Thai Journal of Mathematics : 39-48
Special Issue : Annual Meeting in Mathematics 2019

http://thaijmath.in.cmu.ac.th
Online ISSN 1686-0209

Connections of (m, n)-bi-quasi Hyperideals in Semihyperrings

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Abstract : We introduce the concept of left (resp. right) (m, n)-bi-quasi hyperideals of semihyperrings as a generalization of n-bi-hyperideals, where m and n are positive integers. Then, we characterize regular semihyperrings using their left (resp. right) (m, n)-bi-quasi hyperideals. Moreover, we study the connections between left (resp. right) (m, n)-bi-quasi hyperideals and some many types of hyperideals in semihyperrings.

Keywords : semihyperring; m-bi-hyperideal; (m, n)-quasi hyperideal; (m, n)-bi-quasi hyperideal.
2010 Mathematics Subject Classification : 16Y60; 20M12; 20N20.

1 Introduction

In 1958, Iséki [1] introduced the notion of quasi-ideals for semirings without zero and proved results on semirings using their quasi-ideals. The concept of bi-ideals of associative rings was introduced by Lajos and Szász [2]. Any quasi-ideal is a generalization of a left and a right ideal, while every bi-ideal is a generalization of a quasi-ideal. Later, Chinram [3] introduced a generalization of quasi-ideals in semirings called (m, n)-quasi-ideals and studied characterizations of regular semirings using their (m, n)-quasi-ideals. Then, Munir and Shafiq [4] introduced and

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investigated some properties of the notion of m-bi-ideals in semirings as a generalization of bi-ideals. In 2018, Rao [5] introduced the concepts of left (resp. right) bi-quasi ideals of semirings which are generalizations of bi-ideals and quasi-ideals of semirings.

Algebraic hyperstructure was introduced in 1934 by Marty [6], at the 8^{th} Congress of Scandinavian Mathematicians. In a classical algebraic structure, composition of two elements is an element, while in an algebraic hyperstructure, the composition of two elements is a nonempty set. This theory was studied in the following decades and nowadays by many mathematicians (see, e.g., [7], [8], [9], [10]).

The concept of semihyperrins, which both the sum and the product are hyperoperations, was defined by Vougiouklis [11] as a generalization of semirings. Omidi and Davvaz [12] generalized the notions of m-left and n-right hyperideals in ordered semihyperrings to be (m, n)-quasi-hyperideals. Afterword, Nakkhasen and Pibaljommee [13] introduced the concept of m-bi-hyperideals and characterized regular semihyperrings by their m-bi-hyperideals. In this paper, we introduce the concept of left and right (m, n)-bi-quasi hyperideals of semihyperrings which is a generalization of n-bi-hyperideals and (m, n)-quasi-hyperideals of semihyperrings and characterize regular semihyperrings using left (resp. right) (m, n)-bi-quasi hyperideals. In addition, we investigate the connections between left (resp. right) (m, n)-bi-quasi hyperideals and some many types of hyperideals in semihyperrings.

2 Preliminaries

Let H be a nonempty set. A *hyperoperation* on H is a mapping $\circ : H \times H \to \mathcal{P}^*(H)$, where $\mathcal{P}^*(H)$ denotes the set of all nonempty subsets of H (see, e.g., [7], [8], [9], [10]). Then the structure $(H, \circ)$ is called a *hypergroupoid*. If $A, B \in \mathcal{P}^*(H)$ and $x \in H$, then we denote

$$A \circ B = \bigcup_{a \in A, b \in B} a \circ b, A \circ x = A \circ \{x\} \text{ and } x \circ B = \{x\} \circ B.$$

A hypergroupoid $(H, \circ)$ is called a *semihypergroup* if for every $x, y, z \in H$, $(x \circ y) \circ z = x \circ (y \circ z)$, which means that

$$\bigcup_{u \in x \circ y} u \circ z = \bigcup_{v \in y \circ z} x \circ v.$$

A hyperstructure $(S, +, \cdot)$ is called a *semihyperring* [11] if it satisfies the following conditions:

(i) $(S, +)$ is a semihypergroup;

(ii) $(S, \cdot)$ is a semihypergroup;

(iii) $x \cdot (y + z) = x \cdot y + x \cdot z$ and $(y + z) \cdot x = y \cdot x + z \cdot x$, for all $x, y, z \in S$.

A nonempty subset T of a semihyperring $(S,+,\cdot)$ is called a *subsemihyperring* of S if for all $x,y \in T$, $x+y \subseteq T$ and $x \cdot y \subseteq T$. For more convenient, we write S instead of a semihyperring $(S,+,\cdot)$ and AB instead of $A \cdot B$, for any nonempty subsets A and B of S.

Next, we review some concepts in semihyperrings which will be used in later section. For a semihyperring S and $m \in \mathbb{N}$, we denote $S^m = SS\cdots S$ (m times), in addition, for every $m,n \in \mathbb{N}$ such that $m \geq n$, we conclude that $S^m \subseteq S^n$.

A subsemihyperring A of a semihyperring S is called an *m-left* (resp. *n-right*) *hyperideal* [12] of S if it satisfies $S^m A \subseteq A$ (resp. $AS^n \subseteq A$), where m (resp. n) is a positive integer. A subsemihyperring Q of a semihyperring S is called an *(m,n)-quasi-hyperideal* [12] of S if it satisfies $(S^m Q) \cap (QS^n) \subseteq Q$, where m and n are positive integers. A subsemihyperring B of a semihyperring S is called an *m-bi-hyperideal* [13] of S if it satisfies $BS^m B \subseteq B$, where m is a positive integer.

3 Connections of (m,n)-bi-quasi hyperideals

In this section, we introduce the concept of left (resp. right) (m,n)-bi-quasi hyperideals of semihyperrings. Then, we characterize regular semihyperrings using their left (resp. right) (m,n)-bi-quasi hyperideals, and we present the connections between left (resp. right) (m,n)-bi-quasi hyperideals and some many types of hyperideals in semihyperrings.

Definition 3.1. A subsemihyperring A of a semihyperring S is called a *left* (resp. *right*) *(m,n)-bi-quasi hyperideal* of S if it satisfies $(S^m A) \cap (AS^n A) \subseteq A$ (resp. $(AS^m) \cap (AS^n A) \subseteq A$), where m and n are positive integers.

If A is both a left and a right (m,n)-bi-quasi hyperideal of a semihyperring S, then A is called an *(m,n)-bi-quasi hyperideal* of S.

Example 3.2. Let $S = \left\{ \begin{bmatrix} 0 & u & v & w \\ 0 & 0 & x & y \\ 0 & 0 & 0 & z \\ 0 & 0 & 0 & 0 \end{bmatrix} \mid u,v,w,x,y,z \in \mathbb{N} \cup \{0\} \right\}$. Then $(S,+,\cdot)$ is a semiring under usual addition and multiplication of matrices, see [4]. For every $A,B \in S$, we define

$$A \leq B \text{ iff } a_{ij} \leq b_{ij},$$

where $i,j \in \{1,2,3,4\}$. Next, we define hyperoperations $\oplus$ and $\odot$ on S by letting $A,B \in S$,

$$A \oplus B = \{X \in S \mid X \leq A+B\},$$
$$A \odot B = \{X \in S \mid X \leq A \cdot B\}.$$

We can show that $(S, \oplus, \odot)$ is a semihyperring. Now, let

$$M = \left\{ \begin{bmatrix} 0 & a & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & b \\ 0 & 0 & 0 & 0 \end{bmatrix} \mid a, b \in \mathbb{N} \cup \{0\} \right\}.$$

It is not difficult to check that M is a subsemihyperring of S. We consider

$$M \cdot S \cdot M = \left\{ \begin{bmatrix} 0 & 0 & 0 & m \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \mid m \in \mathbb{N} \cup \{0\} \right\}.$$

It follows that $M \odot S \odot M = \{X \in S \mid X \leq M \cdot S \cdot M\} \nsubseteq M$. Next, we consider

$$S^3 = \left\{ \begin{bmatrix} 0 & 0 & 0 & n \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \mid n \in \mathbb{N} \cup \{0\} \right\}.$$

This implies that

$$S^3 \cdot M = \left\{ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \right\}.$$

Thus,

$$S^3 \odot M = \{X \in S \mid X \leq S^3 \cdot M\} = \left\{ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \right\}.$$

Hence, $(S^3 \odot M) \cap (M \odot S \odot M) \subseteq M$. Therefore, M is a left $(3,1)$-bi-quasi hyperideal of S, but it is not a 1-bi-hyperidea; of S.

Throughout this paper, we always assume that m and n are any positive integers.

Theorem 3.3. *Every n-left hyperideal of a semihyperring S is an (m,n)-bi-quasi hyperideal of S.*

Proof. Let A be an n-left hyperideal of a semihyperring S. Then A is a subsemihyperring of S and $S^n A \subseteq A$. Thus,

$$(S^m A) \cap (AS^n A) \subseteq AS^n A \subseteq AA \subseteq A,$$
$$(AS^m) \cap (AS^n A) \subseteq AS^n A \subseteq AA \subseteq A.$$

Hence, A is an (m,n)-bi-quasi hyperideal of S. □

Theorem 3.4. *Every n-right hyperideal of a semihyperring S is an (m, n)-bi-quasi hyperideal of S.*

Remark 3.5. *Let S be a semihyperring. Then,*

(*i*) *every m-left hyperideal of S is a left (m, n)-bi-quasi hyperideal of S;*

(*ii*) *every m-right hyperideal of S is a right (m, n)-bi-quasi hyperideal of S.*

Theorem 3.6. *Every (m, n)-quasi-hyperideal of a semihyperring S is a left (m, n)-bi-quasi hyperideal of S.*

Proof. Let Q be an (m, n)-quasi-hyperideal of a semihyperring S. Then Q is a subsemihyperring of S and $(S^m Q) \cap (QS^n) \subseteq Q$. So, $(S^m Q) \cap (QS^n Q) \subseteq (S^m Q) \cap (QS^n) \subseteq Q$. Hence, Q is a left (m, n)-bi-quasi hyperideal of S. □

Theorem 3.7. *Every n-bi-hyperideal of a semihyperring S is an (m, n)-bi-quasi hyperideal of S.*

Proof. Let B be an n-bi-hyperideal of a semihyperring S. Then B is a subsemihyperring of S and $BS^n B \subseteq B$. Consider $(S^m B) \cap (BS^n B) \subseteq BS^n B \subseteq B$ and $(BS^m) \cap (BS^n B) \subseteq BS^n B \subseteq B$. Hence, B is an (m, n)-bi-quasi hyperideal of S. □

We note that arbitrary intersection of left (resp. right) (m, n)-bi-quasi hyperideals of a semihyperring S is not empty, then it is also a left (resp. right) (m, n)-bi-quasi hyperideal. If follows that arbitrary intersection of (m, n)-bi-quasi hyperideals of a semihyperring S is not empty, then it is also an (m, n)-bi-quasi hyperideal.

A semihyperring S is called *regular* (see, [14], [15]) if for each $a \in S$, there exists $x \in S$ such that $a \in axa$.

Theorem 3.8 ([14])**.** *Let S be a semihyperring. The following conditions are equivalent:*

(*i*) *S is regular;*

(*ii*) *$a \in aSa$, for every $a \in S$;*

(*iii*) *$A \subseteq ASA$, for all $\emptyset \neq A \subseteq S$.*

Theorem 3.9 ([13])**.** *Let S be a semihyperring and $m_1, m_2 \in \mathbb{N}$ such that $m = \max\{m_1, m_2\}$. Then S is regular if and only if $R \cap L = RS^m L$ for any m_1-left hyperideal L and m_2-right hyperideal R of S.*

Theorem 3.10. *Let S be a semihyperring and $n \geq m$. Then S is regular if and only if $A = (S^m A) \cap (AS^n A)$ for every left (m, n)-bi-quasi hyperideal A of S.*

Proof. Assume that S is regular. Let A be a left (m,n)-bi-quasi hyperideal of S. Then $(S^mA)\cap(AS^nA)\subseteq A$. By Theorem 3.8, we have that

$$A\subseteq (AS)A\subseteq (ASAS)A\subseteq\cdots\subseteq(\underbrace{AS\cdots AS}_{m\text{ terms}})A\subseteq S^mA,$$

$$\begin{aligned}A&\subseteq ASA\subseteq A(SA)SA\subseteq A(SASA)SA\\&\subseteq\cdots\subseteq A(\underbrace{SA\cdots SA}_{n\text{ terms}})SA\subseteq AS^{n+1}A\subseteq AS^nA.\end{aligned}$$

It follows that, $A\subseteq(S^mA)\cap(AS^nA)$. Hence, $A=(S^mA)\cap(AS^nA)$.

Conversely, assume that $A=(S^mA)\cap(AS^nA)$ for every left (m,n)-bi-quasi hyperideal A of S. Let L be an m-left hyperideal and R be an n-right hyperideal of S. By Theorem 3.4 and Remark 3.5, we have that L and R are left (m,n)-bi-quasi hyperideals of S. Then $R\cap L$ is a left (m,n)-bi-quasi hyperideal of S, By assumption, $R\cap L=[S^m(R\cap L)]\cap[(R\cap L)S^n(R\cap L)]\subseteq(R\cap L)S^n(R\cap L)\subseteq RS^nL$. On the other hand, $RS^nL\subseteq R\cap L$. Thus, $R\cap L=RS^nL$. By Theorem 3.9, S is regular. □

Theorem 3.11. *Let S be a semihyperring and $n\geq m$. Then S is regular if and only if $A=(AS^m)\cap(AS^nA)$ for every right (m,n)-bi-quasi hyperideal A of S.*

Theorem 3.12. *Let S be a semihyperring. Then S is regular if and only if $A\cap L\subseteq AS^kL$ for every m-left hyperideal L and (m,n)-bi-quasi hyperideal A of S, where $k=\max\{m,n\}$.*

Proof. Assume that S is regular. Let L be an m-left hyperideal, A be an (m,n)-bi-quasi hyperideal of S and $k=\max\{m,n\}$. Let $a\in A\cap L$. By Theorem 3.8, we have $a\in aSa\subseteq a(Sa)Sa\subseteq a(SaSa)Sa\subseteq\cdots\subseteq a(\underbrace{Sa\cdots Sa}_{k\text{ terms}})Sa\subseteq aS^{k+1}a\subseteq AS^kL$. Hence, $A\cap L\subseteq AS^kL$. Conversely, let L be an m-left hyperideal and R be an n-right hyperideal of S. By Theorem 3.4, we have that R is an (m,n)-bi-quasi hyperideal of S. By assumption, $R\cap L\subseteq RS^kL$, where $k=\max\{m,n\}$. It is easy to show that $RS^kL\subseteq R\cap L$. Thus, $R\cap L=RS^kL$. By Theorem 3.9, S is regular. □

Theorem 3.13. *Let S be a semihyperring. Then S is regular if and only if $R\cap L\subseteq RS^kA$ for every m-right hyperideal R and (m,n)-bi-quasi hyperideal A of S, where $k=\max\{m,n\}$.*

Theorem 3.14. *Let S be a regular semihyperring and A be a nonempty set of S. Then the following statements hold:*

(i) *if A is an m-left hyperideal of S, then A is a right (m,n)-bi-quasi hyperideal of S;*

(ii) *if A is an m-right hyperideal of S, then A is a left (m,n)-bi-quasi hyperideal of S;*

(*iii*) *if A is an (m,n)-quasi-hyperideal of S, then A is a right (m,n)-bi-quasi hyperideal of S.*

Proof. (i) Assume that A is an m-left hyperideal of S. By Theorem 3.8, we have $(AS^m)\cap(AS^nA)\subseteq AS^nA\subseteq(AS)A\subseteq(ASAS)A\subseteq\cdots\subseteq(\underbrace{AS\cdots AS}_{m\text{ terms}})A\subseteq S^mA\subseteq A$. Hence, A is a right (m,n)-bi-quasi hyperideal of S.

(ii) The proof is similar to (i).

(iii) Assume that A is an (m,n)-quasi-hyperideal of S. By Theorem 3.8, we have

$$AS^m\subseteq AS\subseteq AS(AS)\subseteq AS(ASAS)\subseteq\cdots\subseteq AS(\underbrace{AS\cdots AS}_{n\text{ terms}})\subseteq AS^{n+1}\subseteq AS^n,$$

$$AS^nA\subseteq(AS)A\subseteq(ASAS)A\subseteq\cdots\subseteq(\underbrace{AS\cdots AS}_{m\text{ terms}})A\subseteq S^mA.$$

This implies that $(AS^m)\cap(AS^nA)\subseteq(S^mA)\cap(AS^n)\subseteq A$. Therefore, A is a right (m,n)-bi-quasi hyperideal of S. □

Theorem 3.15. *Let S be a regular semihyperring and A be a nonempty subset of S. Then the following statements are equivalent:*

(i) *A is an (m,n)-bi-quasi hyperideal of S;*

(ii) *A is an m-bi-hyperideal of S;*

(iii) *A is an n-bi-hyperideal of S;*

(iv) *A is an (m,n)-quasi-hyperideal of S.*

Proof. $(i)\Rightarrow(ii)$: Assume that A is an (m,n)-bi-quasi hyperideal of S. Clearly, $AS^mS\subseteq S^mA$. By Theorem 3.8, we have $AS^mA\subseteq ASA\subseteq A(SA)SA\subseteq A(SASA)SA\subseteq\cdots\subseteq A(\underbrace{SA\cdots SA}_{n\text{ terms}})SA\subseteq AS^{n+1}A\subseteq AS^nA$. It follows that $AS^mA\subseteq(S^mA)\cap(AS^nA)\subseteq A$. Therefore, A is an m-bi-hyperideal of S.

$(ii)\Rightarrow(iii)$: Assume that A is an m-bi-hyperideal of S. By Theorem 3.8, we have $AS^nA\subseteq ASA\subseteq A(SA)SA\subseteq A(SASA)SA\subseteq\cdots\subseteq A(\underbrace{SA\cdots SA}_{m\text{ terms}})SA\subseteq AS^{m+1}A\subseteq AS^mA\subseteq A$. Hence, A is an n-bi-hyperideal of S.

$(iii)\Rightarrow(iv)$: Assume that A is an n-bi-hyperideal of S. Let $a\in(S^mA)\cap(AS^n)$. By Theorem 3.8, we have $a\in aSa\subseteq(AS^n)S(S^mA)\subseteq AS^nA\subseteq A$. That is, $(S^mA)\cap(AS^n)\subseteq A$. Hence, A is an (m,n)-quasi-hyperideal of S.

$(iv)\Rightarrow(i)$: Assume that A is an (m,n)-quasi-hyperideal of S. By Theorem 3.6, we have that A is a left (m,n)-bi-quasi-hyperideal of S. By Theorem 3.8, we have

$$AS^m\subseteq AS\subseteq AS(AS)\subseteq AS(ASAS)\subseteq\cdots\subseteq AS(\underbrace{AS\cdots AS}_{n\text{ terms}})\subseteq AS^{n+1}\subseteq AS^n,$$

$$AS^nA\subseteq(AS)A\subseteq(ASAS)A\subseteq\cdots\subseteq(\underbrace{AS\cdots AS}_{m\text{ terms}})A\subseteq S^mA.$$

Thus, $(AS^m) \cap (AS^nA) \subseteq (S^mA) \cap (AS^n) \subseteq A$. Hence, A is a right (m,n)-bi-quasi hyperideal of S. Therefore, A is an (m,n)-bi-quasi hyperideal of S. $\square$

The following example, we show that any (m,n)-quasi-hyperideal in a regular semihyperring is not necessary an m-left hyperideal and an n-right hyperideal.

Example 3.16. Let $S = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \mid a,b,c,d \in \mathbb{N} \cup \{0\} \right\}$. Then $(S,+,\cdot)$ is a semiring under usual the matrix addition and the matrix multiplication. For any $A, B \in S$, we define $A \leq B$ iff $a_{ij} \leq b_{ij}$, where $i,j = \{1,2\}$. Next, we define hyperoperations $\oplus$ and $\odot$ on S by letting $A, B \in S$,

$$A \oplus B = \{X \in S \mid X \leq A + B\},$$
$$A \odot B = \{X \in S \mid X \leq A \cdot B\}.$$

We can show that $(S, \oplus, \odot)$ is a semihyperring. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in S$, where $a,b,c,d \in \mathbb{N} \cup \{0\}$. Choose $X = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \in S$. Then

$$A \cdot X \cdot A = \begin{bmatrix} a^2+ab+ac+bc & ab+b^2+ad+bd \\ ac+ad+c^2+cd & bc+bd+cd+d^2 \end{bmatrix}.$$

Consider $a \leq a^2+ab+ac+bc, b \leq ab+b^2+ad+bd, c \leq ac+ad+c^2+cd$, and $d \leq bc+bd+cd+d^2$. This implies that $A \leq A \cdot X \cdot A$. Thus, $A \in A \odot X \odot A$. Hence, S is a regular semihyperring. Now, let

$$Q = \left\{ \begin{bmatrix} 0 & 0 \\ 0 & x \end{bmatrix} \mid x \in \mathbb{N} \cup \{0\} \right\}.$$

It is easy to show that Q is a subsemihyperring of S. Consider

$$S^3Q = \left\{ \begin{bmatrix} 0 & m \\ 0 & n \end{bmatrix} \mid m,n \in \mathbb{N} \cup \{0\} \right\},$$
$$QS^2 = \left\{ \begin{bmatrix} 0 & 0 \\ k & l \end{bmatrix} \mid k,l \in \mathbb{N} \cup \{0\} \right\}.$$

Hence,

$$S^3 \odot Q = \{X \in S \mid X \leq S^3 \cdot Q\} \nsubseteq Q,$$
$$Q \odot S^2 = \{X \in S \mid X \leq Q \cdot S^2\} \nsubseteq Q.$$

On the other hand, $(S^3 \odot Q) \cap (Q \odot S^2) = \left\{ \begin{bmatrix} 0 & 0 \\ 0 & y \end{bmatrix} \mid y \in \mathbb{N} \cup \{0\} \right\} = Q$. Therefore, Q is a $(3,2)$-quasi-hyperideal of S, but it is not a 3-left hyperideal and is not a 2-right hyperideal of S.

Finally, we conclude the connections between left (resp. right) (m,n)-bi-quasi hyperideals and many types of hyperideals in semihyperrings as the following
figure:

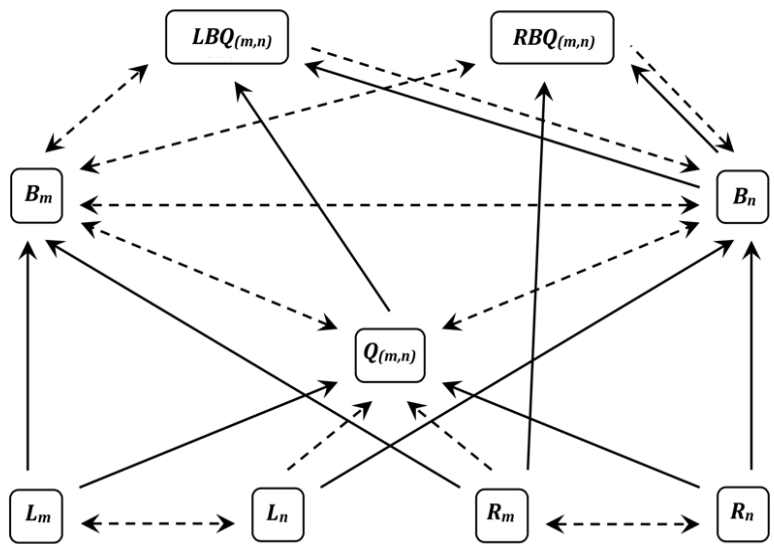

Figure 1: The connections of hyperideals in semihyperrings

where:

$LBQ_{(m,n)}$ denotes the set of all left (m,n)-bi-quasi hyperideals;
$RBQ_{(m,n)}$ denotes the set of all right (m,n)-bi-quasi hyperideals;
B_m denotes the set of all m-bi-hyperideals;
B_n denotes the set of all n-bi-hyperideals;
$Q_{(m,n)}$ denotes the set of all (m,n)-quasi-hyperideals;
L_m denotes the set of all m-left hyperideals;
L_n denotes the set of all n-left hyperideals;
R_m denotes the set of all m-right hyperideals;
R_n denotes the set of all n-right hyperideals;
$\longrightarrow$ denotes the normal implication;
$\dashrightarrow$ denotes the implication with regular semihyperring as an assumption.

References

[1] K. Iséki, Quasiideals in semirings without zero, Proceeding of the Japan Academy 34 (2) (1958) 79–81.

[2] S. Lajos, A. Szász, On the bi-ideals in associative rings, Proceeding of the Japan Academy 46 (6) (1970) 505–507.

[3] R. Chinram, A note on (m,n)-quasi-ideals in semirings, International Journal of Pure and Applied Mathematical 49 (1) (2008) 45–52.

[4] M. Munir, A. Shafiq, A generalization of bi ideals in semirings, Bulletin of the International Mathematical Virtual Institute 8 (2018) 123–133.

[5] M.M.K. Rao, Left bi-quasi ideals of semirings, Bulletin of the International Mathematical Virtual Institute 8 (2018) 45–53.

[6] F. Marty, Sur une generalization de la notion de group, Proceeding of 8^{th} Congress des Mathematician Scandinave (1934) 45–49.

[7] P. Corsini, Prolegomena of hypergroup theory (2^{ed}), Aviani Editore, Italy, 1993.

[8] P. Corsini, V. Leoreanu, Applications of hyperstructure theory, Kluwer Academic Publishers, Netherlands, 2003.

[9] B. Davvaz, V. Leoreanu-Fotea, Hyperring theory and applications, International Academic Press, USA, 2007.

[10] T. Vougiouklis, Hyperstructures and their representations, Hadronic Press, USA, 1994.

[11] T. Vougiouklis, On some representation of hypergroups, Ann. Sci. Univ. Clermont-Ferrand II Math. 26 (1990) 21–29.

[12] S. Omidi, B. Davvaz, Contribution to study special kinds of hyperideals in ordered semihyperrings, Journal of Taibah University for Science 11 (2017) 1083–1094.

[13] W. Nakkhasen, B. Pibaljommee, On m-bi-hyperideals in semihyperrings, Songklanakarin Journal of Science and Technology 41 (6) (2019) 1241–1247.

[14] B. Davvaz, S. Omidi, Basic notions and properties of ordered semihyperrings, Categories and General Algebraic Structures with Applications 4 (1) (2016) 43–62.

[15] X. Haung, Y. Yin, J. Zhan, Characterizations of semihyperrings by their $(\epsilon_\gamma, \epsilon_\gamma \vee q_\delta)$-fuzzy hyperideals, J. Appl. Math. 2013 (2013) 1–13.

(Received 23 May 2019)
(Accepted 24 December 2019)