Thai Journal of Mathematics
Volume 17 (2019) Number 3 : 767–774
http://thaijmath.in.cmu.ac.th
ISSN 1686-0209

Some Matrices in Term of Perrin and Padovan Sequences

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Abstract : Many authors have studied the matrix sequences which are considered in the different types of numbers such as Fibonacci, Lucas, Pell, Padovan, and Perrin. In this paper, the new matrices, which have similar properties to Padovan matrix, are presented. Consequently, we not only provide some matrix formula for Perrin and Padovan sequences but also consider the relation of Perrin and Padovan sequences.

Keywords : Padovan sequence; Perrin sequence; Padovan matrix; matrix formula.

2010 Mathematics Subject Classification : 11B99; 11C20.

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1 Introduction

Many authors [1-8] have studied the matrix sequences which are considered in the different types of numbers such as Fibonacci, Lucas [2,9], Pell, Padovan, Perrin [3-11], (s,t)-Pell and (s,t)-Pell-Lucas [12].

The Padovan sequence (P_n) is the sequences defined by

$$P_n = \begin{cases} 0, & n = 0, 1 \\ 1, & n = 2 \\ P_{n-2} + P_{n-3}, & n \geq 3 \end{cases} \tag{1.1}$$

The first few Padovan sequence are; $0, 0, 1, 0, 1, 1, 1, 2, 2, 3, 4, 5, 7, 9, 12, 16, ...$

The Perrin sequence (R_n) is the sequences defined by

$$R_n = \begin{cases} 3, & n = 0 \\ 0, & n = 1 \\ 2, & n = 2 \\ R_{n-2} + R_{n-3}, & n \geq 3 \end{cases} \tag{1.2}$$

The first few Perrin sequence are; $3, 0, 2, 3, 2, 5, 5, 7, 10, 12, 17, 22, 29, 39, 51, 68, ...$

In 2013, K. Sokhuma [5] studied Padovan Q-matrix such that all entries in Q^n, n^{th} power of Q-matrix, are Padovan number. Moreover, in the same year the matrix formula for Padovan and Perrin numbers,

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}^n \begin{bmatrix} 0 & 3 \\ 0 & 0 \\ 1 & 2 \end{bmatrix}$$

was proposed [6]. That is, the matrix product is a matrix 3×2 that whose entries are Padovan and Perrin numbers. In 2015, P. Seenukul et al. [7] studied and found some new matrices of 3×3 which have similar properties to Padovan Q-Matrix

In this paper, the new matrices, which have similar properties to Padovan Q-matrix, is presented. We also provide some matrix formula for Perrin and Padovan sequence. We are also specially interested in the relation of Perrin and Padovan sequence in these matrix sequences.

The rest of this paper is organized as follows. In the next section, we briefly review some basic fact about Padovan Q-matrix properties and provide the new 3×3 matrices which have similar properties to Padovan Q-matrix. In addition, some new relation of Padovan number are investigated. Section 3 contains some matrix formula for Perrin and Padovan sequence which is described in Section 2. Similarly, the relation of Perrin sequence and Padovan sequence is considered in this section.

2 Preliminaries

Throughout this paper, let N be the set of all positive integer and E be 3×2 matrix defined by

$$E=\begin{bmatrix} 0 & 3 \\ 0 & 0 \\ 1 & 2 \end{bmatrix}.$$

Theorem 2.1. ([7]) *Let P_n be the n^{th} Padovan sequences. For any $n \in N$, we have the following;*

1. $If \;\; A_1=\begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}, then \; A_1^n=\begin{bmatrix} P_{n+2} & P_{n+1} & P_{n+3} \\ P_n & P_{n-1} & P_{n+1} \\ P_{n+1} & P_n & P_{n+2} \end{bmatrix}.$

2. $If \;\; A_2=\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}, then \; A_2^n=\begin{bmatrix} P_{n+2} & P_n & P_{n+1} \\ P_{n+1} & P_{n-1} & P_n \\ P_{n+3} & P_{n+1} & P_{n+2} \end{bmatrix}.$

3. $If \;\; A_3=\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, then \; A_3^n=\begin{bmatrix} P_{n-1} & P_n & P_{n+1} \\ P_{n+1} & P_{n+2} & P_{n+3} \\ P_n & P_{n+1} & P_{n+2} \end{bmatrix}.$

4. $If \;\; A_4=\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, then \; A_4^n=\begin{bmatrix} P_{n+2} & P_{n+3} & P_{n+1} \\ P_{n+1} & P_{n+2} & P_n \\ P_n & P_{n+1} & P_{n-1} \end{bmatrix}.$

5. $If \;\; A_5=\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}, then \; A_5^n=\begin{bmatrix} P_{n+2} & P_{n+1} & P_n \\ P_{n+3} & P_{n+2} & P_{n+1} \\ P_{n+1} & P_n & P_{n-1} \end{bmatrix}.$

Proposition 2.2. ([7]) *For all $m,n \in N$ such that $m<n$. We have the following relations;*

1. $P_n = P_m P_{n-m-1} + P_{m+1} P_{n-m+1} + P_{m+2} P_{n-m}$
2. $P_n = P_{m-1} P_{n-m} + P_m P_{n-m+2} + P_{m+1} P_{n-m+1}$.

Theorem 2.3. ([8]) *Let P_n be the n^{th} Padovan sequences. For any $n \in N$, we have the following;*

1. $If \;\; Q_1=\begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}, then \; Q_1^n=\begin{bmatrix} P_{2n-1} & P_{2n} & P_{2n+1} \\ P_{2n+1} & P_{2n+2} & P_{2n+3} \\ P_{2n} & P_{2n+1} & P_{2n+2} \end{bmatrix}.$

2. $If \;\; Q_2=\begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}, then \; Q_2^n=\begin{bmatrix} P_{2n+2} & P_{2n+1} & P_{2n+3} \\ P_{2n} & P_{2n-1} & P_{2n+1} \\ P_{2n+1} & P_{2n} & P_{2n+2} \end{bmatrix}.$

3. $If \;\; Q_3=\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}, then \; Q_3^n=\begin{bmatrix} P_{2n+2} & P_{2n+1} & P_{2n} \\ P_{2n+3} & P_{2n+2} & P_{2n+1} \\ P_{2n+1} & P_{2n} & P_{2n-1} \end{bmatrix}.$

4. $If\ \ Q_4 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}, then\ Q_4^n = \begin{bmatrix} P_{2n-1} & P_{2n+1} & P_{2n} \\ P_{2n} & P_{2n+2} & P_{2n+1} \\ P_{2n+1} & P_{2n+3} & P_{2n+2} \end{bmatrix}.$

5. $If\ \ Q_5 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}, then\ Q_5^n = \begin{bmatrix} P_{2n+2} & P_{2n} & P_{2n+1} \\ P_{2n+1} & P_{2n-1} & P_{2n} \\ P_{2n+3} & P_{2n+1} & P_{2n+2} \end{bmatrix}.$

6. $If\ \ Q_6 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}, then\ Q_6^n = \begin{bmatrix} P_{2n+2} & P_{2n+3} & P_{2n+1} \\ P_{2n+1} & P_{2n+2} & P_{2n} \\ P_{2n} & P_{2n+1} & P_{2n-1} \end{bmatrix}.$

Proposition 2.4. ([8]) *For all $m, n \in N$ such that $m < n$. We have the following relations;*

1. $P_{2n} = P_{2m-1}P_{2(n-m)} + P_{2m}P_{2(n-m)+2} + P_{2m+1}P_{2(n-m)+1}$
2. $P_{2n} = P_{2m}P_{2(n-m)-1} + P_{2m+1}P_{2(n-m)+1} + P_{2m+2}P_{2(n-m)}$.
3. $P_{2n+1} = P_{2m-1}P_{2(n-m)+1} + P_{2m}P_{2(n-m)+3} + P_{2m+1}P_{2(n-m)+2}$
4. $P_{2n+1} = P_{2m}P_{2(n-m)} + P_{2m+1}P_{2(n-m)+2} + P_{2m+2}P_{2(n-m)+1}$.

Theorem 2.5. ([6]) *Let P_n and R_n be the n^{th} Padovan and Perrin number, respectively.*

$$If\ \ B = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}, then\ B^nE = \begin{bmatrix} P_n & R_n \\ P_{n+1} & R_{n+1} \\ P_{n+2} & R_{n+2} \end{bmatrix}$$

for all $n \in N$.

Proposition 2.6. ([6]) *For all $m, n \in N$ such that $3 \leq m < n$. We have the following relations;*

1. $P_n = P_{m-1}P_{n-m} + P_{m+1}P_{n-m+1} + P_mP_{n-m+2}$
2. $R_n = P_{m-1}R_{n-m} + P_{m+1}R_{n-m+1} + P_mR_{n-m+2}$

Let us add some observation from the proposition above. If we choose $n = 1,000$ and $m = 498$, then we obtain that $P_{1,000} = P_{497}P_{502} + P_{499}P_{503} + P_{498}P_{504}$ and $R_{1,000} = P_{497}R_{502} + P_{499}R_{503} + P_{498}R_{504}$.

Thus the values of $P_{1,000}$ is obtained by using the value of $P_{497}, P_{498}, P_{499}, P_{502}$, P_{503} and P_{504} (not P_{997}, P_{998}). For $R_{1,000}$ we also use the value of $P_{497}, P_{498}, P_{499}$, R_{502}, R_{503} and R_{504} (not R_{997}, R_{998}).

To this end, let us point out some our observations. In [5,7] the authors considered the 3×3 matrices, the entries are 0 and 1, such that five entries are 0 and four entries are 1. Similarly in [8] the authors also considered 3×3 matrices, the entries are 0 and 1, but five entries are 1 and four entries are 0. In the results of Theorem 2.1, Theorem 2.3 and Theorem 2.5, we can see that $A_2 = A_1^T, A_5 = A_4^T, A_3 = B^T, Q_4 = Q_1^T, Q_5 = Q_2^T$ and $Q_6 = Q_3^T$.

3 Main Results

In this section we not only focus on some matrix formula for Perrin and Padovan sequences. But also specially interested in the relation of Perrin and Padovan sequences.

Theorem 3.1. *Let P_n and R_n be the n^{th} Padovan and Perrin number, respectively.*

$$If\ \ Q = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}, then\ Q^n E = \begin{bmatrix} P_{2n+3} & R_{2n+3} \\ P_{2n+1} & R_{2n+1} \\ P_{2n+2} & R_{2n+2} \end{bmatrix}, n \in N. \tag{3.1}$$

Proof. The proof will be done by principle of mathematical induction. For $n = 1$, it is easy to see that

$$Q^1 E = \begin{bmatrix} 1 & 5 \\ 0 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} P_5 & R_5 \\ P_3 & R_3 \\ P_4 & R_4 \end{bmatrix}.$$

That is, we obtain that

$$\begin{bmatrix} P_5 & R_5 \\ P_3 & R_3 \\ P_4 & R_4 \end{bmatrix} = \begin{bmatrix} P_{2(1)+3} & R_{2(1)+3} \\ P_{2(1)+1} & R_{2(1)+1} \\ P_{2(1)+2} & R_{2(1)+2} \end{bmatrix}.$$

Assuming the equation (3.1) holds for all $n = k$, we have that

$$Q^k E = \begin{bmatrix} P_{2k+3} & R_{2k+3} \\ P_{2k+1} & R_{2k+1} \\ P_{2k+2} & R_{2k+2} \end{bmatrix}.$$

Now, let us show that the case also holds for $n = k + 1$.

$$\begin{aligned} Q^{k+1}E = (QQ^k)E = Q(Q^k E) &= \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} P_{2k+3} & R_{2k+3} \\ P_{2k+1} & R_{2k+1} \\ P_{2k+2} & R_{2k+2} \end{bmatrix} \\ &= \begin{bmatrix} P_{2k+3} + P_{2k+2} & R_{2k+3} + P_{2k+2} \\ P_{2k+3} & R_{2k+3} \\ P_{2k+1} + P_{2k+2} & R_{2k+1} + P_{2k+2} \end{bmatrix} \\ &= \begin{bmatrix} P_{2k+5} & R_{2k+5} \\ P_{2k+3} & R_{2k+3} \\ P_{2k+4} & R_{2k+4} \end{bmatrix} \\ &= \begin{bmatrix} P_{2(k+1)+3} & R_{2(k+1)+3} \\ P_{2(k+1)+1} & R_{2(k+1)+1} \\ P_{2(k+1)+2} & R_{2(k+1)+2} \end{bmatrix}. \end{aligned}$$

This complete the proof. ∎

Corollary 3.2. *For all $m, n \in N$ such that $m < n$. We have the following relations;*

1. $P_{2n+1} = P_{2m-1}P_{2(n-m)+1} + P_{2m}P_{2(n-m)+3} + P_{2m+1}P_{2(n-m)+2}$
2. $R_{2n+1} = P_{2m-1}R_{2(n-m)+1} + P_{2m}R_{2(n-m)+3} + P_{2m+1}R_{2(n-m)+2}$
3. $P_{2n+2} = P_{2m}P_{2(n-m)+1} + P_{2m+1}P_{2(n-m)+3} + P_{2m+2}P_{2(n-m)+2}$
4. $R_{2n+2} = P_{2m}R_{2(n-m)+1} + P_{2m+1}R_{2(n-m)+3} + P_{2m+2}R_{2(n-m)+2}$.

Proof. Let $Q = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$. By the laws of exponent, we have $Q^nE = Q^m(Q^{n-m}E)$. From Theorem 2.3(2) and Theorem 3.1. It follows that,

$$\begin{bmatrix} P_{2n+3} & R_{2n+3} \\ P_{2n+1} & R_{2n+1} \\ P_{2n+2} & R_{2n+2} \end{bmatrix} = \begin{bmatrix} P_{2m+2} & P_{2m+1} & P_{2m+3} \\ P_{2m} & P_{2m-1} & P_{2m+1} \\ P_{2m+1} & P_{2m} & P_{2m+2} \end{bmatrix} \begin{bmatrix} P_{2(n-m)+3} & R_{2(n-m)+3} \\ P_{2(n-m)+1} & R_{2(n-m)+1} \\ P_{2(n-m)+2} & R_{2(n-m)+2} \end{bmatrix}.$$

By multiplying the right-hand side matrices and comparing the 2^{nd} rows and 3^{rd} rows entries, we obtain;

$P_{2n+1} = P_{2m-1}P_{2(n-m)+1} + P_{2m}P_{2(n-m)+3} + P_{2m+1}P_{2(n-m)+2}$
$R_{2n+1} = P_{2m-1}R_{2(n-m)+1} + P_{2m}R_{2(n-m)+3} + P_{2m+1}R_{2(n-m)+2}$
$P_{2n+2} = P_{2m}P_{2(n-m)+1} + P_{2m+1}P_{2(n-m)+3} + P_{2m+2}P_{2(n-m)+2}$
and
$R_{2n+2} = P_{2m}R_{2(n-m)+1} + P_{2m+1}R_{2(n-m)+3} + P_{2m+2}R_{2(n-m)+2}$.
This complete the proof. □

Let us point out the useful fact from this corollary. If we choose $n = 499$ and $m = 250$, then we only need $P_{499} - P_{502}$ and $R_{499} - R_{501}$ to obtain $P_{1,000} = P_{500}P_{499} + P_{501}P_{501} + P_{502}P_{500}$ and $R_{1,000} = P_{500}R_{499} + P_{501}R_{501} + P_{502}R_{500}$, which is better than the results obtained from Proposition 2.6.

The proof of Theorems 3.3, 3.5 and Corollaries 3.4, 3.6 are similar to the proof of Theorem 3.1 and Corollary 3.2.

Theorem 3.3. *Let P_n and R_n be the n^{th} Padovan and Perrin number, respectively.*

$$If \;\; Q = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}, then \; Q^nE = \begin{bmatrix} P_{2n} & R_{2n} \\ P_{2n+1} & R_{2n+1} \\ P_{2n+2} & R_{2n+2} \end{bmatrix}, n \in N. \qquad (3.2)$$

Corollary 3.4. *For all $m, n \in N$ such that $m < n$. We have the following relations;*

1. $P_{2n} = P_{2m-1}P_{2(n-m)} + P_{2m}P_{2(n-m)+2} + P_{2m+1}P_{2(n-m)+1}$
2. $R_{2n} = P_{2m-1}R_{2(n-m)} + P_{2m}R_{2(n-m)+2} + P_{2m+1}R_{2(n-m)+1}$
3. $P_{2n+1} = P_{2m}P_{2(n-m)} + P_{2m+1}P_{2(n-m)+2} + P_{2m+2}P_{2(n-m)+1}$
4. $R_{2n+1} = P_{2m}R_{2(n-m)} + P_{2m+1}R_{2(n-m)+2} + P_{2m+2}R_{2(n-m)+1}$.

Theorem 3.5. *Let P_n and R_n be the n^{th} Padovan and Perrin number, respectively.*

$$If\ \ Q = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}, then\ Q^n E = \begin{bmatrix} P_{n+3} & R_{n+3} \\ P_{n+1} & R_{n+1} \\ P_{n+2} & R_{n+2} \end{bmatrix}, n \in N. \tag{3.3}$$

Corollary 3.6. *For all $m, n \in N$ such that $m < n$. We have the following relations;*

1. $P_{n+1} = P_{m-1}P_{n-m+1} + P_m P_{n-m+3} + P_{m+1}P_{n-m+2}$
2. $R_{n+1} = P_{m-1}R_{n-m+1} + P_m R_{n-m+3} + P_{m+1}R_{n-m+2}$
3. $P_{n+2} = P_m P_{n-m+1} + P_{m+1}P_{n-m+3} + P_{m+2}P_{n-m+2}$
4. $R_{n+2} = P_m R_{n-m+1} + P_{m+1}R_{n-m+3} + P_{m+2}R_{n-m+2}$

The following corollary, we use the determinant of matrix in Theorem 2.1 and Theorem 2.3 to give a relation of Padovan sequence.

Corollary 3.7. *Let P_n be the n^{th} Padovan number and $n \in N$, we have the following;*

1. $P_{n-1}P_{n+2}^2 + P_n^2 P_{n+3} + P_{n+1}^3 - P_{n-1}P_{n+1}P_{n+3} - 2P_n P_{n+1}P_{n+2} = 1$
2. $P_{2n-1}P_{2n+2}^2 + P_{2n}^2 P_{2n+3} + P_{2n+1}^3 - P_{2n-1}P_{2n+1}P_{2n+3} - 2P_{2n}P_{2n+1}P_{2n+2} = 1.$

Proof. According to Theorem 2.1(1),

if $A = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$, then we have the following $A^n = \begin{bmatrix} P_{n+2} & P_{n+1} & P_{n+3} \\ P_n & P_{n-1} & P_{n+1} \\ P_{n+1} & P_n & P_{n+2} \end{bmatrix}$.

Clearly, we see that $det(A) = 1$ and $det(A^n) = [det(A)]^n = 1$. Moreover, we observe that $det(A^n) = P_{n-1}P_{n+2}^2 + P_n^2 P_{n+3} + P_{n+1}^3 - P_{n-1}P_{n+1}P_{n+3} - 2P_n P_{n+1}P_{n+2}$, and we conclude that

$$P_{n-1}P_{n+2}^2 + P_n^2 P_{n+3} + P_{n+1}^3 - P_{n-1}P_{n+1}P_{n+3} - 2P_n P_{n+1}P_{n+2} = 1.$$

Similarly, we can use the fact of Theorem 2.3 and determinant of matrix to conclude that,

$$P_{2n-1}P_{2n+2}^2 + P_{2n}^2 P_{2n+3} + P_{2n+1}^3 - P_{2n-1}P_{2n+1}P_{2n+3} - 2P_{2n}P_{2n+1}P_{2n+2} = 1.$$

The proof is now complete. □

4 Conclusion

In this work, we found not only the new three matrices formula for Perrin and Padovan sequences, but also specially interested in some relation of Perrin and Padovan sequences. Moreover, we conjecture that this concept could be extended to $n \times n$ matrices, where $n \geq 4$.

Acknowledgement : The authors would like to thank the referees for useful comments and suggestions on the manuscript.

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(Received 10 May 2018)
(Accepted 13 September 2019)