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On Ordered Fuzzy Points in Ordered Ternary Semigroups

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Abstract : A ternary semigroup is a nonempty set T together with a ternary operation $[\ \ ] : T \times T \times T \to T$, written as $(a, b, c) \mapsto [abc]$ satisfying the associative law $[[abc]uv] = [a[bcu]v] = [ab[cuv]]$ for all $a, b, c, u, v \in T$. A partially ordered ternary semigroup T is called an ordered ternary semigroup if for all $a, b, x, y \in T, a \leq b \Rightarrow [axy] \leq [bxy], [xay] \leq [xby]$, and $[xya] \leq [xyb]$. The concept of ordered ternary semigroup is a natural generalization of ordered semigroups and ternary semigroups. Let $\underline{T}$ be the set of all ordered fuzzy points in an ordered ternary semigroup T. In this paper, we investigate some properties of ordered fuzzy points of ordered ternary semigroups.

Keywords : fuzzy subsets; strongly convex fuzzy subsets; ordered fuzzy points; ordered ternary semigroups.
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1 Introduction

The notion of a ternary algebraic system was introduced by Lehmer [1]. He investigated certain ternary algebraic systems called triplexes which turn out to be ternary groups. The concept of ordered ternary semigroup is a natural generalization of ordered semigroups and ternary semigroups. A number of mathematicians have studied the properties of ordered ternary semigroups. Iampan discussed ordered ideal extensions in ordered ternary semigroups in [2]. In [3], Chinram and Saelee studied fuzzy ideals and fuzzy filters of ordered ternary semigroups. In [4], Changphas initiated the study of m-systems and n-systems in ordered ternary semigroups. Yaqoob, Abdullah, Rehman, and Naeem [5] gave some interesting properties of roughness and fuzziness of ordered ternary semigroups. In [6], Daddi and Pawar studied the properties of ordered quasi-ideals and ordered bi-ideals in an ordered ternary semigroup. In [7], Lekkoksung and Jampachon characterized right weakly regular ordered ternary semigroups in terms of the properties of fuzzy ideals and fuzzy quasi-ideals. Bashir and Du characterized weakly regular ordered ternary semigroups by fuzzy ideals, fuzzy quasi-ideals, and fuzzy bi-ideals in [8].

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In 1965, the fundamental concept of a fuzzy set was introduced by Zadeh in [9]. Since then, it opened up applications in various fields. Fuzzy, rough and rough fuzzy ideals and bi-ideals in ternary semigroups were studied in [10] and [11]. In [12] and [13], Pu and Liu first defined the fuzzy points. Kim [14] considered the semigroup $\underline{S}$ of all fuzzy points of a semigroup S, and discussed the relation between fuzzy interior ideals of S and the subsets of $\underline{S}$. In [15], the relation between some ideals of a semigroup S and the subsets of $\underline{S}$ was discussed by Hamouda. Then he considered the ternary semigroup $\underline{T}$ of all fuzzy points of a ternary semigroup T and discussed the relation between some fuzzy ideals of a ternary semigroup T and the subsets of $\underline{T}$ in [16]. Tang and Xie studied ordered fuzzy points of ordered semigroups in [17] and [18]. Recently, Solano, Suebsung and Chinram [19] extended the concept of fuzzy points to n-ary semigroups. They gave the relation between i-ideals A of an n-ary semigroup S and the subsets $\underline{C_A}$ of the n-ary semigroup $\underline{S}$ of the fuzzy points of an n-ary semigroup S, and ideals A of S and the subsets $\underline{C_A}$ of $\underline{S}$ will be shown.

In this paper, we study fuzzy points in ordered ternary semigroups and give some remarkable properties.

2 Preliminaries

A ternary semigroup is a nonempty set T together with a ternary operation $[\ \ ] : T\times T\times T \to T$, written as $(a, b, c) \mapsto [abc]$ satisfying the associative law $[[abc]uv] = [a[bcu]v] = [ab[cuv]]$ for all $a, b, c, u, v \in T$. A partially ordered ternary semigroup T is called an ordered ternary semigroup if for all $a, b, x, y \in T, a \leq b \Rightarrow [axy] \leq [bxy], [xay] \leq [xby]$, and $[xya] \leq [xyb]$.

Example 2.1. (1) Let $(T, [\ \ ])$ be a ternary semigroup. Then $(T, [\ \ ], id_T)$ is an ordered ternary semigroup where $id_T := \{(a, a) \mid a \in T\}$ is an identity relation on T. This follows that every ternary semigroup can be considered as an ordered ternary semigroup by using a partial order id_T.

(2) Define $[\ \ ] : \mathbb{Z}^- \times \mathbb{Z}^- \times \mathbb{Z}^- \to \mathbb{Z}^-$ by $[abc] = a \cdot b \cdot c$ for all $a, b, c \in \mathbb{Z}^-$ where $\cdot$ is the usual multiplication. Then $(\mathbb{Z}^-, [\ \ ], \leq)$ is an ordered ternary semigroup.

Let T be an ordered ternary semigroup. For a subset H of T, let $(H] := \{t \in T \mid t \leq h$ for some $h \in H\}$. For $H = \{a\}$, we write $(a]$ instead of $(\{a\}]$. For all subsets A, B, C of T, $[ABC] := \{[abc] \mid a \in A, b \in B, c \in C\}$.

Proposition 2.2. *Let A, B and C be subsets of an ordered ternary semigroup T. The following statements are true.*

(1) $A \subseteq (A]$.

(2) *If* $A \subseteq B$, *then* $(A] \subseteq (B]$.

(3) $[(A](B](C]] \subseteq ([ABC]]$.

A fuzzy subset of T is a function from T into the closed interval $[0, 1]$. The ordered ternary semigroup T itself is a fuzzy subset of T such that $T(x) = 1$ for all $x \in T$, denoted also by T. For any two fuzzy subsets f and g of T,

$f \cap g$ is a fuzzy subset of S defined by $(f \cap g)(x) = \min\{f(x), g(x)\} = f(x) \wedge g(x)$ for all $x \in T$.

$f \cup g$ is a fuzzy subset of S defined by $(f \cup g)(x) = \max\{f(x), g(x)\} = f(x) \vee g(x)$ for all $x \in T$.

$f \subseteq g$ if $f(x) \leq g(x)$ for all $x \in T$.

The *characteristic function* of a subset A of T is defined by

$$C_A(x) = \begin{cases} 1 & \text{if } x \in A, \\ 0 & \text{if } x \notin A. \end{cases}$$

for all $x \in T$.

Proposition 2.3. *Let A and B be nonempty subsets of an ordered ternary semigroup T. Then $A \subseteq B$ if and only if $C_A \subseteq C_B$.*

Let $F(T)$ be the set of all fuzzy subsets in an ordered ternary semigroup T. For each $x \in T$, we define $A_x := \{(a_1, a_2, a_3) \in T \times T \times T \mid x \leq [a_1a_2a_3]\}$. For each $f_1, f_2, f_3 \in F(T)$, the ternary product of f_1, f_2 and f_3 is a fuzzy subset $f_1 \circ f_2 \circ f_3$ defined as follows:

$$(f_1 \circ f_2 \circ f_3)(x) = \begin{cases} \bigvee\limits_{(a_1a_2a_3) \in A_x} \{f_1(a_1) \wedge f_2(a_2) \wedge f_3(a_3)\} & \text{if } A_x \neq \emptyset, \\ 0 & \text{otherwise,} \end{cases}$$

for all $x \in T$. Then $F(T)$ is a ternary semigroup with the ternary product $\circ$.

3 Main Results

Let T be an ordered ternary semigroup. For $\alpha \in (0, 1]$ and $x \in T$, an *ordered fuzzy point* x_α of T is a fuzzy subset in T defined by

$$x_\alpha(y) = \begin{cases} \alpha & \text{if } y \in (x], \\ 0 & \text{otherwise,} \end{cases}$$

for all $y \in T$. Clearly, $x_\alpha \circ y_\beta \circ z_\gamma = [xyz]_{\alpha \wedge \beta \wedge \gamma}$ for all fuzzy points $x_\alpha, y_\beta, z_\gamma$ of T.

Let f be a fuzzy subset of T. We define a fuzzy subset $(f]$ by

$$(f](x) = \bigvee_{x \leq y} f(y)$$

for all $x \in T$.

Proposition 3.1. *Let f, g and h be fuzzy subsets of an ordered ternary semigroup T. The following statements are true.*

(1) $f \subseteq (f]$.

(2) If $f \subseteq g$, *then* $(f] \subseteq (g]$.

(3) $(f] \circ (g] \circ (h] \subseteq (f \circ g \circ h]$.

Proof. (1) Let $x \in T$. Since $x \leq x, (f](x) = \bigvee\limits_{x \leq y} f(y) \geq f(x)$. So $f \subseteq (f]$.

(2) Assume that $f \subseteq g$. Then $f(x) \leq g(x)$ for all $x \in T$. So

$$(f](x) = \bigvee_{x \leq y} f(y) \leq \bigvee_{x \leq y} g(y) = (g](x).$$

This implies that $(f] \subseteq (g]$.

(3) Let $x \in T$. If $A_x = \emptyset$, then $(f] \circ (g] \circ (h](x) = 0 \leq (f \circ g \circ h](x)$. If $A_x \neq \emptyset$, then

$$\begin{aligned}
(f] \circ (g] \circ (h](x) &= \bigvee_{(a_1a_2a_3)\in A_x} \{(f](a_1) \wedge (g](a_2) \wedge (h](a_3)\} \\
&= \bigvee_{(a_1a_2a_3)\in A_x} \{\bigvee_{a_1\leq b_1} f(b_1) \wedge \bigvee_{a_2\leq b_2} g(b_2) \wedge \bigvee_{a_3\leq b_3} h(b_3)\} \\
&= \bigvee_{(a_1a_2a_3)\in A_x} \{\bigvee_{a_1\leq b_1, a_2\leq b_2, a_3\leq b_3} f(b_1) \wedge g(b_2) \wedge h(b_3)\} \\
&\leq \bigvee_{(a_1a_2a_3)\in A_x} \{\bigvee_{[a_1a_2a_3]\leq[b_1b_2b_3]} f(b_1) \wedge g(b_2) \wedge h(b_3)\} \\
&\leq \bigvee_{x\leq[a_1a_2a_3]} \{f \circ g \circ h([a_1a_2a_3])\} = (f \circ g \circ h](x).
\end{aligned}$$

□

A fuzzy subset of S is called *strongly convex* if $f = (f]$.

Proposition 3.2. *Let T be an ordered ternary semigroup and f a fuzzy subset of T. Then f is a strongly convex fuzzy subset of T if and only if for all $x, y \in T$, $x \leq y \Rightarrow f(y) \leq f(x)$.*

Proof. Let $x, y \in T$ such that $x \leq y$. Then, by hypothesis,

$$f(x) = (f](x) = \bigvee_{x\leq w} f(w) \geq f(y).$$

Conversely, for any $x \in T$, since $(f](x) = \bigvee_{x\leq y} f(y)$, by hypothesis, we have $f(y) \leq f(x)$ for all $x \leq y$, $(f](x) \leq f(x)$. So, $(f] \subseteq f$. And by Proposition 3.1 (1), therefore $f = (f]$. □

For an ordered fuzzy point x_α and a fuzzy point f, if $x_\alpha \subseteq f$, we say that x_α is contained in f and use the notation $x_\alpha \in f$.

Theorem 3.3. *Let T be an ordered ternary semigroup and x_α an ordered fuzzy point of T.*

(1) If f is a strongly convex fuzzy subset of T, then $f = \bigcup_{x_\alpha\in f} x_\alpha$.

(2) If f is a strongly convex fuzzy subset of T, then $x_\alpha \in f$ if and only if $f(x) \geq \alpha$.

Proof. (1) Assume x_α is an ordered fuzzy point such that $x_\alpha \in f$. Since $x_\alpha(y) \leq f(y)$, we get

$$(\bigcup_{x_\alpha\in f} x_\alpha)(y) = \bigvee_{x_\alpha\in f} x_\alpha(y) \leq \bigvee_{x_\alpha\in f} f(y) = f(y)$$

for all $y \in T$. Hence, $\bigcup_{x_\alpha\in f} x_\alpha \in f$. Conversely, for each $x \in T$, choose $f(x) = \alpha$. Let $y \in T$.

Case 1: If $y \notin (x]$, then $x_\alpha(y) = 0 \leq f(y)$.

Case 2: If $y \in (x]$, then $y \leq x$. Since f is strongly convex and $y \leq x$, $f(y) \geq f(x) = \alpha \geq x_\alpha(y)$.

In both cases, we have $x_\alpha \in f$ and $f(x) = x_\alpha(x) \leq (\bigcup_{x_\alpha\in f} x_\alpha)(x)$. Thus, $f = \bigcup_{x_\alpha\in f} x_\alpha$.

(2) Assume that $x_\alpha \in f$. By (1), $f(x) \geq x_\alpha(x) = \alpha$. Conversely, assume that $f(x) \geq \alpha$. Let $y \in T$.

Case 1 : If $y \notin (x]$, then $x_\alpha(y) = 0 \leq f(y)$.

Case 2 : If $y \in (x]$, then $x_\alpha(y) = \alpha$ and $y \leq x$. By Proposition 3.2, $f(y) \geq x \geq \alpha = x_\alpha(y)$,

In both cases, this means that $x_\alpha \in f$. □

Let $\underline{T}$ be the set of all ordered fuzzy points of an ordered ternary semigroup T. We define an ordered relation $\leq_1$ on $\underline{T}$ as follows: for all $x_\lambda, y_\beta \in \underline{T}$,

$$x_\lambda \leq_1 y_\beta \text{ if } x \leq y \text{ and } \lambda \leq \beta.$$

Theorem 3.4. *Let T be an ordered ternary semigroup. Then $\underline{T}$ is an ordered ternary semigroup with the ternary product $\circ$ and an order $\leq_1$.*

Proof. Let $(T, [\ ], \leq)$ be an ordered ternary semigroup. We obtain $((a_1)_{\alpha_1} \circ (a_2)_{\alpha_2} \circ (a_3)_{\alpha_3}) \circ (a_4)_{\alpha_4} \circ (a_5)_{\alpha_5}$ $= (a_1)_{\alpha_1} \circ ((a_2)_{\alpha_2} \circ (a_3)_{\alpha_3} \circ (a_4)_{\alpha_4}) \circ (a_5)_{\alpha_5} = (a_1)_{\alpha_1} \circ (a_2)_{\alpha_2} \circ ((a_3)_{\alpha_3} \circ (a_4)_{\alpha_4} \circ (a_5)_{\alpha_5})$ for any ordered fuzzy points $(a_1)_{\alpha_1}, (a_2)_{\alpha_2}, (a_3)_{\alpha_3}, (a_4)_{\alpha_4}, (a_5)_{\alpha_5} \in \underline{T}$. Hence, $(\underline{T}, \circ)$ is a ternary semigroup. Moreover, $(\underline{T}, \circ, \leq_1)$ is an ordered ternary semigroup. Since "$\leq$" is an ordered relation on T, then "$\leq_1$" is an ordered relation on $\underline{T}$. In the remainder, let $(a_1)_{\alpha_1}, (a_2)_{\alpha_2}, x_\lambda, y_\beta \in \underline{T}$ such that $(a_1)_{\alpha_1} \leq_1 (a_2)_{\alpha_2}$. So, $a_1 \leq a_2$ and $\alpha_1 \leq \alpha_2$. Then $[a_1xy] \leq [a_2xy]$ and $\alpha_1 \wedge \lambda \wedge \beta \leq \alpha_2 \wedge \lambda \wedge \beta$. This implies that $[a_1xy]_{\alpha_1 \wedge \lambda \wedge \beta} \leq_1 [a_2xy]_{\alpha_2 \wedge \lambda \wedge \beta}$, that is, $(a_1)_{\alpha_1} \circ x_\lambda \circ y_\beta \leq_1 (a_2)_{\alpha_2} \circ x_\lambda \circ y_\beta$. Similarly, $x_\lambda \circ (a_1)_{\alpha_1} \circ y_\beta \leq_1 x_\lambda \circ (a_2)_{\alpha_2} \circ y_\beta$ and $x_\lambda \circ y_\beta \circ (a_1)_{\alpha_1} \leq_1 x_\lambda \circ y_\beta \circ (a_2)_{\alpha_2}$. Thus, $(\underline{T}, \circ, \leq_1)$ is an ordered ternary semigroup. □

Let $\underline{T}$ be the set of all ordered fuzzy points in an ordered ternary semigroup T. Thus, $\underline{T}$ is an ordered ternary subsemigroup of $F(T)$. For any $f \in F(T)$, $\underline{f}$ denotes the set of all ordered fuzzy points contained in f, that is,

$$\underline{f} = \{x_\alpha \in \underline{T} \mid x_\alpha \in f\}.$$

For any $f_1, f_2, f_3 \in F(T)$, we define the ternary product of $\underline{f_1}, \underline{f_2}$ and $\underline{f_3}$ as follows:

$$\underline{f_1} \circ \underline{f_2} \circ \underline{f_3} = \{(a_1)_{\alpha_1} \circ (a_2)_{\alpha_2} \circ (a_3)_{\alpha_3} \mid (a_i)_{\alpha_i} \in f_i\}.$$

Theorem 3.5. *Let f_1, f_2, f_3 be strongly convex fuzzy subsets in an ordered ternary semigroup T. Then*

(1) $\underline{f_1 \cup f_2 \cup f_3} = \underline{f_1} \cup \underline{f_2} \cup \underline{f_3}$.

(2) $\underline{f_1 \cap f_2 \cap f_3} = \underline{f_1} \cap \underline{f_2} \cap \underline{f_3}$.

Proof. (1) Let $x_\alpha \in \underline{f_1 \cup f_2 \cup f_3}$. By Theorem 3.3 (2), $(f_1 \cup f_2 \cup f_3)(x) \geq \alpha$. This implies $f_1(x) \geq \alpha$ or $f_2(x) \geq \alpha$ or $f_3(x) \geq \alpha$. Hence, $x_\alpha \in \underline{f_1} \cup \underline{f_2} \cup \underline{f_3}$. Therefore $\underline{f_1 \cup f_2 \cup f_3} \subseteq \underline{f_1} \cup \underline{f_2} \cup \underline{f_3}$. Conversely, let $x_\alpha \in \underline{f_1} \cup \underline{f_2} \cup \underline{f_3}$. By Theorem 3.3 (2), $f_1(x) \geq \alpha$ or $f_2(x) \geq \alpha$ or $f_3(x) \geq \alpha$. So, $(f_1 \cup f_2 \cup f_3)(x) \geq \alpha$. Then this implies $x_\alpha \in \underline{f_1 \cup f_2 \cup f_3}$. Therefore, $\underline{f_1} \cup \underline{f_2} \cup \underline{f_3} \subseteq \underline{f_1 \cup f_2 \cup f_3}$. Thus, $\underline{f_1 \cup f_2 \cup f_3} = \underline{f_1} \cup \underline{f_2} \cup \underline{f_3}$.

(2) Let $x_\alpha \in \underline{f_1 \cap f_2 \cap f_3}$. By Theorem 3.3 (2), $(f_1 \cap f_2 \cap f_3)(x) \geq \alpha$. This implies $f_1(x) \geq \alpha$ and $f_2(x) \geq \alpha$ and $f_3(x) \geq \alpha$. Hence, $x_\alpha \in \underline{f_1} \cap \underline{f_2} \cap \underline{f_3}$. Therefore $\underline{f_1 \cap f_2 \cap f_3} \in \underline{f_1} \cap \underline{f_2} \cap \underline{f_3}$. Conversely, let $x_\alpha \in \underline{f_1} \cap \underline{f_2} \cap \underline{f_3}$. By Theorem 3.3 (2), $f_1(x) \geq \alpha$ and $f_2(x) \geq \alpha$ and $f_3(x) \geq \alpha$. So, $(f_1 \cap f_2 \cap f_3)(x) \geq \alpha$. This implies $x_\alpha \in \underline{f_1 \cap f_2 \cap f_3}$. Therefore, $\underline{f_1} \cap \underline{f_2} \cap \underline{f_3} \subseteq \underline{f_1 \cap f_2 \cap f_3}$. Thus, $\underline{f_1 \cap f_2 \cap f_3} = \underline{f_1} \cap \underline{f_2} \cap \underline{f_3}$. □

Theorem 3.6. *Let f_1, f_2, f_3 be strongly convex fuzzy subsets in an ordered ternary semigroup T. Then* $\underline{f_1} \circ \underline{f_2} \circ \underline{f_3} \subseteq \underline{f_1 \circ f_2 \circ f_3}$.

Proof. Let $x_\alpha \in \underline{f_1} \circ \underline{f_2} \circ \underline{f_3}$. Then $x_\alpha = (a_1)_{\alpha_1} \circ (a_2)_{\alpha_2} \circ (a_3)_{\alpha_3}$ for some $(a_i)_{\alpha_i} \in \underline{f_i}$. By Theorem 3.3 (2), this implies that $x_\alpha = [a_1a_2a_3]_{\alpha_1 \wedge \alpha_2 \wedge \alpha_3}$ and $(f_i)(a_i) \geq \alpha_i$ for all i. So, $x = [a_1a_2a_3]$ and $\alpha = \alpha_1 \wedge \alpha_2 \wedge \alpha_3$. Therefore, $(f_i)(a_i) \geq \alpha_i \geq \alpha$ for all i. Hence, $(f_1 \circ f_2 \circ f_3)(x) \geq \alpha$. Therefore, by Theorem 3.3 (2), $x_\alpha \in \underline{f_1 \circ f_2 \circ f_3}$. □

Theorem 3.7. *Let A be a nonempty subset of an ordered ternary semigroup T. Then $(A] \subseteq A$ if and only if C_A is strongly convex.*

Proof. Assume that $(A] \subseteq A$. Let $x.y \in T$ such that $x \leq y$. If $x \in A$, then $C_A(x) = 1 \geq C_A(y)$. If $x \notin A$, this implies that $y \notin A$, then $C_A(x) \geq 0 = C_A(y)$. By Proposition 3.2, C_A is strongly convex. Conversely, assume that C_A is strongly convex. Let $x \in (A]$. Then there exists $y \in A$ such that $x \leq y$. By Proposition 3.2, $C_A(x) \geq C_A(y) = 1$. This implies that $C_A(x) = 1$, hence $x \in A$. □

Theorem 3.8. *Let A be a nonempty subset of an ordered ternary semigroup T such that $(A] \subseteq A$. Then $x_\alpha \in \underline{C_A}$ if and only if $x \in A$.*

Proof. Assume that $x_\alpha \in \underline{C_A}$. By Theorem 3.3 (2), we have $C_A(x) \geq \alpha$. Hence, $C_A(x) = 1$. This implies $x \in A$. Conversely, assume that $x \in A$. Then $C_A(x) = 1 \geq \alpha$ for all $\alpha \in (0, 1]$. By Theorem 3.3 (2), this implies that $x_\alpha \in \underline{C_A}$. □

Theorem 3.9. *Let A and B be nonempty subsets an ordered ternary semigroup T such that $(A] \subseteq A$ and $(B] \subseteq B$, then $A \subseteq B$ if and only if $\underline{C_A} \subseteq \underline{C_B}$.*

Proof. Assume that $A \subseteq B$. Let $x_\alpha \in \underline{C_A}$. By Theorem 3.8, $x \in A$. Since $A \subseteq B$, $x \in B$. By Theorem 3.8, $x_\alpha \in \underline{C_B}$. Thus, $\underline{C_A} \subseteq \underline{C_B}$. Conversely, assume that $\underline{C_A} \subseteq \underline{C_B}$. Let $x \in A$. By Theorem 3.8, $x_\alpha \in \underline{C_A}$. Since $\underline{C_A} \subseteq \underline{C_B}$, $x_\alpha \in \underline{C_B}$. By Theorem 3.8, $x \in B$. Thus, $A \subseteq B$. □

Theorem 3.10. *For any strongly convex fuzzy subsets f and g of an ordered ternary semigroup T, then $f \subseteq g$ if and only if $\underline{f} \subseteq \underline{g}$.*

Proof. Assume that $f \subseteq g$. Thus, $f(x) \leq g(x)$ for all $x \in T$. Let $x_\alpha \in \underline{f}$. Then $x_\alpha \in f$. By Theorem 3.3 (2), we have $g(x) \geq f(x) \geq \alpha$. Hence, $x_\alpha \in \underline{g}$ by Theorem 3.3 (2). Conversely, assume that $\underline{f} \subseteq \underline{g}$. Let $x \in T$. If $f(x) = 0$, then $f(x) \leq g(x)$. Assume $f(x) \neq 0$ and let $\alpha = f(x)$. Then $x_\alpha \in \underline{f}$. So $x_\alpha \in \underline{g}$. Hence, $g(x) \geq \alpha = f(x)$ by Theorem 3.3 (2). So $f \subseteq g$. □

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