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A Note on Multipliers of Weighted Lebesgue Spaces

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Abstract : In this paper, it is solved that the spaces $M\left(L_w^{p^/}(G), L_{w^/}^{\infty}(G)\right)$ and $L_w^P(G)$ can be topologically and algebraically identified, where $1 \leq p^/ < \infty$, $\frac{1}{p} + \frac{1}{p^/} = 1$ and w a Beurling weight on a locally compact Abelian group G. Also it is proved that the spaces $M\left(L_w^1 \cap L_w^p(G), L_w^1(G)\right)$ can be identified with the weighted spaces of bounded measures $M_w(G)$.

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1 Introduction

Throughout this paper, G is a locally compact Abelian group and dx is a Haar measure on G. The unit of G shall be denoted by e. The translation by $a \in G$ of a measurable function f is defined by the formula $L_a f(x) = f(x-a)$. We denote by $C_0(G)$ the space of continuous functions vanishing at infinity and by $C_c(G)$ the space of continuous compactly supported functions. If $1 \leq p < \infty$, then $L^p(G)$ shall denote the space of functions f such that $|f|^p$ is integrable [1–3]. A Beurling weight on G is a measurable locally bounded function w satisfying, for each $x, y \in G$, the following two properties: $w(x) \geq 1$ and $w(x+y) \leq w(x)w(y)$. From this definition of w, it is deduced easily that wdx is a positive measure on

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G. We denote by $L_w^p(G), 1 \leq p < \infty$, the Banach spaces of equivalence classes of complex-valued measurable functions on G under the system of norm

$$\|f\|_{p,w} = \left(\int_G |f(x)|^p w(x)\, dx \right)^{\frac{1}{p}} < \infty.$$

We represent by $L_w^\infty(G)$ the Banach space of all measurable functions f on G such that

$$\|f\|_{\infty,w} = ess\sup_{x\in G} \{|f(x)| w(x)\}.$$

We express by l_w^p the discrete version of $L_w^p(G)$. Again we have the space

$$M_w(G) = \left\{ \mu : \mu \text{ is a bounded measure and } \|\mu\|_w = \int w|\mu| < \infty \right\}.$$

All these spaces are Banach spaces. One can find more about these spaces in [2–6].

Let E and F be two Banach spaces of measurable functions and assume that E and F are stable by translations. A multiplier on E to F is a bounded linear operator commuting with all translations. We denote by $M(E,F)$ the space of all multipliers on E to F. It is known that a translation operator is an isometry on $L^p(G)$, while it is not in general an isometry on $L_w^p(G)$. This fact is closely related to multiplier problems for $L_w^p(G)$, [6–9].

The conjugate space of $L_w^p(G)$ is the $L_{w^/}^{p^/}(G)$, where $w^/ = w^{1-p^/}$ and $\frac{1}{p} + \frac{1}{p^/} = 1$ or $p^/ = \frac{p}{p-1}$.

It can be easily seen that $L_w^p(G)$ is a reflexive Banach space. $L_w^1(G)$ is a Banach algebra under convolution,

$$L_w^p * L_w^1 \subset L_w^p,$$

and by [7, 9] there exists the inequality

$$\|g * f\|_{p,w} \leq \|g\|_{1,w} \|f\|_{p,w}.$$

2 Main Results

2.1 Multipliers Space of $\left(L_{w^/}^{p^/}(G), L_{w^/}^\infty(G)\right), p^/ > 1$

In this section, we will give the space of multipliers acting on some Beurling weighted spaces. We denote by $M(L_w^p(G), L_w^q(G))$ the space of all multipliers of $(L_w^p(G), L_w^q(G))$ and $\|T\|_{p,q,w}$ the operator norm of each $T \in M(L_w^p(G), L_w^q(G))$. Furthermore we denote by $M\left(L_w^1(G)\right)$ the space of all multipliers on $L_w^1(G)$ and $\|T\|_{1,w}$ the operator norm of each $T \in M\left(L_w^1(G)\right)$. Denote $\langle f, g\rangle_w = \int_G f(t)\overline{g(t)}w(t)\,dt$ for $f \in L_w^p(G),\ g \in L_{w^/}^q(G)$. If $\mu \in M_w(G)$, then we write $\|\mu\|_w = \int w|\mu|$ (see [3] for the definition of $|\mu|$). Before starting to define multipliers spaces, we will give the following definition and theorems, whose proofs can be found in [5].

Definition 2.1. Let $T : L^p_w(G) \to L^q_w(G)$ be a bounded linear transformation, where $1 \le p, q \le \infty$. T is said to be a multiplier of $(L^p_w(G), L^q_w(G))$ if T commutes with every translation operator.

Theorem 2.2. *Let $T : L^1_w(G) \to L^p_w(G)$ be a bounded linear transformation with $p > 1$. Then*

(i) $T \in M\left(L^1_w(G), L^p_w(G)\right)$ *if and only if there exists a unique function* $g \in L^p_w(G)$ *such that* $T = T_g : f \to g * f,\ f \in L^1_w(G)$.

(ii) There exists a constant $c \ge 1$ dependent only on the weight function w such that

$$\|Tg\|_{1,p,w} \le \|g\|_{p,w} \le c\,\|Tg\|_{1,p,w}\,.$$

(iii) $M\left(L^1_w(G), L^p_w(G)\right)$ *and* $L^p_w(G)$ *are topologically and algebraically identified by the mapping of part (i).*

Theorem 2.3. *Assume that the weight w is continuous. Let T be a bounded linear operator on $L^1_w(G)$. Then*

(i) $T \in M\left(L^1_w(G)\right)$ *if and only if there exists a unique measure μ such that* $T = T_\mu : f \to \mu * f,\ f \in L^1_w(G)$.

(ii) $w(e)\,\|\mu\|_w = \|T_\mu\|$.

(iii) $M\left(L^1_w(G)\right)$ *and* $M_w(G)$ *are topologically and algebraically identified.*

The proofs can be found [5].

A characterization of the elements in $M\left(L^{p'}_w(G), L^\infty_w(G)\right)$ can be readily obtained by examing the adjoints of these multipliers in the light of the results of the previous two theorem.

Lemma 2.4. *Let G be a noncompact, locally compact Abelian group. Then*

(i) If $f \in L^p_w(G),\ 1 \le p < \infty$, *then* $\lim\limits_{s\to\infty} \|f + L_s f\|_{p,w} = 2^{\frac{1}{p}}\,\|f\|_{p,w}$,

(ii) If $f \in C_{0,w'}(G)$, *then* $\lim\limits_{s\to\infty} \|f + L_s f\|_{\infty,w} = \|f\|_{\infty,w}$.

Proof. Let f be in $L^p_w(G)$. Then $fw^{\frac{1}{p}} \in L_p$ and since $\overline{C_c(G)} = L^p(G)$, for all $\varepsilon > 0$, there exists $g_\varepsilon \in C_c(G)$ such that

$$\left\|fw^{\frac{1}{p}} - g_\varepsilon\right\|_p < \frac{\varepsilon}{2 + 2^{\frac{1}{p}}}. \tag{1}$$

If $\sup pg = K$ is said, then since K is compact, so is KK^{-1}. Thus using that $C_c(G)$ is translation invariant, we have $L_s g \in C_c(G)$ and hence $K \cap K_s = \varnothing$, where $\sup pL_s g = sK = K_s$ for $s \notin KK^{-1}$. Indeed if $K \cap K_s \ne \varnothing$, then there exists a $t \in K \cap K_s$ such that $t = k_1$ and $t = sk_2$ with $k_1, k_2 \in K$. Thus $k_1 = sk_2$

and hence $s = k_1 k_2^{-1}$, and we have $s \in KK^{-1}$ as a contradiction by choosing of s. It is again known that

$$\left\| g + L_s g \right\|_p = 2^{\frac{1}{p}} \left\| g \right\|_p \tag{2}$$

for all $g \in C_c(G)$ [10]. Using that the space $L^p(G)$ is translation invariant and (1), (2), we obtain

$$\begin{aligned} \left| \left\| f w^{\frac{1}{p}} + L_s\left(f w^{\frac{1}{p}} \right) \right\|_p - \left\| g + L_s g \right\|_p \right| &\leq \left\| f w^{\frac{1}{p}} + L_s\left(f w^{\frac{1}{p}} \right) - g - L_s g \right\|_p \\ &\leq \left\| f w^{\frac{1}{p}} - g \right\|_p + \left\| L_s\left(f w^{\frac{1}{p}} - g \right) \right\|_p \\ &= 2 \left\| f w^{\frac{1}{p}} - g \right\|_p < 2 \frac{\varepsilon}{2 + 2^{\frac{1}{p}}}, \end{aligned}$$

and for $s \notin KK^{-1}$, using this last inequality and (2)

$$\begin{aligned} \left| \left\| f w^{\frac{1}{p}} + L_s\left(f w^{\frac{1}{p}} \right) \right\|_p - 2^{\frac{1}{p}} \left\| f w^{\frac{1}{p}} \right\|_p \right| &\leq \left| \left\| f w^{\frac{1}{p}} + L_s\left(f w^{\frac{1}{p}} \right) \right\|_p - \left\| g + L_s g \right\|_p \right| \\ &\quad + \left| \left\| g + L_s g \right\|_p - 2^{\frac{1}{p}} \left\| g \right\|_p \right| \\ &\quad + \left| 2^{\frac{1}{p}} \left\| g \right\|_p - 2^{\frac{1}{p}} \left\| f w^{\frac{1}{p}} \right\|_p \right| \\ &< 2 \frac{\varepsilon}{2 + 2^{\frac{1}{p}}} + 2^{\frac{1}{p}} \left\| f w^{\frac{1}{p}} - g \right\|_p \\ &< 2 \frac{\varepsilon}{2 + 2^{\frac{1}{p}}} + 2^{\frac{1}{p}} \frac{\varepsilon}{2 + 2^{\frac{1}{p}}} = \varepsilon. \end{aligned}$$

□

Lemma 2.5. *If $f \in L^p_w(G)$ and $g \in L^{p^{/}}_{w^{/}}(G)$, where $\frac{1}{p} + \frac{1}{p^{/}} = 1$ and $w^{/} = w^{1-p^{/}}$, then*

$$\left\| f * g \right\|_{\infty, w^{/}} \leq \left\| f \right\|_{p,w} \left\| g \right\|_{p^{/}, w^{/}}. \tag{3}$$

Proof. If we show that

$$\left\| f * g \right\|_{\infty, w^{/}} \leq \left\| \left| f \right| w^{\frac{1}{p}} * \left| g \right| \left(w^{/} \right)^{\frac{1}{p^{/}}} \right\|_{\infty}, \tag{4}$$

then one can easily seen (3). Let $x \in G$ be abritrary and fixed. Then, for $y \in G$

we have

$$\begin{aligned}
\left|f*g\left(x\right)w^{/}\left(x\right)\right| &= \left|\int_G f\left(x-y\right)w^{\frac{1}{p}}\left(x-y\right)\frac{w^{1-p^{/}\left(x\right)}}{w^{\frac{1}{p}}\left(x-y\right)}g\left(y\right)dy\right| \\
&\leq \int_G \left(\left(\left|f\right|w^{\frac{1}{p}}\right)\left(x-y\right)w^{1-p^{/}\left(x\right)}\frac{w^{\frac{1}{p}}\left(x\right)}{w^{\frac{1}{p}}\left(y\right)}\left|g\right|\left(y\right)\right)dy \\
&= \int_G \left(\left(\left|f\right|w^{\frac{1}{p}}\right)\left(x-y\right)\left|g\right|\left(y\right)w^{-\frac{1}{p}}\left(y\right)w^{1-p^{/}}\left(x\right)w^{\frac{1}{p}}\left(x\right)\right)dy \\
&= \int_G \left(\left(\left|f\right|w^{\frac{1}{p}}\right)\left(x-y\right)\left(\left(\left|g\right|w^{-\frac{1}{p}}\right)\left(y\right)\right)w^{2-\left(p^{/}+\frac{1}{p^{/}}\right)}\right)dy \\
&\leq \int_G \left(\left(\left|f\right|w^{\frac{1}{p}}\right)\left(x-y\right)\left(\left(\left|g\right|w^{-\frac{1}{p}}\right)\left(y\right)\right)\right)dy \\
&= \int_G \left(\left(\left|f\right|w^{\frac{1}{p}}\right)*\left(\left|g\right|w^{-\frac{1}{p}}\right)\right)dy.
\end{aligned}$$

This satisfies (4). Finally (3) can be easily obtained from (4). □

Theorem 2.6. *Let $T: L^{p^{/}}_{w^{/}}\left(G\right) \to L^{\infty}_{w^{/}}\left(G\right)$ be a bounded linear transformation, where $p>1, \frac{1}{p}+\frac{1}{p^{/}}=1$, w is a Beurling weight function and $w^{/}=w^{1-p^{/}}$. Then*

(a) $T: L^{p^{/}}_{w^{/}}\left(G\right) \to L^{\infty}_{w^{/}}\left(G\right)$ if and only if there exists a unique function $g \in L^p_w\left(G\right)$ such that $T=T_g: f \to g*f$, $f \in L^{p^{/}}_{w^{/}}\left(G\right)$, where $\frac{1}{p}+\frac{1}{p^{/}}=1$ and $w^{/}=w^{1-p^{/}}$,

(b) There exists a constant $c \geq 1$ dependent only on the weight function w such that

$$\left\|T_g\right\|_{p^{/},\infty,w^{/}} \leq \left\|g\right\|_{p,w} \leq c\left\|T_g\right\|_{p^{/},\infty,w^{/}},$$

(c) $M\left(L^{p^{/}}_{w^{/}}\left(G\right), L^{\infty}_{w^{/}}\left(G\right)\right)$ and $L^p_w\left(G\right)$ are topologically and algeabrically identified by the mapping of part (a).

Proof. If $g \in L^p_w\left(G\right)$ and we set $Tf = g*f$ for each $f \in L^{p^{/}}_{w^{/}}\left(G\right)$, then by using Lemma 2.2 we obtain $\left\|g*f\right\|_{\infty,w^{/}} \leq \left\|g\right\|_{p,w}\left\|f\right\|_{p^{/},w^{/}}$. Therefore $T \in M(L^{p^{/}}_{w^{/}}\left(G\right), L^{\infty}_{w^{/}}\left(G\right))$ and

$$\left\|T\right\|_{p^{/},\infty,w^{/}} \leq \left\|g\right\|_{p,w}. \tag{5}$$

Conversely suppose $T \in M\left(L^{p^{/}}_{w^{/}}\left(G\right), L^{\infty}_{w^{/}}\left(G\right)\right)$ and denote by T^* the adjoint of T, that is, the continuous linear transformation from $L^{\infty}_{w^{/}}\left(G\right)^*$ to $L^{p^{/}}_{w^{/}}\left(G\right)^* = L^p_w\left(G\right)$, $\frac{1}{p}+\frac{1}{p^{/}}=1$, $w^{/}=w^{1-p^{/}}$. Since $L^1_w\left(G\right) \subset L^{\infty}_{w^{/}}\left(G\right)^*$ we can write

$\langle Tf,h\rangle_w = \langle f,T^*h\rangle_w$ for each $f \in L^{p'}_{w'}(G)$, $h \in L^1_w(G)$. Moreover, for each $s \in G$ we have

$$\begin{aligned}\langle f,T^*L_sh\rangle_w &= \langle Tf,L_sh\rangle_w = \langle L_sTf,h\rangle_w = \langle TL_sf,h\rangle_w \\ &= \langle L_sf,T^*h\rangle_w = \langle f,L_sT^*h\rangle_w ,\end{aligned}$$

where $f \in L^{p'}_{w'}(G)$, $h \in L^1_w(G)$. Consequently, by Theorem 2.1, there exists a unique $g \in L^p_w(G)$ such that $T^*h = g * h$ for each $h \in L^1_w(G)$. An elementary computation reveals for each $f \in L^{p'}_{w'}(G)$ and $h \in L^1_w(G)$ that

$$\langle Tf,h\rangle_w = \langle f,T^*h\rangle_w = \langle f,g*h\rangle_w = \langle g*f,h\rangle_w .$$

Therefore $Tf = g*f$ for each $f \in L^{p'}_{w'}(G)$. On the other hand from the form of T^* and Theorem 2.1, we see that

$$\|g\|_{p,w} \leq c\,\|T^*\|_{1,w} \leq c\,\|T^*\|_{w',\infty} = c\,\|T\|_{p',\infty,w'} ,$$

where $\|T^*\|_{1,w}$ denotes the norm of T^* restricted to $L^1_w(G)$. Thus

$$\|g\|_{p,w} \leq c\,\|T\|_{p',\infty,w'} . \tag{6}$$

This combined with (5) and (6) completes the proof. of Theorem2.3. □

2.2 Multipliers of $(L^1_w(G) \cap L^p_w(G), L^1_w(G))$.

Note that $L^1_w(G) \cap L^p_w(G)$ is a Banach space with the norm $.\|.\|_{1,p,w} = \|.\|_{1,w} + \|.\|_{p,w}$ [8]. Now we denote by $M\left(L^1_w(G) \cap L^p_w(G), L^1_w(G)\right)$ the space of all multipliers from $L^1_w(G) \cap L^p_w(G)$ to $L^1_w(G)$ and $\|T\|$ the operator norm of each $M\left(L^1_w(G) \cap L^p_w(G), L^1_w(G)\right)$. If $\mu \in M_w(G)$, then we define $\|\mu\|_w = \int_G w\,|\mu|$, and if $w = 1$, then we use the symbol $\|\mu\| = \int_G |\mu|$.

The following theorem is a generalized version of the Theorem 3.5.1 in [8].

Theorem 2.7. *Let G be a noncompact locally compact Abelian group, $1 < p < \infty$, w is continuous at the unit e of G and $w(e) = 1$. If $T : L^1_w(G) \cap L^p_w(G) \to L^1_w(G)$ is a continuous linear transformation, then the following are equivalent:*

(i) $T \in M\left(L^1_w(G) \cap L^p_w(G), L^1_w(G)\right)$,

(ii) There exists a unique measure $\mu \in M_w(G)$ such that $Tf = \mu * f$ for each $f \in L^1_w(G) \cap L^p_w(G)$.

Moreover the correspondece between T and μ defines an isometric algebra isomorphism of $M\left(L^1_w(G) \cap L^p_w(G), L^1_w(G)\right)$ onto $M_w(G)$.

Proof. If $\mu \in M_w(G)$ and $Tf = \mu * f$ for each $f \in L_w^1(G) \cap L_w^p(G)$, then

$$\begin{aligned} \|Tf\|_{1,w} &= \|\mu * f\|_{1,w} = \int_G \left| \int_G f(t-s)\,\mu(s)\,ds \right| w(t)\,dt \\ &\le \int_G \left(\int_G |f(t-s)|\,|\mu(s)|\,ds \right) w(t)\,dt \\ &\le \int_G \left(\int_G |f(t)|\,|\mu(s)|\,ds \right) w(t+s)\,dt \\ &\le \int_G \left(\int_G |f(t)|\,w(t)\,dt \right) w(s)\,|\mu(s)|\,ds \\ &\le \|f\|_{1,w}\,\|\mu\|_w \le \|f\|_{1,p,w}\,\|\mu\|_w . \end{aligned}$$

Thus $T \in M\left(L_w^1(G) \cap L_w^p(G), L_w^1(G)\right)$ and $\|T\| \le \|\mu\|_w$.

Conversely, suppose that $T \in M\left(L_w^1(G) \cap L_w^p(G), L_w^1(G)\right)$. Then for each $f \in L_w^1(G) \cap L_w^p(G)$ we have $\|Tf\|_{1,w} \le \|T\| \left(\|f\|_{1,w} + \|f\|_{p,w} \right)$. Combining this estimate with Lemma 2.1 (i), we deduce that

$$\begin{aligned} 2\|Tf\|_{1,w} &= \lim_{s\to\infty} \|Tf + L_sTf\|_{1,w} = \lim_{s\to\infty} \|T(f + L_sf)\|_{1,w} \\ &\le \lim_{s\to\infty} \|T\| \left(\|f + L_sf\|_{1,w} + \|f + L_sf\|_{p,w} \right) \\ &= \|T\| \left(2\|f\|_{1,w} + 2^{\frac{1}{p}} \|f\|_{p,w} \right) \end{aligned}$$

for each $f \in L_w^1(G) \cap L_w^p(G)$. Thus

$$\|Tf\|_{1,w} \le \|T\| \left(\|f\|_{1,w} + 2^{\frac{1}{p}-1} \|f\|_{p,w} \right), \ f \in L_w^1(G) \cap L_w^p(G).$$

Repeating this process n times, we see that

$$\|Tf\|_{1,w} \le \|T\| \left(\|f\|_{1,w} + 2^{n\left(\frac{1}{p}-1\right)} \|f\|_{p,w} \right).$$

Since $p > 1$ we have $\lim_n 2^{n\left(\frac{1}{p}-1\right)} = 0$, and so we conclude that

$$\|Tf\|_{1,w} \le \|T\|\,\|f\|_{1,w} .$$

Hence T is continuous on $L_w^1(G) \cap L_w^p(G)$, considered as a subspace of $L_w^1(G)$. Thus T defines a continuous linear transformation from $L_w^1(G) \cap L_w^p(G)$ as a subspace of $L_w^1(G)$ to $L_w^1(G)$ which commutes with translation. Since $L_w^1(G) \cap L_w^p(G)$ is dense in $L_w^1(G)$, T determines a unique element $T^/$ of $M\left(L_w^1(G)\right)$ and $\left\|T^/\right\| \le \|T\|$. By Theorem 2.2 in [5], there exists a unique a element $\mu \in M_w(G)$ such that $T^/ f = \mu * f$ for each $f \in L_w^1(G)$ and $\|\mu\|_w = \left\|T^/\right\|$. Consequently $Tf = \mu * f$ for each $f \in L_w^1(G) \cap L_w^p(G)$ and $\|\mu\|_w \le \|T\|$. Hence (i) and (ii) are equivalent. It is evident that the correspondence between T and μ defines isometric algebra isomorphism from $M\left(L_w^1(G) \cap L_w^p(G), L_w^1(G)\right)$ onto $M_w(G)$. □

The spaces $M\left(L_w^p(G), L_w^q(G)\right)$ are Banach spaces of continuous linear transformations from $L_w^p(G)$ to $L_w^q(G)$. The norm of an element $T \in M(L_w^p(G), L_w^q(G))$ will be denoted by $\|T\|_{p,q,w}$. Our first result shows that certain of these spaces may be identified with each other.

Theorem 2.8. *Let G be a locally compact Abelian group and suppose that $1 \leq p < \infty$, $1 < q \leq \infty$, $\frac{1}{p} + \frac{1}{p^{/}} = 1$, $\frac{1}{q} + \frac{1}{q^{/}} = 1$, that w is a weight function on G, $w^{/} = w^{1-p^{/}}$. Then there exists an isometric linear isomorphism of $M\left(L_w^p(G), L_w^q(G)\right)$ onto $M\left(L_{w^{/}}^{q^{/}}(G), L_{w^{/}}^{p^{/}}(G)\right)$.*

Proof. Let $T \in M\left(L_w^p(G), L_w^q(G)\right)$. If $1 < p, q < \infty$, then we define $T^* : L_{w^{/}}^{q^{/}}(G) \to L_{w^{/}}^{p^{/}}(G)$ to be the operator adjoint to T, that is, the linear operator determined by the equation

$$\langle f, T^* g\rangle_w = \langle Tf, g\rangle_w \qquad \left(f \in L_w^p(G), g \in L_{w^{/}}^{q^{/}}(G)\right).$$

Clearly T^* is continuous. Morever, $T^* L_s = L_s T^*$ for each $s \in G$, since as usual we have for $f \in L_w^p(G)$ and $g \in L_{w^{/}}^{q^{/}}(G)$ that

$$\begin{aligned} \langle f, T^* L_s g\rangle_w &= \langle Tf, L_s g\rangle_w = \langle L_s Tf, g\rangle_w = \langle T L_s f, g\rangle_w \\ &= \langle L_s f, T^* g\rangle_w = \langle f, L_s T^* g\rangle_w . \end{aligned}$$

Thus $T^* \in M\left(L_{w^{/}}^{q^{/}}(G), L_{w^{/}}^{p^{/}}(G)\right)$ and $\|T\|_{p,q,w} = \|T^*\|_{q^{/},p^{/},w}$. The reflexivity of $L_w^p(G)$ and $L_w^q(G)$ shows immediately that the mapping $T \to T^*$ is surjective. Hence this mapping defines an isometric linear isomorphism from $M(L_w^p(G), L_w^q(G))$ onto $M\left(L_{w^{/}}^{q^{/}}(G), L_{w^{/}}^{p^{/}}(G)\right)$, when $1 < p, q < \infty$. □

The assertion of the theorem for the cases $p = 1$, $1 < q \leq \infty$ and $1 \leq q < \infty$, $q = \infty$, follows immediately from Theorem 2.1 and Theorem 2.3.

References

[1] E. Hewit, K.A. Ross, Abstract Harmonic Analysis, Vol. 1, Springer-Verlag, Berlin, 1963.

[2] H. Rieter, J.D. Stegeman, Classical Harmonic Analysis and Locally Compact Groups, London Mathematical Society, New Sreies, Oxford Science Publication, 2000.

[3] W. Rudin, Fourier Analysis on Groups, Interscience Publisheri in Pure and Applied Mathematics (Second Printing), 1966.

[4] J.J. Benedetto, Harmonic Analysis and Applications, Boca Raton, Florida: CRC Press, 1997.

[5] A. Bourouihiya, Beurling Weighted Spaces, Product-Convolution Operators, and the Tensor Product of Frames, P.h.D. Thesis, University of Maryland, 2006.

[6] G.I. Gaudry, Multipliers of weighted Lebesgue and measure spaces, Proc. London Math. Soc. 19 (1969) 327-340.

[7] C.E. Heil, Wiener Amalgam Space in Generalized Harmonic Analysis and Wavelet Theory, P.h.D. Thesis, University of Maryland, 1990.

[8] R. Larsen, An Introduction to the Theory of Multipliers, Die Grundlehren der mathematischen wissenschaften, 1971.

[9] G.N.K. Murthy, K.R. Unni, Multipliers on weighted spaces, Func. Analysis and its Applications, Lecture Notes in Math. 399, Springer-Verlag, Berlin (1973), 273-291.

[10] L. Hörmander, Estimates for translation invariant operators in L^p-spaces, Acta Math. 104 (1960) 93-140.

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