Thai Journal of Mathematics : (2019) 201–215
Special Issue: Structural Change Modeling
and Optimization in Econometrics 2018

http://thaijmath.in.cmu.ac.th
ISSN 1686-0209

Forecasting GDP in ASIAN Countries Using Relevant Vector Machines

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Abstract : The relevance vector machine(RVM) is applied to predict GDP, the highly important measurement of national economic growth, of some ASEAN countries (Malaysia, Thailand and Singapore) for comparison with the autoregressive model (AR(p)). The results show that RVM dominates the AP(p) model by measuring the error (MAE, MAPE, MSE and RMSE) from both training data and validation data.

Keywords : GDP; relevant vector machine; autoregressive model.

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2010 Mathematics Subject Classification : 35K05; 91G20.

1 Introduction

Gross Domestic Product (GDP) is an overall estimate of the size of an economy, in terms of total productive output. As such, it is of great importance to policy makers and central banks to know its trend and foresee possible changes. This facilitates accurate assessment of the future state of the economy whether it be heading toward expansion or contraction such that preemptive actions can be taken as appropriate. Reliable forecasting of GDP is a very important tool in the macroeconomics toolbox (Roush and Hu [1]).

GDP growth forecast has always been one of the most popular and important research topics and many forecasting models have been developed by economists, econometricians and statisticians. To make accurate GDP growth forecast, forecasting models need to address the following two issues, reasonable selection of predictors and efficient utilization of data with different frequencies (Koenig et al.[2],Armesto et al. [3], Andreou et al. [4]).

In the past, the forecast Auto Regressive Integrated Moving Average (ARIMA) technique has been widely used for time series forecasting. However, ARIMA is a general univariate model and it is developed based on the assumption that the time series being forecasted are linear and stationary. This model has in form:

$$y_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i y_{t-i} + \sum_{j=1}^{q} \beta_j \varepsilon_{t-j} + \varepsilon_t, t = 1, 2, \cdots, T, \tag{1.1}$$

where $y_1, y_2, \cdots, y_T$ are the time series data, $\alpha_0, \alpha_1, \cdots, \alpha_p, \beta_1, \cdots, \beta_q$ are the parameter in ARIMA model.

In the literature, the forecast shows that several studies. For instance, Quinonero-Candela and Hansen [5] studies time series prediction based on the relevance vector machine with adaptive kernels. Quinonero-Candela et al. [7] explain to prediction at an uncertain input for Gaussian processes and relevance vector machines-application to multiple-step ahead time-series forecasting. Furte rmore Wabomba et al. [8] forecast Kenyan GDP using Autoregressive Integrated Moving Average (ARIMA) Models. Thianpaen et al. [9] forecast the Thailand GDP growth rate using AR-based belief function model. In the most recently Navapan et al. [10] studies forecast the growth of total debt service ratio used ARIMA and State Space model. Boonyakunakorn et al. [11] they apply five models include Autoregressive (AR) model, a linear model, followed by four non-linear models, Self-exciting autoregressive (SETAR), Logistic STAR (LSTAR), Markov switching Autoregressive (MSAR) and Kink Autoregressive (AR) models to forecast Thailand's exports to ASEAN.

In recent years, a novel neural network technique called 'Relevance Vector Machine' (RVM) has found useful applications in time-series analysis and forecasting, The RVM is a specific instance of this model, which is intended to mirror the structure of the support vector machine. In particular, the basic functions are given by kernels, with one kernel associated with each of the data points from the training set.

$$f(x) = \sum_{i=1}^{N} \omega_i K(x) + \omega_0, \tag{1.2}$$

where $K(\cdot)$ is a kernel function and ω_0 is a constant and $\omega_1, \cdots, \omega_n$ are the weight of model. However, the subsequent analysis is valid for arbitrary choices of basis functions, and for generality we shall work with the general form. In contrast to the SVM, there is no restriction to positive-definite kernels, nor are the basis functions tied in either number or location to the training data points.

Therefore, in this study, we investigating the forecasting GDP in ASEAN countries include Thailand, Malaysia and Singapore using Relevant Vector Machines. RVM was proposed by Tipping [12]. The RVM is a probabilistic sparse kernel model identical in functional form to the SVM. RVM is based on a Bayesian formulation of a linear model with an appropriate prior that results in a sparse representation than that achieved by SVM. RVM is based on a hierarchical prior, where an independent Gaussian prior is defined on the weight parameters in the first level, and an independent Gamma hyper prior is used for the variance parameters in the second level. This results in an overall student-t prior on the weight parameters, which leads to model sparseness.

The structure of the rest of this paper is as follows. The next section provides the model used in this study. Section 3 presents data,Section 4 presents the empirical results. Finally, the conclusion is provided in Section 5.

2 Methodology

2.1 Auto Regressive Model (AR(p))

This model is the special case of ARIMA has in form

$$y_t = f(y_{t-1}, y_{t-2}, \cdots, y_{t-p}) + \varepsilon_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i y_{t-i} + \varepsilon_t \tag{2.1}$$

The parameter in AP(p) model can estimated by using ordinary least square (OLS) or maximum likelihood method (MLE). Thr AR(p) is stationary process if the roots of the polynomial $z^p - \sum_{i=1}^{p} \alpha_i z^i$ inside the unit circle and $|z_i| < 1, \forall i$.

2.2 Relevant Vector Machines (RVM)

Given a dataset of input-target pairs $(x_n, t_n), n = 1, \cdots, N$, we follow the standard formulation and assume $p(t|x)$ is Gaussian distribution $N(t, y(x), \sigma^2)$. The mean of this distribution for a given x is modeled by $y(x)$ as defined in (1.2) for the SVM. The likelihood of the dataset can be written as

$$p(t|w, \sigma^2) = \frac{1}{(2\pi\sigma^2)^{N/2}} \exp\left[-\frac{1}{2\sigma^2}\|t - \Phi w\|^2\right], \tag{2.2}$$

where $t = (t_1, \cdots, t_N), w = (w_0, w_1, \cdots, w_N)$ and $\Phi(\cdot)$ is the $N \times (N+1)$ 'design' matrix with $\Phi_{nm} = K(x_n, x_{m-1})$ and $\Phi_{n1} = 1$. Maximum-likelihood estimation of w and σ^2 from (2.2) will generally lead to severe overfitting, so we encode a preference for smoother functions by defining and ARD Gaussian prior MacKay [13], Neal [6] over the weights:

$$p(w, \alpha) = \prod_{i=0}^{N} N(w_i \left| 0, \frac{1}{\alpha_i} \right.), \tag{2.3}$$

with α_i a vector of $N+1$ hyperparameters. This introduction of an individual hyperparameter for every weight is the key feature of the model, and is ultimately responsible for its sparsity properties. The posterior over the weights is then obtained from Bayes' rule:

$$p(w \left| t, \alpha, \sigma^2 \right.) = (2\pi)^{-(N+1)/2} \left| \Sigma \right|^{-1/2} \exp\left[-\frac{1}{2}(w-\mu)^T \Sigma^{-1} (w-\mu) \right], \tag{2.4}$$

with

$$\Sigma = \left(\Phi^T B \Phi + A \right)^{-} 1 \tag{2.5}$$

$$\mu = \Sigma \Phi^T B t, \tag{2.6}$$

where we defined $A = diag(\alpha_0, \alpha_1, \cdots, \alpha_N)$ and $B = \sigma^{-2} I_N$. (Note that σ^2 is also treated as a hyperparameter, which may be estimated from the data.) By integrating out the weights, we obtain the marginal likelihood, or evidence (Mackay [13]), for the hyperparameters:

$$p(t \left| \alpha, \sigma^2 \right.) = (2\pi)^{-(N+1)/2} \left| B^{-1} + \Phi A^{-1} \Phi^T \right|^{-1/2} \exp\left[-\frac{1}{2} t^T \left(B^{-1} + \Phi A^{-1} \Phi^T \right)^{-1} t \right], \tag{2.7}$$

For ideal Bayesian inference, we should define hyperpriors over α and σ^2 , and integrate out the hyperparameters too. However, such marginalisation cannot be performed in closed-form here, so we adopt a pragmatic procedure, based on that of MacKay [13], and optimise the marginal likelihood (2.7) with respect to α and σ^2 , which is essentially the type II maximum likelihood method see Berger [14]. This is equivalent to finding the maximum of $p(\alpha, \sigma^2 \left| t \right.)$, assuming a uniform (and thus improper) hyperprior. We then make predictions, based on (2.4), using these maximizing values.

In this paper we use RVM and write the model similar to AR(p) model:

$$y_t = f(y_{t-1}, y_{t-2}, \cdots, y_{t-p}) = \sum_{j=1}^{p} \omega_j K(y_{t-j}) + \omega_0 \tag{2.8}$$

2.3 Performance Criteria

The inspection of the predict result is the key of the forecasting performance of the model, because we can acquire the information of the characteristic of the different forecasting methods, and this is very useful for the people to choose and use the variety of the

forecasting methods. The prediction performance is evaluated using the following statistical metrics, namely, the normalized mean absolute error (MAE), mean absolute percent error (MAPE), mean squared error (MSE) and root mean square error (RMSE). The definitions of these criteria is following
1. Mean absolute error (MAE)

$$\text{MAE} = \frac{1}{n}\sum_{i=1}^{n}\left|y_i - \hat{y}_i\right|. \tag{2.9}$$

2. Mean absolute percent error (MAPE)

$$\text{MAPE} = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{y_i - \hat{y}_i}{y_i}\right|, y_i \neq 0\ \forall i. \tag{2.10}$$

3. Mean squared error (MSE)

$$\text{MSE} = \frac{1}{n}\sum_{i=1}^{n}\left(y_i - \hat{y}_i\right)^2. \tag{2.11}$$

4. Root-mean-square error (RMSE)

$$\text{RMSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(y_i - \hat{y}_i\right)^2}. \tag{2.12}$$

The smaller the values of the four indexes, the higher are the prediction accuracy. Where: y_i represents the actual values; $\hat{y}_i$ represents the forecasting values; n represents the number of the sample.

3 Data Description

In this section, we describe the data used in this study. The annual data for 3 countries (i.e. Thailand, Malaysia and Singapore) over the period 1961-2017 are collected. The source of this data is the World Bank database. All variables are transformed into natural log.

The summary of the descriptive statistics is illustrated in Table 1. In this study, we use Minimum Bayes factor (MBF) as the tool for checking the significant result. This MBF can be considered as an alternative of p-value (Held and Ott, 2016). If $1 < MBF < 1/3$, $1/3 < MBF < 1/10$, $1/10 < MBF < 1/30$, $1/30 < MBF < 1/100$, $1/100 < MBF < 1/300$ and $MBF < 1/300$, there is a chance that the MBF favors the weak evidence, moderate evidence, substantial evidence, strong evidence, very strong evidence and decisive evidence for respectively. All data series are stationary as shown by the MBF values. The results of Jarque-Bera test lead to the rejection of the null hypothesis of normality for all variables, thus indicating the non-normality of the unconditional distribution of all the variables. Moreover, the Augmented DickeyFuller test (ADF) unit root test is conducted and it shows that there are decisive evidence for stationary data.

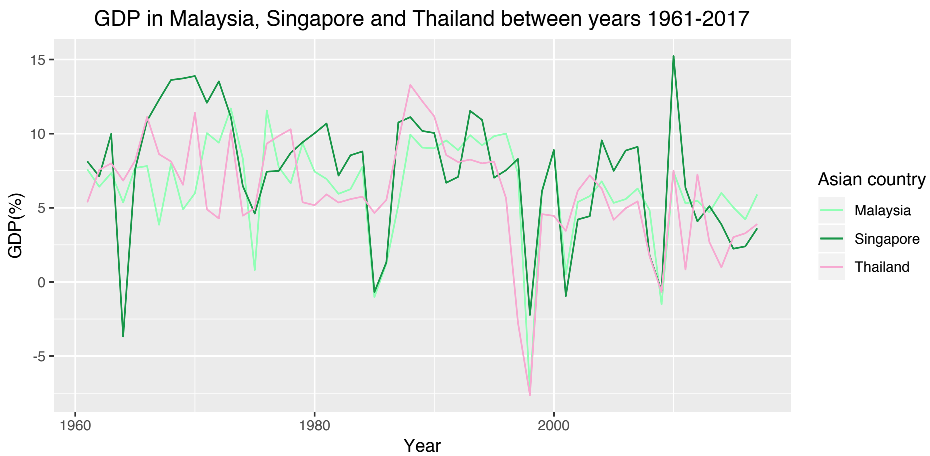

Figure 1: depicts that GDP in ASEAN countries(Thailand, Malaysia and Singapore) gently fluctuate with nonlinearity during the period 1961-2017.

Table 1: Descriptive statistics

	Thailand	Malaysia	Singapore
Mean	6.0179	6.3795	7.3866
Median	5.7524	6.6539	7.5972
Maximum	13.2881	11.7011	15.2404
Minimum	-7.6337	-7.3594	-3.6805
Std. Dev.	3.6303	3.3471	4.2602
Skewness	-0.9656	-1.5998	-0.6028
Kurtosis	5.5036	6.9986	2.968
Jarque-Bera	23.7438 [0.0000]	62.2886 [0.0000]	3.4542 [0.0025]
Observations	57	57	57
Unit Root test	-4.1324 [0.0001]	-6.2895 [0.0000]	-5.2307 [0.0000]

Note: [ ] MBF is Minimum Bayes factor, computed by , where is -value (see, Held and Ott [15]).

4 Empirical Results

In the results shows that the model selection criterion and evaluation indices. The contrasts between the observed value of the raw series and the predicted values obtained through the four methods were compared to determine the efficacy of the four forecasting methods used in the present study. The mean absolute error (MAE), mean absolute percentage error (MAPE), Mean squared error (MSE), and the root mean square error (RMSE) were selected as the measures of evaluation because as empirical methods they are widely used in combining and selecting forecasts for measuring bias and accuracy of models. Furthermore, the results shows the fluctuate of training data and validation data in figure2-4.

From Table 4 AR(p) we can that for training data AR(3) is the best model if we select by using MAPE but for validation data AR(4) is better that AR(3) model by the same performance measure. From Table 5 RVM (3) is best model in training data and the validation data by using MAPE.

But If we compare between AR(p) and RVM(p) for $p = 1, 2, \cdots, 5$. We can see that all of performance measure in RVM(p) lower than AR(p) in training data and validation data. So that RVM(p) dominate AR(p) for all p in GDP of Malaysia.

From Table 6 AR(p) we can that for training data AR(4) is the best model if we select by using MAPE but for validation data AR(1) is better that AR(4) model by the same performance measure. From Table 7 RVM (1) is best model in training data by using MAPE but in the validation data RVM(3) have the lower MAPE.

But If we compare between AR(p) and RVM(p) for $p = 1, 2, \cdots, 5$. We can see that all of performance measure in RVM(p) lower than AR(p) in training data and validation data. So that RVM(p) dominate AR(p) for all p in GDP of Singapore.

	Training				
	AR(1) RVM(1)	AR(2) RVM(2)	AR(3) RVM(3)	AR(4) RVM(4)	AR(5) RVM(5)
MAE	2.1131 (1.2884)	2.1211 (1.9555)	2.0953 (1.5826)	2.12 (1.3705)	2.1393 (2.2935)
MAPE	0.3535 (**0.2286**)	0.3578 (0.3242)	**0.3493** (0.2768)	0.3536 (0.2446)	0.3583 (0.3855)
MSE	9.0087 (3.1002)	8.9591 (6.1356)	8.8719 (4.5874)	8.8097 (3.2371)	8.9811 (8.5127)
RMSE	3.0014 (1.7607)	2.9932 (2.477)	2.9786 (2.1418)	2.9681 (1.7992)	2.9968 (2.9177)

Note: ( ) RVM is Relevant Vector Machines.

Table 2: Thailand: Performance measurement of AR(p) and RVM(p) model in training period

	Validation				
	AR(1) RVM(1)	AR(2) RVM(2)	AR(3) RVM(3)	AR(4) RVM(4)	AR(5) RVM(5)
MAE	5.2864 (3.3098)	5.0016 (2.9055)	5.9186 (2.8067)	5.5015 (2.7127)	5.6159 (2.3639)
MAPE	3.6583 (1.5232)	**3.4573** (0.9774)	3.6622 (1.1060)	3.9238 (0.8544)	3.7323 (**0.8168**)
MSE	35.7284 (17.3119)	32.4813 17.5888	41.1182 (13.6884)	41.8034 (13.4915)	37.0558 (11.3789)
RMSE	5.9773 (4.1608)	5.6992 (4.1939)	6.4123 (3.6998)	6.4656 (3.6731)	6.0873 (3.3733)

Note: ( ) RVM is Relevant Vector Machines.

Table 3: Thailand: Performance measurement of AR(p) and RVM(p) model in validation period

	Training				
	AR(1) RVM(1)	AR(2) RVM(2)	AR(3) RVM(3)	AR(4) RVM(4)	AR(5) RVM(5)
MAE	2.3315 (1.9262)	2.3915 (1.7713)	2.3938 (1.5221)	2.4469 (1.7983)	2.4658 (2.7639)
MAPE	0.9432 (0.6921)	0.9577 (0.5789)	**0.8756** (**0.4006**)	0.873 (0.5271)	0.8888 (0.7808)
MSE	12.0519 (7.7763)	12.2192 (5.9786)	12.0728 (3.8970)	12.0838 (6.1957)	12.3274 (13.024)
RMSE	3.4716 (2.7886)	3.4956 (2.4451)	3.4746 (1.9741)	3.4762 (2.4891)	3.511 (3.6089)

Note: ( ) RVM is Relevant Vector Machines.

Table 4: Malaysia: Performance measurement of AR(p) and RVM(p) model in training period

	Validation				
	AR(1) RVM(1)	AR(2) RVM(2)	AR(3) RVM(3)	AR(4) RVM(4)	AR(5) RVM(5)
MAE	2.6738 (2.9094)	2.1889 (3.1892)	3.0356 (2.1753)	2.5565 (2.0685)	2.6357 (2.2859)
MAPE	0.8681 (0.8594)	0.7729 (0.8839)	0.9538 (**0.6840**)	**0.8532** (0.7186)	0.8647 (0.762)
MSE	11.2491 (13.5673)	9.4088 (15.1655)	13.8758 (10.506)	11.3984 (8.8782)	11.519 (10.2886)
RMSE	3.354 (3.6834)	3.0674 (3.8943)	3.725 (3.2413)	3.3761 (2.9796)	3.394 (3.2076)

Note: ( ) RVM is Relevant Vector Machines.

Table 5: Malaysia: Performance measurement of AR(p) and RVM(p) model in validation period

	Training				
	AR(1) RVM(1)	AR(2) RVM(2)	AR(3) RVM(3)	AR(4) RVM(4)	AR(5) RVM(5)
MAE	2.7866 (2.2343)	2.8252 (2.6595)	2.8282 (2.9797)	3.8996 (2.5946)	2.5846 (2.3361)
MAPE	0.9574 (**0.5839**)	0.976 (0.8062)	0.9657 (0.8670)	**0.8793** (0.8441)	0.8918 (0.7896)
MSE	14.8781 (9.0278)	15.2022 (11.9878)	15.2067 (13.8363)	11.4428 (11.1889)	11.2473 (9.3123)
RMSE	3.8572 (3.0046)	3.899 (3.4623)	3.8996 (3.7197)	3.3827 (3.3450)	3.3537 (3.0516)

Note: ( ) RVM is Relevant Vector Machines.

Table 6: Singapore: Performance measurement of AR(p) and RVM(p) model in training period

	Validation				
	AR(1) RVM(1)	AR(2) RVM(2)	AR(3) RVM(3)	AR(4) RVM(4)	AR(5) RVM(5)
MAE	5.9062 (3.7305)	5.9274 (4.3605)	5.978 (3.1647)	5.6508 (3.107)	5.2782 (3.1338)
MAPE	**2.7814** (1.6223)	3.139 (1.7119)	3.1235 (**1.3915**)	3.231 (1.4601)	2.9652 (1.6173)
MSE	40.2639 (30.0582)	44.7216 (23.6553)	43.8145 (17.5625)	45.0624 (15.572)	38.9669 (15.4224)
RMSE	6.3454 (5.4825)	6.6874 (4.8637)	6.6193 (4.1908)	6.7129 (3.9461)	6.2424 (3.9271)

Note: ( ) RVM is Relevant Vector Machines.

Table 7: Singapore: Performance measurement of AR(p) and RVM(p) model in validation period

5 Conclusion

The paper focuses on forecasting GDP in ASEAN countries. The time series data sets of GDP from monthly over the period 1961 to 2017, totally are 57 observations. In this paper, we employ two different models namely Auto Regressive model (AR(p)) model, Relevant Support Vector Machines (RVM) to forecast GDP in ASEAN countries.

According to the results, we compared to determine the efficacy of the four forecasting methods used in the present study. The mean absolute error (MAE), mean absolute percentage error (MAPE), Mean squared error (MSE), and the root mean square error (RMSE) were selected as the measures of evaluation. Thus, the empirical results are based on four evaluation indices. However, the present study shows that the RVM has the best forecasting performance in terms of lowest MAPE and RVM all of performance measure in RVM(p) lower than AR(p) in training data and validation data. So that RVM(p) dominate AR(p) for all p in GDP in Thailand, Malaysia and Singapore.
For the next future research, we will apply RVM to other economics problem and compare with another econometric model.

Figure 2: THALAND GDP forecasting

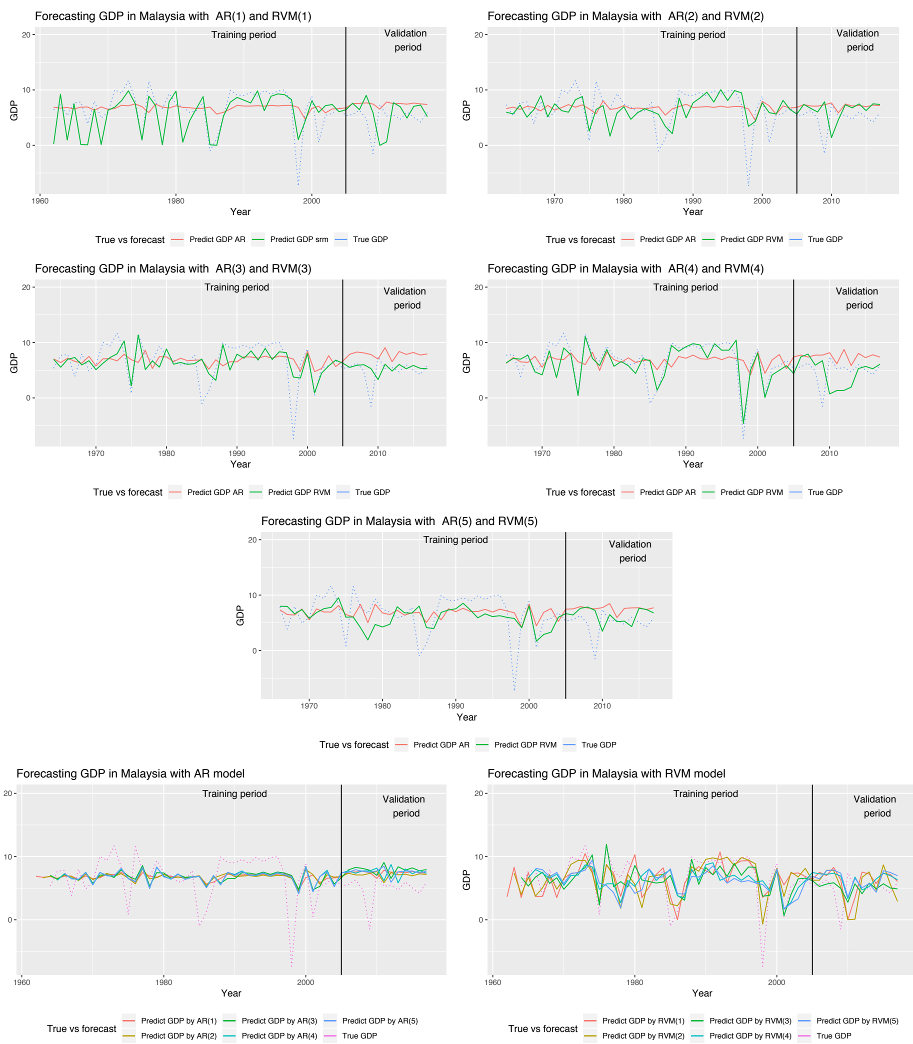

Figure 3: Malaysia GDP forecasting

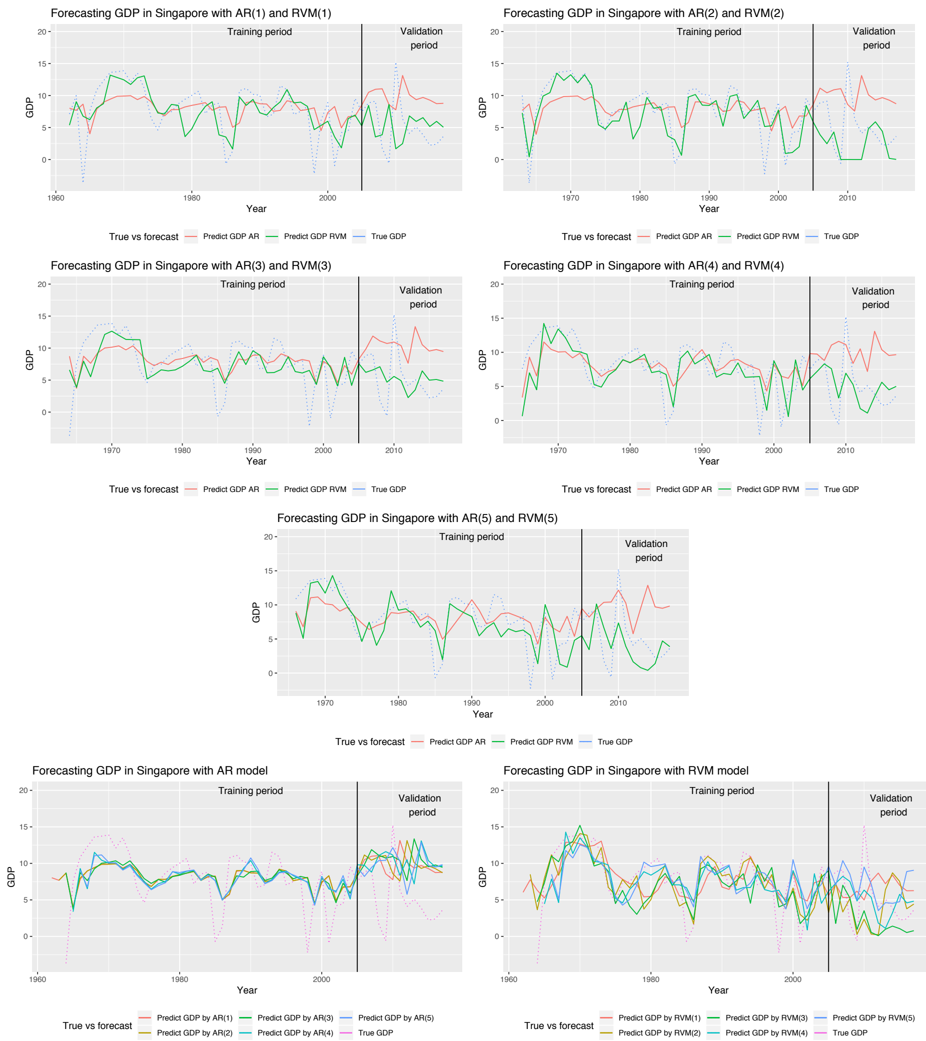

Figure 4: Singapore GDP forecasting

Acknowledgements : We are grateful for financial support from Centre of Excellence in Econometrics, Faculty of Economics, Chiang Mai University. We thanks referee for valuable comment to improve this paper.

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(Received 25 November 2018)
(Accepted 5 February 2019)