Thai Journal of Mathematics
Volume 17 (2019) Number 1 : 253–261
http://thaijmath.in.cmu.ac.th
ISSN 1686-0209

Fixed Points of $\alpha\eta{-}\xi\theta$-Expansive Mappings

Saurabh Manro[†,1] **and Peyman Salimi**[‡]

[†]School of Mathematics and Computer Applications
Thapar University, Patiala, Punjab, India
e-mail : sauravmanro@hotmail.com

[‡]Young Researchers and Elite Club, Rasht Branch
Islamic Azad University, Rasht, Iran
e-mail : salimipeyman@gmail.com

Abstract : The aim of this paper is to introduce a new class of expansive mappings - $\alpha\eta - \xi\theta$-expansive mappings and establish fixed point theorems for such mappings in a complete metric space. The results presented in this paper substantially generalize and extend several comparable results in the existing literature. As an application, we prove new fixed point results for graphic weak $\xi\theta$-expansive mappings.

Keywords : metric spaces; $\alpha\eta - \xi\theta$-expansive mapping; fixed point.
2010 Mathematics Subject Classification : 47H10; 54H25.

1 Introduction

Fixed point theory is one of the most powerful and fruitful tools in nonlinear analysis. The Banach contraction principle is the simplest and one of the most versatile elementary results in fixed point theory. In 1984, Wang et al. [1] presented expansion mappings in metric spaces and proved some fixed point theorems using these mappings. Rhoades [2] generalized the results of Wang et al. [1] for a pair of expansive mappings. Recently, Samet et al. [3] introduced $\alpha - \psi-$contractive type mappings via admissible mappings and established fixed point theorems for such mappings in complete metric spaces. The paper written by Samet et al. [1] is due to the inspiration derived from a large number of papers like [4–6] that

[1]Corresponding author.

employ the use of such mappings directly or indirectly in various spaces and their applications.

In this paper, we introduce the new class of $\alpha\eta - \xi\theta$-expansive mappings in a metric space and prove fixed point results for this new class of expansive mappings. Our results substantially generalize and extend several comparable results in the existing literature. As an application, we prove new fixed point results for graphic weak $\xi\theta$-expansive mappings.

2 Preliminaries

Let Θ denote the set of all functions $\theta : R^+ \times R^+ \times R^+ \times R^+ \to R^+$ satisfying:
(Θ_1) θ is continuous;
(Θ_2) $\theta(t_1, t_2, t_3, t_4) = 0$ iff $t_1.t_2.t_3.t_4 = 0$.

Example 2.1. Let $\theta(t_1, t_2, t_3, t_4) = t_1.t_2.t_3.t_4$. Clearly, $\theta \in \Theta$.

Other examples of θ appear in paper [4].

Definition 2.2. Let χ denote the set of all functions $\xi : [0, \infty) \to [0, \infty)$ which satisfy the following properties:
(i) ξ is non-decreasing;
(ii) $\Sigma_{n=1}^{\infty}\xi^n(a) < \infty$ for each $a > 0$.

We now introduce the following:

Definition 2.3. Let (X, d) be a metric space and f be a self-mapping on X. Then,
$(\star)$ we say that f is an $\alpha\eta - \xi\theta$-expansive mapping if

$$\eta(fx, x) \leq \alpha(x, y) \Rightarrow \xi(d(fx, fy)) + \theta(d(x, fx), d(y, fy), d(x, fy), d(y, fx)) \geq d(x, y)$$

where $\theta \in \Theta$, $\xi \in \chi$ and $\alpha, \eta : X \times X \to [0, \infty)$ are two functions.
$(\star\star)$ We say, f is an $\alpha - \xi\theta$-expansive mapping if,

$$\alpha(x, y) \geq 1 \Rightarrow \xi(d(fx, fy)) + \theta(d(x, fx), d(y, fy), d(x, fy), d(y, fx)) \geq d(x, y)$$

where $\theta \in \Theta$, $\xi \in \chi$ and $\alpha : X \times X \to [0, \infty)$ is a function.
$(\star\star\star)$ We say, f is an $\alpha - \xi\theta$-expansive mapping if,

$$\xi(d(fx, fy)) + \theta(d(x, fx), d(y, fy), d(x, fy), d(y, fx)) \geq \alpha(x, y)d(x, y)$$

where $\theta \in \Theta$, $\xi \in \chi$ and $\alpha : X \times X \to [0, \infty)$ is a function.

3 Main Results

The main result of the paper is the following theorem.

Theorem 3.1. *Let (X, d) be a complete metric space and $f : X \to X$ be a bijective $\alpha\eta - \xi\theta$-expansive mapping satisfying the following conditions:*
(i) f^{-1} is $\alpha-$admissible with respect to η :
(ii) there exist $x_0 \in X$ such that

$$\alpha(x_0, f^{-1}x_0) \geq \eta(x_0, f^{-1}x_0);$$

(iii) f is continuous.
Then f has a fixed point.

Proof. Let $x_0 \in X$ such that $\alpha(x_0, f^{-1}x_0) \geq \eta(x_0, f^{-1}x_0)$. Define the sequence $\{x_n\}$ in X by $x_n = fx_{n+1}$ for all n = 1,2,3,....

If $x_n = x_{n+1}$ for some n, then x_n is a fixed point of f.

Assume that $x_n \neq x_{n-1}$ for all n. It is given that

$$\alpha(x_0, f^{-1}x_0) \geq \eta(x_0, f^{-1}x_0);$$

that is,

$$\alpha(x_0, x_1) \geq \eta(x_0, x_1).$$

Since f^{-1} is α admissible with respect to η, we have

$$\alpha(f^{-1}x_0, f^{-1}x_1) = \alpha(x_1, x_2) \geq \eta(f^{-1}x_0, f^{-1}x_1) = \eta(x_1, x_2).$$

This gives,

$$\alpha(x_1, x_2) \geq \eta(x_1, x_2).$$

In general, we have for all n,

$$\alpha(x_n, x_{n+1}) \geq \eta(x_n, x_{n+1}) = \eta(fx_{n+1}, x_{n+1}).$$

Since f is an $\alpha\eta - \xi\theta$-expansive mapping then we get,

$$\xi(d(fx_{n+1}, fx_n)) + \theta(d(x_{n+1}, x_n), d(x_n, fx_n), d(x_{n+1}, fx_n), d(x_n, fx_{n+1})) \geq d(x_{n+1}, x_n);$$

that is,

$$\xi(d(x_n, x_{n-1})) + \theta(d(x_{n+1}, x_n), d(x_n, x_{n-1}), d(x_{n+1}, x_{n-1}), d(x_n, x_n)) \geq d(x_{n+1}, x_n),$$

and so

$$\xi(d(x_n, x_{n-1})) \geq d(x_{n+1}, x_n).$$

By repeated application of above inequality, we get

$$d(x_n, x_{n+1}) \leq \xi^n(d(x_{n-1}, x_n))$$

for all n. For any $n > m \geq 0$, we have

$$d(x_m, x_n) \leq d(x_m, x_{m+1}) + d(x_{m+1}, x_{m+2}) + ... + d(x_{n-1}, x_n)$$

$$\leq \xi^m(d(x_0,x_1)) + ... + \xi^{n-1}(d(x_0,x_1)).$$

Since $\sum \xi^n(a) < \infty$ for each $a > 0$, it gives, $\{x_n\}$ is a Cauchy sequence in X. As (X,d) is a complete metric space, there exists a $u \in X$ such that $x_n \to u$ as $n \to \infty$. The continuity of f gives, $x_n = fx_{n+1} \to fu$ as $n \to \infty$. By uniqueness of the limit we get $fu = u$; that is, u is a fixed point of f. □

If, in Theorem 3.1, we take $\eta(x,y) = 1$ for all $x,y \in X$, then we deduce the following corollary.

Corollary 3.2. *Let (X,d) be a complete metric space and $f : X \to X$ be a bijective $\alpha - \xi\theta$-expansive mapping satisfying the following conditions:*
(i) f^{-1} is $\alpha-$admissible;
(ii) there exist $x_0 \in X$ such that $\alpha(x_0, f^{-1}x_0) \geq 1$;
(iii) f is continuous.
Then f has a fixed point.

In analogy to the main result, but omitting the continuity hypothesis of f, we can state the following theorem.

Theorem 3.3. *Let (X,d) be a complete metric space and $f : X \to X$ be a bijective $\alpha\eta - \xi\theta$-expansive mapping satisfying the following conditions:*
(i) f^{-1} is an $\alpha-$admissible with respect to η;
(ii) there exists $x_0 \in X$ such that $\alpha(x_0, f^{-1}x_0) \geq \eta(x_0, f^{-1}x_0)$;
(iii) if $\{x_n\}$ is a sequence in X such that $\alpha(x_n, x_{n+1}) \geq \eta(x_n, x_{n+1})$ for all $n \in N \cup \{0\}$ and $x_n \to x$ as $n \to \infty$ then $\alpha(f^{-1}x_n, f^{-1}x) \geq \eta(f^{-1}x, x)$ for all $n \in N \cup \{0\}$.
Then f has a fixed point.

Proof. By Theorem 3.1, $\{x_n\}$ is a Cauchy sequence in X. Since X is complete, there exist $u \in X$ such that $x_n \to u$ as $n \to \infty$. Also, as in Theorem 3.1, we have, $\alpha(x_n, x_{n+1}) \geq \eta(x_n, x_{n+1})$ for all n.
Taking the limit as $n \to \infty$, by condition (iii), we have

$$\alpha(x_{n+1}, f^{-1}u) = \alpha(f^{-1}x_n, f^{-1}u) \geq \eta(f^{-1}u, u) = \eta(f^{-1}u, f(f^{-1}u))$$

for all $n \in N \cup \{0\}$.

Now since, f is a modified $\alpha\eta - \xi\theta$-expansive mapping,

$$\begin{aligned}&\xi(d(ff^{-1}u, fx_{n+1}))\\&+ \theta(d(f^{-1}u, ff^{-1}u), d(x_{n+1}, fx_{n+1}), d(f^{-1}u, fx_{n+1}), d(x_{n+1}, ff^{-1}u))\\&\geq d(f^{-1}u, x_{n+1})\end{aligned}$$

which implies that

$$\xi(d(u,x_n)) + \theta(d(f^{-1}u,u), d(x_n,x_n), d(f^{-1}u,x_n), d(x_n,u)) \geq d(f^{-1}u, x_{n+1})$$

or

$$0 = \lim_{n\to\infty} d(x_{n+1}, f^{-1}u) = d(u, f^{-1}u).$$

i.e., $u = f^{-1}u$, so $u = fu$. Hence f has a fixed point. □

To ensure the uniqueness of the fixed point in above theorems, we consider the condition:
(A) for all $x, y \in Fix(f)$ with $\alpha(x,y) < \eta(x,x)$, there exists $w \in X$ such that $\alpha(x,w) \geq \eta(x,w)$ and $\alpha(y,w) \geq \eta(y,w)$;
(B) for all $u \in Fix(f)$ and all $x \in X$ we have, $\eta(u,x) \geq \eta(x,fx)$.

If in Theorem 3.1, we take $\eta(x,y) = 1$ for all $x, y \in X$, then we deduce the following corollary.

Corollary 3.4. *Let (X,d) be a complete metric space and $f : X \to X$ be a bijective $\alpha - \xi\theta$-expansive mapping satisfying the following:*
(i) f^{-1} is $\alpha-$admissible;
(ii) there exists an $x_0 \in X$ such that $\alpha(x_0, f^{-1}x_0) \geq 1$;
(iii) if $\{x_n\}$ is a sequence in X such that $\alpha(x_n, x_{n+1}) \geq 1$ for all n and as $n \to \infty$ then $\alpha(f^{-1}x_n, f^{-1}x) \geq 1$ for all $n \in N \cup \{0\}$.
Then f has a fixed point.

Theorem 3.5. *Adding the conditions (A) and (B) to the hypotheses of Theorem 3.1 (resp. Theorem 3.3, Corollary 3.2 and Corollary 3.4), we obtain the uniqueness of the fixed point of f.*

Proof. To prove uniqueness, let u and v be two fixed points of f such that $u \neq v$.

Step 1: Let $\alpha(u,v) \geq \eta(v,v) = \eta(v,fv)$. Since f is an $\alpha\eta - \xi\theta$-expansive mapping, we have,

$$\xi(d(fv,fu)) + \theta(d(v,fv), d(u,fu), d(v,fu), d(u,fv)) \geq d(v,u) \tag{3.1}$$

which implies that

$$\xi(d(fv,fu)) > \xi(d(v,u)) = \xi(d(fv,fu)) \geq d(v,u)$$

a contradiction. Hence, $u = v$.

Step 2: Let $\alpha(u,v) < \eta(v,v)$. Then from (A), there exists a $w \in X$ such that $\alpha(u,w) \geq \eta(u,w)$ and $\alpha(v,w) \geq \eta(v,w)$.
As f^{-1} is α-admissible with respect to η, we get

$$\alpha(u, f^{-1}w) \geq \eta(u, f^{-1}w)$$

and

$$\alpha(v, f^{-1}w) \geq \eta(v, f^{-1}w),$$

which gives,

$$\alpha(u, f^{-n}w) \geq \eta(u, f^{-n}w) \text{ and } \alpha(v, f^{-n}w) \geq \eta(v, f^{-n}w).$$

On the other hand, from (B) we get,

$$\eta(u, f^{-n}w) \geq \eta(f^{-n}w, f(f^{-n}w)) \text{ and } \eta(v, f^{-n}w) \geq \eta(f^{-n}w, f(f^{-n}w)).$$

Therefore,

$$\alpha(u, f^{-n}w) \geq \eta(f^{-n}w, f(f^{-n}w)) \text{ and } \alpha(v, f^{-n}w) \geq \eta(f^{-n}w, f(f^{-n}w)).$$

As f be an $\alpha\eta - \xi\theta$-expansive mapping, for all n

$$\xi(d(fu, ff^{-n}w)) + \theta(d(u, fu), d(f^{-n}w, ff^{-n}w), d(u, ff^{-n}w), d(f^{-n}w, fu)) \\ \geq d(u, f^{-n}w),$$

and so

$$\xi(d(u, ff^{-n}w)) \geq d(u, f^{-n}w).$$

Repeating this process, we get

$$\xi^n(d(u, w)) \geq d(u, f^{-n}w)$$

for all n. Thus, $f^{-n}w \to u$ as $n \to \infty$. Similarly, we get $f^{-n}w \to v$ as $n \to \infty$. From the uniqueness of a limit point, we get $u = v$. Hence, f has unique fixed point in X. □

We now present some examples to illustrate the validity of our results.

Example 3.6. Let $X = [0, \infty)$ together with the metric $d(x, y) = max\{x, y\}$ if $x \neq y$ and $d(x, y) = 0$ if $x = y$. Define the self mappings f and ξ by $fx = x^2$ and $\xi t = t/2$, respectively, for all $x, t \in X$. Consider the mappings $\alpha, \eta : X \times X \to [0, \infty)$ defined by $\alpha(x, y) = 1$ if $x = y, \alpha(x, y) = 0$ otherwise, and $\eta(x, y) = 1$ for all $x, y \in X$. Also, let $\theta(t_1, t_2, t_3, t_4) = t_1.t_2.t_3.t_4$. Clearly,
(i) $\theta \in \Theta$;
(ii) f is bijective, continuous and an $\alpha\eta - \xi\theta$-expansive mapping;
(iii) f^{-1} is an $\alpha-$admissible with respect to η;
(iv) there exist $x_0 = 0 \in X$ such that $\alpha(x_0, f^{-1}x_0) \geq \eta(x_0; f^{-1}x_0)$.
Since all of the conditions of Theorem 3.1 are satisfied, f has a fixed point.

The following example establishes the validity of Theorem 3.3.

Example 3.7. Let $X = [0, \infty)$ together with usual metric d. Define self mappings f and ξ by $fx = \frac{x}{2}$ if $x \in [0, 1)$, $fx = 0$ otherwise, and $\xi t = \frac{t}{2}$ respectively, for all $t \in X$. Consider the mappings $\alpha, \eta : X \times X \to [0, \infty)$ defined by $\alpha(x, y) = 1$ if $x, y \geq 1$, $\alpha(x, y) = 0$ otherwise, and $\eta(x, y) = 1$ for all x, y. Also, let $\theta(t_1, t_2, t_3, t_4) = t_1.t_2.t_3.t_4$. Clearly,
(i) f^{-1} is an $\alpha-$admissible with respect to η;
(ii) there exists an $x_0 = 1 \in X$ such that $\alpha(x_0, f^{-1}x_0) \geq \eta(x_0, f^{-1}x_0)$;
(iii) if $\{x_n\}$ is a sequence in X such that $\alpha(x_n, x_{n+1}) \geq \eta(x_n, x_{n+1})$ for all $n \in N \cup \{0\}$ and $x_n \to x$ as $n \to \infty$, then $\alpha(f^{-1}x_n, f^{-1}x) \geq \eta(f^{-1}x, x)$ for all $n \in N \cup \{0\}$;
(iv) f is bijective, and an $\alpha\eta - \xi\theta-$expansive mapping.
All the conditions of Theorem 3.3 are satisfied, and hence f has a fixed point.

4 Graphic $\xi\theta$-Expansive Mappings

Let (X, d) be a metric space. Consider a directed graph G such that the set $V(G)$ of its vertices coincides with X and the set $E(G)$ of its edges contains all loops. We assume that G has no parallel edges, so we can identify G with the pair $(V(G), E(G))$.

Definition 4.1. Let (X, d) be a metric space endowed with a graph G and $f : X \to X$ be a bijective self mapping. We say that f is a Graphic $\xi\theta-$ expansive mapping if

$$(x, y) \in E(G) \Rightarrow (f^{-1}x, f^{-1}y) \in E(G)$$

and

$$(x, y) \in E(G) \Rightarrow \xi(d(fx, fy)) + \theta(d(x, fx), d(y, fy), d(x, fy), d(y, fx)) \geq d(x, y),$$

where $\theta \in \Theta$ and $\xi \in \chi$.

Theorem 4.1. *Let (X, d) be a complete metric space endowed with a graph G and f be a bijective Graphic $\xi\theta$-expansive mapping. Suppose that the following assertions hold:*
(i) there exists an $x_0 \in X$ such that $(x_0, f^{-1}x_0) \in E(G)$;
(ii) f is G-continuous.
Then f has a fixed point.

Proof. Define $\alpha : X \times X \to (-\infty, +\infty)$ by $\alpha(x, y) = 1$ if $(x, y) \in E(G)$ and $\alpha(x, y) = 0$, otherwise. We first show that f^{-1} is an $\alpha-$admissible mapping. Let $\alpha(x, y) \geq 1$. Then $(x, y) \in E(G)$. As f is a Graphic $\xi\theta$-expansive mapping, we have $(f^{-1}, f^{-1}y) \in E(G)$; that is, $\alpha(f^{-1}x, f^{-1}y) \geq 1$. Hence f^{-1} is an $\alpha-$admissible mapping. Secondly, we show that f is continuous (or α continuous on (X, d)). Since f is G-continuous on X, $x_n \to x$ as $n \to \infty$, and $(x_n, x_{n+1}) \in E(G)$ for all n imply that $fx_n \to fx$. Thus $x_n \to x$ as $n \to \infty$, and $\alpha(x_n, x_{n+1}) \geq 1$ for all n imply that $fx_n \to fx$ which implies that f is α continuous on X. From (i),there exists an $x_0 \in X$ such that $(x_0, f^{-1}x_0) \in E(G)$, which gives, $\alpha(x_0, f^{-1}x_0) \geq 1$. Since f is a Graphic $\xi\theta$-expansive mapping,
$(x, y) \in E(G)$ or $\alpha(x, y) \geq 1$ and this gives

$$\xi(d(fx, fy)) + \theta(d(x, fx), d(y, fy), d(x, fy), d(y, fx)) \geq d(x, y),$$

which gives the result that f is a bijective $\alpha - \xi\eta$-expansive mapping.

Thus all of the conditions of Corollary 3.2 are satisfied, and f has a fixed point. □

Theorem 4.2. *Let (X, d) be a complete metric space endowed with a graph G and f be a bijective Graphic $\xi\theta$-expansive mapping. Suppose that the following assertions hold:*
(i) there exists an $x_0 \in X$ such that $(x_0, f^{-1}x_0) \in E(G)$;
(ii) if $\{x_n\}$ is a sequence in X such that $(x_n, x_{n+1}) \in E(G)$ for all $n \in N \cup 0$ and $x_n \to x$ as $n \to \infty$ then $(f^{-1}x_n, f^{-1}x) \in E(G)$.
Then f has a fixed point.

Proof. Define $\alpha : X \times X \to (-\infty, +\infty)$ as in proof of Theorem 4.1. Let $\{x_n\}$ be a sequence in X such that $\alpha(x_n, x_{n+1}) \geq 1$ for all $n \in N \cup \{0\}$ and $x_n \to x$ as $n \to \infty$; i.e., $\{x_n\}$ is a sequence in X such that $(x_n, x_{n+1}) \in E(G)$ for all $n \in N \cup \{0\}$ and $x_n \to x$ as $n \to \infty$. Then by (ii), $(f^{-1}x_n, f^{-1}x) \in E(G)$ for all $n \in N \cup \{0\}$ or $\alpha(f^{-1}x_n; f^{-1}x) \geq 1$ for all $n \in N \cup \{0\}$; which implies condition (iii) of Corollary 3.2. All other conditions of Corollary 3.2 follow as in the proof of Theorem 4.1 and consequently f has a fixed point. □

To ensure the uniqueness of the fixed point in each of the above theorems, we consider the condition:
(C) for all $x, y \in Fix(f)$ with (x, y) not in $E(G)$, there exists a $w \in X$ such that $(x, w) \in E(G)$ and $(y, w) \in E(G)$.

Remark 4.3. *Adding condition* (C) *to the hypotheses of Theorem* 4.2 (*resp. Theorem* 4.1)*, one obtains the uniqueness of the fixed point of* f*.*

Let $(X, d, \preceq)$ be a partial ordered metric space. Define the graph G by

$$E(G) = (x, y) \in X \times X : x \preceq y.$$

For this graph the condition $(x, y) \in E(G) \Rightarrow (f^{-1}x, f^{-1}y) \in E(G)$ of Definition 4.1 means that f is a nondecreasing map. From Theorems 4.1 and 4.2, we derive following important results in partially ordered metric spaces.

Theorem 4.4. *Let* $(X, d, \preceq)$ *be a partial ordered metric space and* f *be a bijective self mapping on* X*. Suppose that the following assertions hold:*
(i) *f is a nondecreasing map*;
(ii) $x \preceq y \Rightarrow \xi(d(fx, fy)) + \theta(d(x, fx), d(y, fy), d(x, fy), d(y, fx)) \geq d(x, y)$, *where* $\theta \in \Theta$ *and* $\xi \in \chi$;
(iii) *there exists an* $x_0 \in X$ *such that* $x_0 \preceq f^{-1}x_0$;
(iv) *f is continuous.*
Then f has a fixed point.

Theorem 4.5. *Let* $(X, d, \preceq)$ *be a partial ordered metric space and* f *be a bijective self-mapping on* X*. Suppose that the following assertions hold:*
(i) *f is a nondecreasing map*;
(ii) $x \preceq y \Rightarrow \xi(d(fx, fy)) + \theta(d(x, fx), d(y, fy), d(x, fy), d(y, fx)) \geq d(x, y)$, *where* $\theta \in \Theta$ *and* $\xi \in \chi$;
(iii) *there exist an* $x_0 \in X$ *such that* $x_0 \preceq f^{-1}x_0$;
(iv) *if* $\{x_n\}$ *is a sequence in* X *such that* $x_n \preceq x_{n+1}$ *for all* $n \in N \cup \{0\}$ *and* $x_n \to x$ *as* $n \to \infty$ *then* $f^{-1}x_n \preceq f^{-1}x$ *for all* $n \in N \cup \{0\}$.
Then f has a fixed point.

Acknowledgement : Authors would like to thank the referee for his comments and suggestions on the manuscript.

References

[1] S.Z. Wang, B.Y. Li, Z.M. Gao, K. Iseki, Some fixed point theorems on expansion mappings, Math.Jpn. 29 (1984) 631-636.

[2] B.E. Rhoades, Some fixed point theorems for pairs of mappings, Jnanabha 15 (1985) 151-156.

[3] B. Samet, C. Vetro, P. Vetro, Fixed point theorems for α, ϕ-contractive type mappings, Nonlinear Anal., Theory Methods Appl. 75 (2012) 2154-2165.

[4] N. Hussain, S. Al-Mezel, P. Salimi, Fixed points for ψ-graphic contractions with application to integral equations, Abstr. Appl. Anal. 2013 (2013) Article ID 575869.

[5] E. Karapinar, B. Samet, Generalized $\alpha - \psi$-contractive type mappings and related fixed point theorems with applications, Abstr. Appl. Anal. 2012 (2012) Article ID 793486.

[6] E. Karapinar, P. Shahi, J. Kaur, S. Bhatia, Generalized (ξ, α)-expansive mappings and related fixed-point theorems, Journal of Inequalities and Applications (2014) https://doi.org/10.1186/1029-242X-2014-22.

(Received 27 June 2014)
(Accepted 28 December 2014)