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Generalization of Suzuki's Method on Partial Metric Spaces

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Abstract : We establish common fixed point theorems for two mappings satisfying nonlinear contractive conditions in partial metric space. The presented work generalize the Suzuki's method [1] for multivalued contractive in partial metric space. Our study also generalize some well- known results in the literature.

Keywords : multivalued mapping; common fixed point; Suzuki's method; partial metric space.
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1 Introduction

The notion of partial metric is one of the most useful and interesting generalizations of the classical concept of metric. The partial metric spaces were introduced in 1994 by Matthews [2]. Based on this notion, Matthews [2, 3], Oltra and Valero [4], Ilic et al. [5, 6], Kadelburg et al. [7], Di Bari et al. [8], Hemant Kumar Nashine et al. [9] obtained some very interesting fixed point theorems for mappings satisfying different contractive conditions.

On the other hand, in order to generalize the well-known Banach contraction theorem in complete metric space many authors have introduced various type of contraction. In 2008, Suzuki introduced a new method [1] and then this method was extended by some authors [10–13].
Very recently this method extended to the partial metric space in [14]

The purpose of this work is to provide a new condition for two multivalued mappings which guarantees the existence of common fixed point.

Our results generalize some old results see for example [13,14]. In this way, we

consider the set $\mathcal{R}$ of continuous function $g : [0, 1) \to [0, 1)$ satisfying the following properties:
(a) $g(1, 1, 1, 2, 0) = g(1, 1, 1, 0, 2) = h \in (0, 1)$,
(b) g is sub-homogeneous, that is, $g(\alpha x_1, \alpha x_2, \alpha x_3, \alpha x_4, \alpha x_5) < \alpha g(x_1, x_2, x_3, x_4 x_5)$ for all $\alpha \geq 0$ and all $(x_1, x_2, x_3, x_4, x_5) \in [0, 1)^5$,
(c) If $x_i, y_i \in [0, 1)$ and $x_i < y_i$ for $i = 1, 2, 3, 4$, then $g(x_1, x_2, x_3, x_4, 0) < g(y_1, y_2, y_3, y_4, 0)$ and $g(x_1, x_2, x_3, 0, x_4) < g(y_1, y_2, y_3, 0, y_4)$.

We appeal the following result in the sequel.

Proposition 1.1. [15] *If $g \in \mathcal{R}$ and $u, v \in [0, 1)$ are such that*

$$u \leq \max\{g(v, v, u, v+u, 0), g(v, v, u, 0, v+u), g(v, u, v, v+u, 0), g(v, u, v, 0, v+u)\},$$

then $u \leq hv$.

2 Preliminaries

Definition 2.1. [2] A partial metric on a nonempty set X is a mapping $p : X \times X \to \mathbb{R}^+$ such that for all $x, y, z \in X$ the following conditions are satisfied:
(i) $x = y \Leftrightarrow p(x, x) = p(x, y) = p(y, y)$,
(ii) $p(x, x) \leq p(x, y)$,
(iii) $p(x, y) = p(y, x)$,
(iv) $p(x, y) \leq p(x, z) + p(z, y) - p(z, z)$.

A partial metric space is a pair (X, p) such that X is a nonempty set and p is a partial metric on X. Each partial metric p on X generates a T_0 topology τ_p on X which has as a base, the family of open p-balls $\{B_p(x, \epsilon); x \in X, \epsilon > 0\}$, where

$$B_p(x, \epsilon) = \{y \in X : p(x, y) < p(x, x) + \epsilon\},$$

for all $x \in X$ and $\epsilon > 0$.

If p is a partial metric on X, then the mapping $d_p : X \times X \to \mathbb{R}^+$ given by

$$d_p(x, y) = 2p(x, y) - p(x, x) - p(y, y),$$

is a metric on X.

Definition 2.2. [3,16] Let (X, p) be a partial metric space. Then a sequence $\{x_n\}$ in X is called
(i) convergent if there exists a point $x \in X$ such that $p(x, x) = \lim_{n\to\infty} p(x_n, x)$,
(ii) Cauchy sequence if there exists (and is finite) $\lim_{n,m\to\infty} p(x_n, x_m)$.

Definition 2.3. [3,16] A partial metric space (X, p) is said to be complete if every Cauchy sequence $\{x_n\}$ in X converges, with respect to τ_p, to a point $x \in X$ such that $p(x, x) = \lim_{n,m\to\infty} p(x_n, x_m)$.

Lemma 2.4. [3,16] *Let (X, p) be a partial metric space. Then*
(i) $\{x_n\}$ is a Cauchy sequence in (X, p) if and only if it is a Cauchy sequence in the metric space (X, d_p),
(ii) (X, p) is complete if and only if the metric space (X, d_p) is complete. Furthermore, $\lim_{n\to\infty} d_p(x_n, x) = 0$ if and only if $p(x, x) = \lim_{n\to\infty} p(x_n, x) = \lim_{n,m\to\infty} p(x_n, x_m)$.

Let $CB^p(X)$ be a family of all nonempty, closed and bounded subsets of the partial metric space (Xp). Note that closedness is taken from (X, τ_p) (τ_p is the topology induced by p) and boundedness is given as follows: A, is a bounded subset in (X, p) if there exist $x_0 \in X$ and $M \geq 0$ such that for all $a \in A$, we have $a \in B_p(x_0, M)$, that is, $p(x_0, a) < p(a, a) + M$.
For $A, B \in CB^p(X)$ and $x \in X$, we defined

$$p(x, A) = \inf\{p(x, y) : y \in A\},$$

$$\delta_p(A, B) = \sup\{p(a, B) : a \in A\},$$

$$\delta_p(B, A) = \sup\{p(A, b) : b \in B\},$$

and

$$H_p(A, B) = \max\{\delta_p(A, B), \delta_p(B, A)\}.$$

It is immediate to check that $p(x, A) = 0$ implies that $d_p(x, A) = 0$, where $d_p(x, A) = \inf\{d_p(x, a) : a \in A\}$.

Remark 2.5. [17] *Let (X, p) be a partial metric space and A be any nonempty set in (X, p), then $a \in A$ if and only if $p(a, A) = p(a, a)$, where A denotes the closure of A with respect to the partial metric p. Note that A is closed in (X, p) if and only if $A = \overline{A}$.*

Proposition 2.6. [18] *Let (X, p) be a partial metric space. For any $A, B, C \in CB^p(X)$, we have the following:*
(i) $\delta_p(A, A) = \sup\{p(a, a) : a \in A\}$,
(ii) $\delta_p(A, A) \leq \delta_p(A, B)$,
(iii) $\delta_p(A, B) = 0 \Leftrightarrow A \subseteq B$,
(iv) $\delta_p(A, B) \leq \delta_p(A, C) + \delta_p(C, B) - \inf_{c\in C} p(c, c)$.

Proposition 2.7. [18] *Let (X, p) be a partial metric space. For all $A, B, C \in CB^p(X)$, we have*
(h1) $H_p(A, A) \leq H_p(A, B)$,
(h2) $H_p(A, B) = H_p(B, A)$,
(h3) $H_p(A, B) \leq H_p(A, C) + H_p(C, B) - \inf_{c\in C} p(c, c)$,
(h4) $H_p(A, B) = 0 \Leftrightarrow A = B$.

The mapping $H_p : CB^p(X) \times CB^p(X) \to [0, \infty)$, is called the partial Hausdorff metric induced by p. It is easy to show that any Hausdorff metric is a partial Hausdorff metric. The converse is not true see Example 2.6 in [18].

3 Main Results

Now, we give the following result about common fixed points of two multivalued mappings.

Theorem 3.1. *Let X denote a complete partial metric space and $T, S : X \to CB^p(X)$ two multivalued mappings. Suppose that there exits $\alpha \in (0,1)$ and $g \in \mathcal{R}$ such that $\alpha(h+1) < 1$ and $\alpha p(x,Tx) \leq p(x,y)$ or $\alpha p(y,Sy) \leq p(x,y)$ implies*

$$H_p(Tx,Sy) \leq g(p(x,y), p(x,Tx), p(y,Sy), p(x,Sy) - p(x,x), p(y,Tx) - p(y,y)),$$

for all $x, y \in X$. Then $F(T) = F(S)$ and $F(T)$ is nonempty.

Proof. Let x_0 be a arbitrary point in X and $1 > r > h$. choose $x_1 \in Tx_0$ such that $\alpha p(x_0, Tx_0) \leq p(x_0, x_1)$. Then, we have

$$\begin{aligned} p(x_1, Sx_1) &\leq H_p(Tx_0, Sx_1) \\ &\leq g(p(x_0,x_1), p(x_0,Tx_0), p(x_1,Sx_1), p(x_0,Sx_1) - p(x_0,x_0), \\ &\quad p(x_1,Tx_0) - p(x_1,x_1)) \\ &\leq g(p(x_0,x_1), p(x_0,x_1), p(x_1,Sx_1), p(x_0,x_1) + p(x_1,Sx_1), 0). \end{aligned}$$

By using Proposition 1.1, we have

$$p(x_1, Sx_1) \leq hp(x_0, x_1) < rp(x_0, x_1).$$

Now we choose a number μ such that $\inf_{y \in Sx_1} p(x_1, y) = p(x_1, Sx_1) < \mu < rp(x_0, x_1)$. Thus, there exists $x_2 \in Sx_1$ such that $p(x_1, x_2) < \mu < rp(x_0, x_1)$. Since $\alpha p(x_1, Sx_1) < p(x_1, x_2)$, we get,

$$\begin{aligned} p(x_2, Tx_2) &\leq H_p(Tx_2, Sx_1) \\ &\leq g(p(x_1,x_2), p(x_2,Tx_2), p(x_1,Sx_1), p(x_2,Sx_1) - p(x_2,x_2), \\ &\quad p(x_1,Tx_2) - p(x_1,x_1)) \\ &\leq g(p(x_1,x_2), p(x_2,Tx_2), p(x_1,x_2), 0, p(x_1,x_2) + p(x_2,Tx_2)). \end{aligned}$$

By using Proposition 1.1, we have

$$p(x_2, Tx_2) \leq hp(x_1, x_2) < rp(x_1, x_2).$$

Now by using a similar method, we can find $x_3 \in Tx_2$ such that

$$p(x_2, x_3) \leq rp(x_1, x_2) \leq r^2 p(x_0, x_1),$$

Continuing this process, we can find a sequence $\{x_n\}$ in X such that $x_{2n-1} \in Tx_{2n-2}$, $x_{2n} \in Sx_{2n-1}$, and we have,
(i) $p(x_n, x_{n+1}) < r^n p(x_0, x_1)$,
(ii) $p(x_{2n}, Tx_{2n}) \leq hp(x_{2n-1}, x_{2n})$ and $p(x_{2n-1}, Sx_{2n-1}) \leq hp(x_{2n-2}, x_{2n-1})$.
If $x_n = x_m$ for some $m \geq 1$, then T and S have a common fixed point.

Suppose that $x_n \neq x_{n+1}$ for all $n \geq 1$. By using (i), we show that $\{x_n\}$ is a cauchy sequence. Obviously we have,

$$\begin{aligned} p(x_n, x_{n+m}) &\leq p(x_n, x_{n+1}) + p(x_{n+1}, x_{n+2}) + ... + p(x_{n+m-1}, x_{n+m}) \\ &\leq r^n p(x_0, x_1) + r^{n+1} p(x_0, x_1) + ... + r^{n+m-1} p(x_0, x_1) \\ &\leq (r^n + r^{n+1} + ... + r^{n+m-1}) p(x_0, x_1) \\ &\frac{r^n}{1 - r_n} p(x_0, x_1) \to 0. \end{aligned}$$

By the definition of d_p, we get,

$$d_p(x_n, x_{n+m}) \leq 2p(x_n, x_{n+m}) \to 0,$$

as $n \to \infty$, which implies that $\{x_n\}$ is a Cauchy sequence in (X, d_p). Since (X, p) is complete, hence (X, d_p) is complete, so we have $\lim_{n\to\infty} d_p(x_n, x) = 0$, for some $x \in X$. Now by Lemma 2.4 we get

$$p(x, x) = \lim_{n\to\infty} p(x_n, x) = \lim_{n,m\to\infty} p(x_n, x_m) = 0.$$

Now we claim that for each $n \geq 1$ on of the relation $\alpha p(x_{2n}, Tx_{2n}) \leq p(x_{2n}, x)$ and $\alpha p(x_{2n+1}, Sx_{2n+1}) \leq p(x_{2n+1}, x)$ hold.
If $\alpha p(x_{2n}, Tx_{2n}) > p(x_{2n}, x)$ and $\alpha p(x_{2n+1}, Sx_{2n+1}) > p(x_{2n+1}, x)$ for some $n \geq 1$, then we obtain

$$\begin{aligned} p(x_{2n}, x_{2n+1}) &\leq p(x_{2n}, x) + p(x, x_{2n+1}) - p(x, x) \\ &\leq p(x_{2n}, x) + p(x, x_{2n+1}) \\ &< \alpha p(x_{2n}, Tx_{2n}) + \alpha p(x_{2n+1}, Sx_{2n+1}) \\ &\leq \alpha p(x_{2n}, x_{2n+1}) + \alpha h p(x_{2n}, x_{2n+1}). \end{aligned}$$

Thus, $\alpha(1 + h) > 1$, which is a contradiction. Therefore our claim is proved. Now by using the assumption for each $n \geq 1$ either

$$\begin{aligned} H_p(Tx_{2n}, Sx) \leq g(&p(x_{2n}, x), p(x_{2n}, Tx_{2n}), p(x, Sx), p(x_{2n}, Sx) - p(x_{2n}, x_{2n}) \\ &, p(x, Tx_{2n}) - p(x, x)), \end{aligned}$$

or

$$\begin{aligned} H_p(Tx_{2n+1}, Sx) \leq g(&p(x_{2n+1}, x), p(x_{2n+1}, Tx_{2n+1}), p(x, Sx), p(x, Tx_{2n+1}) - p(x, x) \\ &, p(x_{2n+1}, Sx) - p(x_{2n+1}, x_{2n+1})), \end{aligned}$$

hold, Therefore, we have one of the following cases:
(i) In first case we have

$$\begin{aligned} p(x_{2n+1}, Sx) \leq H_p(Tx_{2n}, Sx) \leq g(&p(x_{2n}, x), p(x_{2n}, Tx_{2n}), p(x, Sx), \\ &p(x, Tx_{2n}) - p(x, x), p(x_{2n}, Sx) - p(x_{2n}, x_{2n})), \end{aligned}$$

for all $n \in \mathbb{N}$. Therefore we have

$$\begin{aligned}
p(x,Sx) &\leq p(x,x_{2n+1}) + p(x_{2n+1},Sx) - p(x_{2n+1},x_{2n+1}) \\
&\leq p(x,x_{2n+1}) + g(p(x_{2n},x),p(x_{2n},Tx_{2n}),p(x,Sx),p(x,Tx_{2n}) - p(x,x), \\
&\quad p(x_{2n},Sx) - p(x_{2n},x_{2n})) \\
&\leq p(x,x_{2n+1}) + g(p(x_{2n},x),p(x_{2n},x_{2n+1}),p(x,Sx),p(x,x_{2n+1}) - p(x,x), \\
&\quad p(x_{2n},x) + p(x,Sx) - p(x_{2n},x_{2n})),
\end{aligned}$$

for all $n \in \mathbb{N}$. Since g is continuous letting $n \to \infty$ we obtain

$$\begin{aligned}
p(x,Sx) &\leq p(x,x) + g(p(x,x),p(x,x),p(x,Sx),p(x,x) - p(x,x), \\
&\qquad p(x,x) + p(x,Sx) - p(x,x)) \\
&= g(0,0,p(x,Sx),0,p(x,Sx)).
\end{aligned}$$

Now by using Proposition 1.1 we have $p(x,Sx) = 0$ and so $x \in Sx$.
(ii) In the second case we have,

$$\begin{aligned}
p(Tx,x_{2n+2}) \leq H_p(Tx,Sx_{2n+1}) &\leq g(p(x,x_{2n+1}),p(x,Tx),p(x_{2n+1},Sx_{2n+1}) \\
&,p(x_{2n+1},Tx) - p(x_{2n+1},x_{2n+1}),p(x,Sx_{2n+1}) - p(x,x)),
\end{aligned}$$

for all $n \in \mathbb{N}$. Therefore,

$$\begin{aligned}
p(x,Tx) &\leq p(x,x_{2n+2}) + p(x_{2n+2},Tx) - p(x_{2n+2},x_{2n+2}) \\
&\leq p(x,x_{2n+2}) + g(p(x,x_{2n+1}),p(x,Tx),p(x_{2n+1},Sx_{2n+1}),p(x_{2n+1},Tx) \\
&- p(x_{2n+1},x_{2n+1}),p(x,Sx_{2n+1}) - p(x,x)) \\
&\leq p(x,x_{2n+2}) + g(p(x,x_{2n+1}),p(x,Tx),p(x_{2n+1},x_{2n+2}),p(x_{2n+1},x) \\
&+ p(x,Tx) - p(x,x),p(x,x_{2n+2}) - p(x,x)),
\end{aligned}$$

for all $n \in \mathbb{N}$. Since g is continuous letting $n \to \infty$ we obtain

$$p(x,Tx) \leq g(0,p(x,Tx),0,p(x,Tx),0).$$

Now by using Proposition 1.1 we have $p(x,Tx) = 0$ and so $x \in Tx$. Therefore in all cases we have $F(T)$ is non-empty.
Next we show that $F(T) = F(S)$. Let $x \in Tx$, then $\alpha d(x,Tx) \leq d(x,x)$, therefore we have,

$$\begin{aligned}
p(x,Sx) &\leq H_p(Tx,Sx) \\
&\leq g(p(x,x),p(x,Tx),p(x,Sx),p(x,Sx) - p(x,x),p(x,Tx) - p(x,x)) \\
&\leq g(p(x,x),p(x,x),p(x,Sx),p(x,x) + p(x,Sx),0).
\end{aligned}$$

Now by using Proposition 1.1 we have $p(x,Sx) \leq hp(x,Sx)$. This implies that $p(x,Sx) = 0$ and so $x \in Sx$. Thus $F(T) \subseteq F(S)$. Similarly we can show that $F(S) \subseteq F(T)$. This completes the proof. □

The following result is a consequence of Theorem 3.1.

Theorem 3.2. *Let X denote a complete partial metric space and $T : X \to CB^p(X)$ be a multivalued mapping. Suppose that there exits $\alpha \in (0, 1)$ and $g \in \mathcal{R}$ with $h = g(1, 1, 1, 2, 0)$ such that $\alpha(h + 1) \leq 1$ and $\alpha p(x, Tx) \leq p(x, y)$ implies*

$$H_p(Tx, Ty) \leq g(p(x, y), p(x, Tx), p(y, Ty), p(x, Ty) - p(x, x), p(y, Tx) - p(y, y)),$$

for all $x, y \in X$. Then T has a fixed point.

Theorem 3.3. [14] *Define a strictly decreasing function θ from $[0, 1)$ onto $(\frac{1}{2}, 1]$ by $\theta(r) = \frac{1}{1+r}$. Let (X, p) be a complete partial metric space and $T : X \to CB^p(X)$ be a multivalued mapping. Assume that there exists $r \in [0, 1)$ such that $\theta(r)p(x, Tx) \leq p(x, y)$ implies $H_p(Tx, Ty) \leq rp(x, y)$ for all $x, y \in X$. Then T has a fixed point.*

Proof. Define $g \in \mathcal{R}$ by $g(x_1, x_2, x_3, x_4, x_5) = rx_1$. put $\alpha = \theta(r)$. Since $h = r$ and $\alpha(1 + h) \leq 1$, by using Theorem 3.2, T has a fixed point. □

Theorem 3.4. *Let X be a complete partial metric space and $T : X \to CB^p(X)$ be a multivalued mapping. Assume that there exist $a, b, c \in [0, 1)$ such that $a+b+c < 1$ and $\frac{1-b-c}{1+a}p(x, Tx) \leq p(x, y)$ implies $H_p(Tx, Ty) \leq ap(x, y)+bp(x, Tx)+cp(y, Ty)$ for all $x, y \in X$. Then T has a fixed point.*

Proof. Define $g \in \mathcal{R}$ by $g(x_1, x_2, x_3, x_4, x_5) = ax_1 + bx_2 + cx_3$. Put $\alpha = \frac{1-b-c}{1+a}$. Since $h = a+b+c$ and $\alpha(1+h) \leq 1$, by using Theorem 3.2, T has a fixed point. □

Theorem 3.5. *Let X be a complete metric space and $T : X \to CB^P(X)$ be a multivalued mapping. Assume that there exists $r \in [2^{\frac{-1}{1}}, 1)$ such that $\theta(r)p(x, Tx) \leq p(x, y)$ implies $H(Tx, Ty) \leq r \max\{p(x, y), p(x, Tx), p(y, Ty)\}$ for all $x, y \in X$. Then T has a fixed point.*

Proof. Define $g \in \mathcal{R}$ by $g(x_1, x_2, x_3, x_4, x_5) = r \max\{x_1, x_2, x_3\}$. Put $\alpha = \theta(r)$. Since $h = r$ and $\alpha(1 + h) \leq 1$, by using Theorem 3.2, T has a fixed point. □

Theorem 3.6. *Let X be a complete partial metric space and $T : X \to CB^p(X)$ be a multivalued mapping. Assume that there exist $\beta, \gamma \in [0, 1)$ such that $\frac{1}{2\beta+\gamma+1}p(x, Tx) \leq p(x, y)$ implies $H_p(Tx, Ty) \leq \gamma p(x, y) + \beta p(x, Tx) + \beta p(y, Ty)$ for all $x, y \in X$. Then T has a fixed point.*

Proof. Define $g \in \mathcal{R}$ by $g(x_1, x_2, x_3, x_4, x_5) = \gamma x_1 + \beta x_2 + \beta x_3$. put $\alpha = \frac{1}{2\beta+\gamma+1}$. Since $h = 2\beta+1$ and $\alpha(1+h) \leq 1$, by using Theorem 3.2, T has a fixed point. □

Theorem 3.7. *Let X be a complete partial metric space and $T : X \to CB^p(X)$ be a multivalued mapping. Assume that there exist $r \in [0, 1)$, and $L \in [0, 1)$ such that $\frac{1}{1+r+L}p(x, Tx) \leq p(x, y)$ implies*

$$H_p(Tx, Ty) \leq rp(x, y) + L \min\{p(x, Ty) - p(x, x), p(y, Tx) - p(y, y)\},$$

for all $x, y \in X$. Then T has a fixed point.

Proof. Define $g \in \mathcal{R}$ by $g(x_1, x_2, x_3, x_4, x_5) = rx_1 + L\min\{x_4, x_5\}$. Put $\alpha = \frac{1}{1+r+L}$. Since $h = r$ and $\alpha(1 + h) \leq 1$, by using Theorem 3.2, T has a fixed point. □

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