpha_0\alpha_1^2\left(pb_1 + qb_2\right)\left(\phi'\right)^2 - 3a\alpha_1\left(c^2 + k^2\right)\phi''\phi' = 0, \tag{4.33}$$

$$\begin{aligned} \phi^{-1} : &a\alpha_1\left(c^2 + k^2\right)\phi''' \\ &- \left(-6\left(pb_1 + qb_2\right)\alpha_0^2 + a\left(p^2 + q^2\right) + \omega\right)\alpha_1\left(\phi'\right) = 0, \end{aligned} \tag{4.34}$$

$$\phi^0 : -\left(-2\left(pb_1 + qb_2\right)\alpha_0^2 + a\left(p^2 + q^2\right) + \omega\right)\alpha_0 = 0. \tag{4.35}$$

Solving Eqs. (4.32) and (4.35), we get

$$\alpha_0 = \pm\sqrt{\frac{a\left(p^2+q^2\right)+\omega}{2(pb_1+qb_2)}},\ \alpha_1 = \pm\sqrt{-\frac{a\left(c^2+k^2\right)}{pb_1+qb_2}}. \tag{4.36}$$

Eqs. (4.33) and (4.34) can be rewritten as

$$\phi' = \frac{a\left(c^2+k^2\right)}{2\alpha_1\alpha_0\left(pb_1+qb_2\right)}\phi'', \tag{4.37}$$

and

$$\phi' = \frac{a\left(c^2+k^2\right)}{a\left(p^2+q^2\right)-6\left(pb_1+qb_2\right)\alpha_0^2+\omega}\phi''', \tag{4.38}$$

respectively. Using Eqs. (4.37) and (4.38), we have the following ODE

$$\frac{\phi'''}{\phi''} = \frac{a\left(p^2+q^2\right)-6\left(pb_1+qb_2\right)\alpha_0^2+\omega}{2\alpha_1\alpha_0\left(pb_1+qb_2\right)}. \tag{4.39}$$

Integrating Eq. (4.39) with respect to ξ, it yields

$$\phi'' = \varsigma_3 \exp\left(\frac{\left(a\left(p^2+q^2\right)-6\left(pb_1+qb_2\right)\alpha_0^2+\omega\right)\xi}{2\alpha_1\alpha_0\left(pb_1+qb_2\right)}\right), \tag{4.40}$$

where ς_3 is a constant of integration. Replacing ϕ'' in Eq. (4.40) into Eq. (4.37), we obtain

$$\phi' = \frac{a\left(c^2+k^2\right)\varsigma_3}{2\alpha_1\alpha_0\left(pb_1+qb_2\right)}\exp\left(\frac{\left(a\left(p^2+q^2\right)-6\left(pb_1+qb_2\right)\alpha_0^2+\omega\right)\xi}{2\alpha_1\alpha_0\left(pb_1+qb_2\right)}\right). \tag{4.41}$$

Integration Eq. (4.41) once, we have that

$$\begin{aligned}\phi =& \frac{a\left(c^2+k^2\right)\varsigma_3}{a\left(p^2+q^2\right)-6\left(pb_1+qb_2\right)\alpha_0{}^2+\omega} \\ &\times \exp\left(\frac{\left(a\left(p^2+q^2\right)-6\left(pb_1+qb_2\right)\alpha_0^2+\omega\right)\xi}{2\alpha_1\alpha_0\left(pb_1+qb_2\right)}\right)+\varsigma_4,\end{aligned} \tag{4.42}$$

where ς_4 is a constant of integration.

Next, we substitute $\phi(\xi)$ in Eq. (4.42) and $\phi'(\xi)$ in Eq. (4.41) into Eq. (4.31). Then, we obtain

$$u(\xi) = \alpha_0 + \frac{a\left(c^2+k^2\right)\varsigma_3 \exp\left(\frac{\left(a\left(p^2+q^2\right)-6(pb_1+qb_2)\alpha_0^2+\omega\right)\xi}{2\alpha_1\alpha_0(pb_1+qb_2)}\right)}{2\alpha_0\left(pb_1+qb_2\right)\left(\frac{a(c^2+k^2)\varsigma_3\exp\left(\frac{\left(a\left(p^2+q^2\right)-6(pb_1+qb_2)\alpha_0^2+\omega\right)\xi}{2\alpha_1\alpha_0(pb_1+qb_2)}\right)}{a(p^2+q^2)-6\alpha_0^2(b_1p+qb_2)+\omega}+\varsigma_4\right)}. \tag{4.43}$$

Substituting the values of α_0, α_1 from Eq. (4.36) into Eq. (4.43) and then using Eq. (4.27), we obtain the exact solution of Eq. (4.25) as follows

$$\begin{aligned}\Psi_1(x,y,t) &= \pm\sqrt{\frac{a\left(p^2+q^2\right)+\omega}{2\left(pb_1+qb_2\right)}} \\ &\times\left(1+\frac{a\left(c^2+k^2\right)\varsigma_3\exp\left(\pm\frac{\xi\sqrt{-2\,a(p^2+q^2)-2\,\omega}}{\sqrt{a(c^2+k^2)}}\right)}{\left(a\left(p^2+q^2\right)+\omega\right)\left(-\frac{a(c^2+k^2)\varsigma_3\exp\left(\pm\frac{\xi\sqrt{-2\,a(p^2+q^2)-2\,\omega}}{\sqrt{a(c^2+k^2)}}\right)}{2\,a(p^2+q^2)+2\,\omega}+\varsigma_4\right)}\right) \\ &\times\exp\left(i\left(p\frac{x^{\gamma_1}}{\gamma_1}+\omega\frac{t^{\gamma_2}}{\gamma_2}+q\frac{y^{\gamma_3}}{\gamma_3}+\varphi\right)\right),\end{aligned} \tag{4.44}$$

where $\xi = c\frac{x^{\gamma_1}}{\gamma_1}+k\frac{y^{\gamma_3}}{\gamma_3}-2a\left(cp+kq\right)\frac{t^{\gamma_2}}{\gamma_2}$.

In particular, if we first choose $\varsigma_4 = \frac{a\left(c^2+k^2\right)\varsigma_3}{(p^2+q^2)a-6(pb_1+qb_2)\alpha_0^2+\omega}$ for Eq. (4.44), then the solitary wave solution (4.44) can be written in terms of the hyperbolic function solution with $\xi = c\frac{x^{\gamma_1}}{\gamma_1}+k\frac{y^{\gamma_3}}{\gamma_3}-2a\left(cp+kq\right)\frac{t^{\gamma_2}}{\gamma_2}$ as follows

$$\begin{aligned}\Psi_2(x,y,t) = &\pm\sqrt{\frac{a\left(p^2+q^2\right)+\omega}{2\left(pb_1+qb_2\right)}}\tanh\left(\sqrt{\frac{-\left(a\left(p^2+q^2\right)+\omega\right)}{2a\left(c^2+k^2\right)}}\xi\right) \\ &\times\exp\left(i\left(p\frac{x^{\gamma_1}}{\gamma_1}+\omega\frac{t^{\gamma_2}}{\gamma_2}+q\frac{y^{\gamma_3}}{\gamma_3}+\varphi\right)\right),\end{aligned} \tag{4.45}$$

in which the dark soliton solution exists when $\frac{a\left(p^2+q^2\right)+\omega}{pb_1+qb_2}>0$ and $\frac{a\left(p^2+q^2\right)+\omega}{a}<0$. From Eq. (4.45), we can equivalently obtain the trigonometric function solution of (4.45) as

$$\begin{aligned}\Psi_3(x,y,t) = &\pm\sqrt{-\frac{a\left(p^2+q^2\right)+\omega}{2\left(pb_1+qb_2\right)}}\tan\left(\sqrt{\frac{a\left(p^2+q^2\right)+\omega}{2a\left(c^2+k^2\right)}}\xi\right) \\ &\times\exp\left(i\left(p\frac{x^{\gamma_1}}{\gamma_1}+\omega\frac{t^{\gamma_2}}{\gamma_2}+q\frac{y^{\gamma_3}}{\gamma_3}+\varphi\right)\right),\end{aligned} \tag{4.46}$$

where $\xi = c\frac{x^{\gamma_1}}{\gamma_1}+k\frac{y^{\gamma_3}}{\gamma_3}-2a\left(cp+kq\right)\frac{t^{\gamma_2}}{\gamma_2}$ in which the singular-periodic solution occurs when $\frac{a\left(p^2+q^2\right)+\omega}{pb_1+qb_2}<0$ and $\frac{a\left(p^2+q^2\right)+\omega}{a}>0$.

Furthermore, if we set $\varsigma_4 = \frac{-a\left(c^2+k^2\right)\varsigma_3}{(p^2+q^2)a-6(pb_1+qb_2)\alpha_0{}^2+\omega}$, then the solitary wave solution (4.44) can be written in terms of the hyperbolic function solution as

follows

$$\Psi_4(x,y,t) = \pm\sqrt{\frac{a\left(p^2+q^2\right)+\omega}{2\left(pb_1+qb_2\right)}}\coth\left(\sqrt{\frac{-\left(a\left(p^2+q^2\right)+\omega\right)}{2a\left(c^2+k^2\right)}}\xi\right) \times \exp\left(i\left(p\frac{x^{\gamma_1}}{\gamma_1}+\omega\frac{t^{\gamma_2}}{\gamma_2}+q\frac{y^{\gamma_3}}{\gamma_3}+\varphi\right)\right), \tag{4.47}$$

where $\xi = c\frac{x^{\gamma_1}}{\gamma_1} + k\frac{y^{\gamma_3}}{\gamma_3} - 2a\left(cp+kq\right)\frac{t^{\gamma_2}}{\gamma_2}$ in which the singular soliton solution exists and is valid if $\frac{a\left(p^2+q^2\right)+\omega}{pb_1+qb_2} > 0$ and $\frac{a\left(p^2+q^2\right)+\omega}{a} < 0$. Alternatively, we can obtain the trigonometric function solution of Eq. (4.25) using Eq. (4.47). The converted solution is

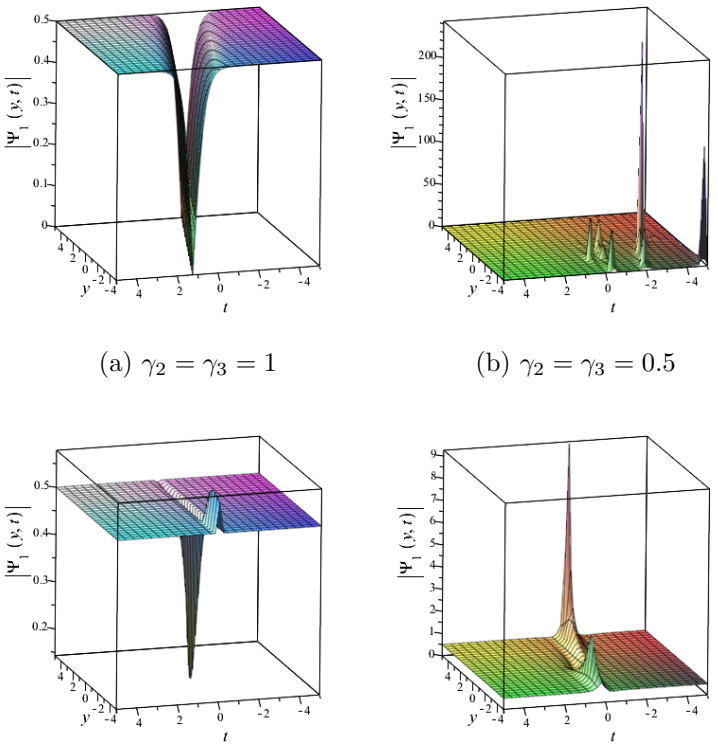
$$\Psi_5(x,y,t) = \pm\sqrt{-\frac{a\left(p^2+q^2\right)+\omega}{2\left(pb_1+qb_2\right)}}\cot\left(\sqrt{\frac{a\left(p^2+q^2\right)+\omega}{2a\left(c^2+k^2\right)}}\xi\right) \times \exp\left(p\frac{x^{\gamma_1}}{\gamma_1}+\omega\frac{t^{\gamma_2}}{\gamma_2}+q\frac{y^{\gamma_3}}{\gamma_3}+\varphi\right), \tag{4.48}$$

where $\xi = c\frac{x^{\gamma_1}}{\gamma_1} + k\frac{y^{\gamma_3}}{\gamma_3} - 2a\left(cp+kq\right)\frac{t^{\gamma_2}}{\gamma_2}$ in which the singular-periodic solution occurs and is valid for $\frac{a\left(p^2+q^2\right)+\omega}{pb_1+qb_2} < 0$ and $\frac{a\left(p^2+q^2\right)+\omega}{a} > 0$.

In the following part, we will give some graphical representations of the selected exact solutions of the conformable space-time (2+1)-dimensional chiral nonlinear Schrödinger's equation (4.25). For plotting, the target domain is defined as $-5 \le x \le 5$, $-5 \le y \le 5$ and $-5 \le t \le 5$ and the parameter values are $k = 1$, $\omega = 3$, $c = 1$, $p = 1$, $q = 1$, $a = -1$, $b_1 = 1$, $b_2 = 1$, $\varsigma_3 = 1$ and $\varsigma_4 = 1$. Setting $x = 0$ and using the following variations of the fractional orders γ_2 and γ_3: $\{\gamma_2 = \gamma_3 = 1\}$, $\{\gamma_2 = \gamma_3 = 0.5\}$, $\{\gamma_2 = 0.1, \gamma_3 = 0.8\}$ and $\{\gamma_2 = 0.8, \gamma_3 = 0.6\}$, the absolute values of the exact solution (4.44) are plotted in Figure 3. In Figure 4, we depict the absolute values of the exact solution (4.45) with $y = 0$ using the following sets of fractional orders γ_1 and γ_2: $\{\gamma_1 = \gamma_2 = 1\}$, $\{\gamma_1 = \gamma_2 = 0.1\}$, $\{\gamma_1 = 0.8, \gamma_2 = 0.9\}$ and $\{\gamma_1 = 0.8, \gamma_2 = 0.5\}$. All of the graphs in Figure 4 are generated using $k = 1$, $\omega = 3$, $c = 1$, $p = 1$, $q = 1$, $a = -1$, $b_1 = 1$, $b_2 = 1$.

5 Conclusions

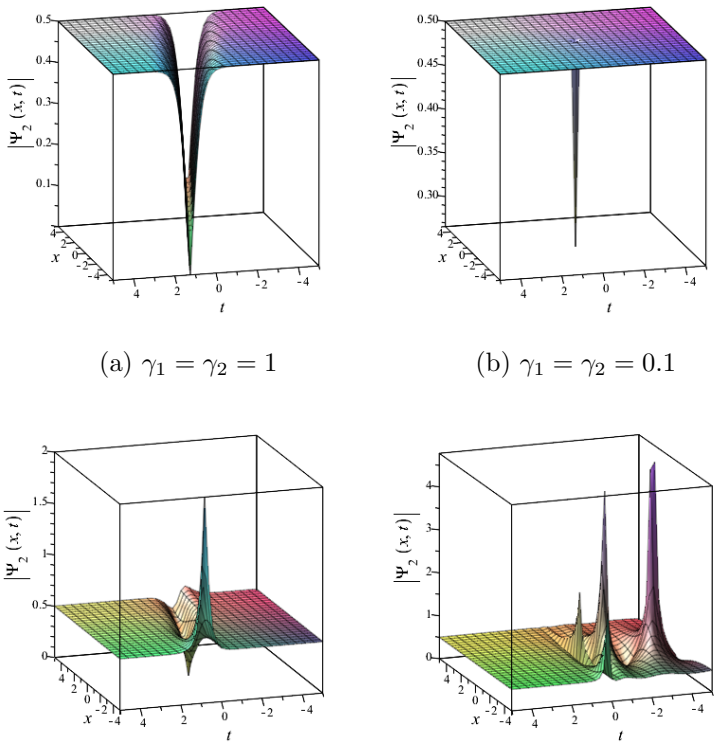
In this work, we have employed the modified simple equation method to obtain exact traveling wave solutions for the conformable space-time (1+1) and (2+1)-dimensional chiral nonlinear Schrödinger's equations using the conformable derivative. For each of the problem, the method has generally constructed the exponential function solutions which can be reduced to the hyperbolic function solutions and trigonometric function solutions depending upon choices of the constants of integration. The obtained solutions have been physically characterized as the soliton solutions and singular-periodic solutions. The constraint conditions for



(a) $\gamma_2 = \gamma_3 = 1$ (b) $\gamma_2 = \gamma_3 = 0.5$

(c) $\gamma_2 = 0.1$, $\gamma_3 = 0.8$ (d) $\gamma_2 = 0.8$, $\gamma_3 = 0.6$

Figure 3: Graphs of the absolute value of the solution $\Psi_1(x, y, t)$ in Eq. (4.44) by setting $x = 0$ and using $k = 1$, $\omega = 3$, $c = 1$, $p = 1$, $q = 1$, $b_1 = 1$, $b_2 = 1$, $\varsigma_3 = 1$, $\varsigma_4 = 1$ on $-5 \leq y \leq 5$ and $-5 \leq t \leq 5$.



(a) $\gamma_1 = \gamma_2 = 1$ (b) $\gamma_1 = \gamma_2 = 0.1$

(c) $\gamma_1 = 0.8, \gamma_2 = 0.9$ (d) $\gamma_1 = 0.8, \gamma_2 = 0.5$

Figure 4: Graphs of the absolute value of the solution $\Psi_2(x, y, t)$ in Eq. (4.45) by setting $y = 0$ and using $k = 1$, $\omega = 3$, $c = 1$, $p = 1$, $q = 1$, $a = -1$, $b_1 = 1$, $b_2 = 1$ on $-5 \leq x \leq 5$ and $-5 \leq t \leq 5$.

occurring dark soliton, singular soliton and singular-periodic solutions have been established. In addition, we have demonstrated some graphical representations of the chosen exact solutions of the problems by means of varying their fractional-order values. All of the obtained solutions have been verified by substituting them back into the associated problems with the help of the Maple 17 package program. Even though, the method cannot give bright soliton solutions to the problems, but it is still simple, powerful and reliable for obtaining the exact solutions for a considerable number of real-world problems which are written in terms of NPDEs including conformable NPDEs.

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