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Bayesian Estimation for Fully Shifted Panel AR(1) Time Series Model

Jitendra Kumar[1] **and Varun Agiwal**

Department of Statistics, Central University of Rajasthan
Bandersindri, Ajmer, India
e-mail : vjitendrav@gmail.com (J. Kumar)
varunagiwal.stats@gmail.com (V. Agiwal)

Abstract : Present manuscript proposed a shifted panel autoregressive (PAR) model through structural break assumption. A Bayesian estimation method is developed considering known from of prior information. Since expression of posterior distribution under different loss functions is in complicated form, therefore Gibbs sampler technique is used to obtain the conditional posterior distribution. A simulation and empirical study for proposed shifted panel AR(1) model is carried out to record the performance of the Bayes estimators and compared with the classical procedures such as maximum likelihood and least square estimator. A realization of real data set is also explored to illustrate the prospective interpretation of the proposed model.

Keywords : Bayesian estimator; information criterion; panel autoregressive model; structural break.
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1 Introduction

In statistics, parameter estimation is a concept to make implication about unknown quantities of interest related to a real data set. It has equal importance

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[1]Corresponding author email.

in time series model associated with structural break. The issue of estimation in time dependent model contains a vast amount of literature in statistics, time series, econometric etc. Albert and Chib [1] and Barnett et al. [2] proposed a Bayesian method via Gibbs sampling not only to estimate the parameters in autoregressive model but also enforcement of stationarity, identification of outliers and missing observations. Bai and Perron [3] and Wang and Zivot [4] addressed the issue of estimation and testing of multiple structural breaks occurring in linear and autoregressive model respectively. Meligkotsidou et al. [5] applied Bayesian approach to test the unit root hypothesis in autoregressive model when structural break is present in level, trend and error variance.

For the development of estimation procedure and model selection in panel data model related to structural break(s) mainly came into picture after 20^{th} century. De Wachter and Tzavalis [6] introduced GMM model selection criterion like AIC, BIC, HOIC and classical hypothesis test approach to detect the break point in panel data model and found that GMM-HQIC criterion is performs similar as classical hypothesis test. Bai [7] developed consistency and limiting distribution of break point in panel data model and estimated the parameters by least square and quasi maximum likelihood method. Lee and Yu [8] estimated the spatial panel autoregressive model including individual effect using maximum likelihood and observed that parameter consistency is based on data transformation technique and quasi maximum likelihood estimator. Kim [9] applied sum of square residuals method to estimate the deterministic time trend break in large panels. Safadi et al. [10] studied Bayesian methodology in panel AR(p) model and obtained predictive distribution with application in genetic evaluation of beef cattle. Lin and Ng [11] proposed two-step Pseudo threshold and K-means clustering estimators for estimating the parameter in panel data heterogeneity model.

More work regarding estimation, identification and testing the structural break in the panel data model has been carried out by many authors like De Watcher and Tzavalis [12], Chan et al. [13] etc. under the condition that model is heterogeneous or homogeneous in presence of endogenous and exogenous variables. Jirata et al. [14] had discussion on the application of panel data estimation using two-stage and GLS estimator for heterogeneity as well as endogenous explanatory variables. Lee and Yu [15] studied space-time filter in linear panel data model for estimation of coefficients and disturbance errors. Kessler and Munkin [16] proposed Bayesian estimation via Gibbs sampling procedure for non linear panel data model with fractional dependent and endogenous variable. Baltagi et al. [17] studied change point estimation in large dimensional panel data model with stationary or non-stationary regressor. Qian and Su [18] proposed penalized least squares and penalized method of moments to estimate panel data model with unknown number of breaks.

In the above literature, parameters have been estimated by classical methods such as ordinary least square, quasi maximum likelihood, penalized principal component etc. Some of the parameters estimation have been extended by using Gibbs sampler and Metropolis-Hastings (MH) algorithm in Bayesian inference under the availability of some prior information. In this paper, we have estimated the parameters of panel autoregressive model in consideration of break point in all parameters i.e. autoregressive coefficient, mean and error variance. A Bayesian approach is used to estimate the model parameters and compared with the classical techniques using the mean squared error. For this, we have derived conditional posterior distribution using known prior distribution under the Gibbs sampler technique. A simulation study is prepared to know the performance. Also, an application on agriculture production of fertilizer is illustrated. A realization of empirical study is also completed for better interpretation of the proposed estimation procedure for the proposed model.

2 Model Specification

The form of our model is extension of univariate AR(1) model having break point in autoregressive coefficient, intercept and error variance as considered by Meligkotsidou et al. [19]. Let us consider the following panel data time series model of order one (PAR(1)) with n cross sectional units and T time-periods in which a single structural break point may occur at any time-point T_B.

$$y_{it} = \begin{cases} \mu_{i1} + u_{it} & \text{for } t = 1, 2, ..., T_B \\ \mu_{i2} + u_{it} & \text{for } t = T_B + 1, ..., T \end{cases} \tag{2.1}$$

where $i = 1, 2, .., n$, stochastic error u_{it} follows PAR(1) process (see Equation (2.2)) and $\varepsilon_{it}'s$ are assumed to be identical and independently distributed (i.i.d.) normal random variable.

$$u_{it} = \begin{cases} \rho_1 u_{i,t-1} + \sigma_1 \varepsilon_{it} & \text{for } t = 1, 2, ..., T_B \\ \rho_2 u_{i,t-1} + \sigma_2 \varepsilon_{it} & \text{for } t = T_B + 1, ..., T \end{cases} \tag{2.2}$$

Utilizing Equation (2.2) in Equation (2.1), we get the following model

$$y_{it} = \begin{cases} \rho_1 y_{it-1} + (1-\rho_1)\mu_{i1} + \sigma_1 \varepsilon_{it} & \text{for } t = 1, 2, ..., T_B \\ \rho_2 y_{it-1} + (1-\rho_2)\mu_{i2} + \sigma_2 \varepsilon_{it} & \text{for } t = T_B + 1, ..., T \end{cases} \tag{2.3}$$

The main interest of study is to estimate the parameters $\rho_1, \rho_2, \mu_{i1}, \mu_{i2}, \sigma_1$ and σ_2 under Bayesian framework and compare with classical estimators. It can be seen that this model can nest different models that consider break point in only one or two parameters or no break point. In a model comparison setting, we may consider various models obtained from Equation (2.3) under the condition that the parameters are having break or no break. First, we assume that these is no break point in autoregressive coefficient (Bai [7]), Model (2.3) is reduced to

$$y_{it} = \begin{cases} \rho y_{it-1} + (1-\rho)\mu_{i1} + \sigma_1 \varepsilon_{it} & \text{for } t = 1, 2, ..., T_B \\ \rho y_{it-1} + (1-\rho)\mu_{i2} + \sigma_2 \varepsilon_{it} & \text{for } t = T_B + 1, ..., T \end{cases} \tag{2.4}$$

Further, consider that there is break in intercept term only, then model reduces to (Bai [7])

$$y_{it} = \begin{cases} \rho y_{it-1} + (1-\rho)\mu_{i1} + \sigma\varepsilon_{it} & \text{for } t = 1, 2, ..., T_B \\ \rho y_{it-1} + (1-\rho)\mu_{i2} + \sigma\varepsilon_{it} & \text{for } t = T_B + 1, ..., T \end{cases} \tag{2.5}$$

On the other hand, we can consider the case where intercept term is equal in both segments, while error variance and autoregressive coefficient are different (Pesaran [20]), then Model (2.3) becomes

$$y_{it} = \begin{cases} \rho_1 y_{it-1} + (1-\rho_1)\mu_i + \sigma_1\varepsilon_{it} & \text{for } t = 1, 2, ..., T_B \\ \rho_2 y_{it-1} + (1-\rho_2)\mu_i + \sigma_2\varepsilon_{it} & \text{for } t = T_B + 1, ..., T \end{cases} \tag{2.6}$$

There is break only in variance, then Model 2.6 becomes (Bai [7])

$$y_{it} = \begin{cases} \rho y_{it-1} + (1-\rho)\mu_i + \sigma_1\varepsilon_{it} & \text{for } t = 1, 2, ..., T_B \\ \rho y_{it-1} + (1-\rho)\mu_i + \sigma_2\varepsilon_{it} & \text{for } t = T_B + 1, ..., T \end{cases} \tag{2.7}$$

If there is no structural break in the error variance, Model (2.3) is considering break in autoregressive coefficient as well as intercept term (De Wachter & Tzavalis [12]), i.e.,

$$y_{it} = \begin{cases} \rho_1 y_{it-1} + (1-\rho_1)\mu_{i1} + \sigma\varepsilon_{it} & \text{for } t = 1, 2, ..., T_B \\ \rho_2 y_{it-1} + (1-\rho_2)\mu_{i2} + \sigma\varepsilon_{it} & \text{for } t = T_B + 1, ..., T \end{cases} \tag{2.8}$$

and further ignoring break in intercept term (Liu et al. [21]), the model becomes

$$y_{it} = \begin{cases} \rho_1 y_{it-1} + (1-\rho_1)\mu_i + \sigma\varepsilon_{it} & \text{for } t = 1, 2, ..., T_B \\ \rho_2 y_{it-1} + (1-\rho_2)\mu_i + \sigma\varepsilon_{it} & \text{for } t = T_B + 1, ..., T \end{cases} \tag{2.9}$$

Finally, if no break is considered in any parameter (Levin et al. [22]), then model (2.3) reduces to standard panel autoregressive model, i.e.,

$$y_{it} = \rho y_{it-1} + (1-\rho)\mu_i + \sigma\varepsilon_{it} \qquad \text{for } t = 1, 2, ..., T \tag{2.10}$$

3 Bayesian Inference

Bayesian inference is an approach that apply Bayes rule in order to make probability estimate of a hypothesis which utilize the available prior information. Prior distribution gives us information about unknown parameter, one may use both informative and non-informative priors. For present study, we are considered the following prior distribution given in Schotman and Van Dijk [23] and Phillips [24].

$$P(\rho_j) = \frac{1}{1 - l_j}; \quad l_j < \rho_j < 1 \text{ and } l_j > -1 \tag{3.1}$$

$$P(\sigma_j^2) = \frac{d_j^{c_j}}{\Gamma(c_j)} (\sigma_j^2)^{-c_j-1} exp\left[-\frac{d_j}{\sigma_j^2}\right]; \quad c_j, d_j > 0 \tag{3.2}$$

$$P(\mu_{ij}) = \frac{1}{(2\pi)^{\frac{1}{2}}\tau_j\sigma_j} exp\left[-\frac{1}{2\tau_j^2\sigma_j^2}\left(\mu_{ij}-\vartheta_{ij}\right)^2\right]; \quad -\infty < \vartheta < \infty; \tau > 0 \tag{3.3}$$

In prior distribution, some unknown constant is also there, known as hyper parameter. For simplification, we have assumed all hyper-parameters to be known and chosen such that it appropriately prcised the form of unknown parameters. Using the likelihood function that contains sample information with available probabilistic parametric information in form of joint prior distribution, the posterior distribution is given as

$$\pi\left(\Theta|y\right) = K\left(\frac{(\sigma_1^2)^{-\left(\frac{nT_B+n}{2}+c_1+1\right)}(\sigma_2^2)^{-\left(\frac{n(T-T_B)+n}{2}+c_2+1\right)}d_1^{c_1}d_2^{c_2}}{(2\pi)^{\frac{nT}{2}+n}\tau_1^{-n}\tau_2^{-n}(1-l_2)(1-l_2)\Gamma(c_1)\Gamma(c_2)}\right)$$
$$\exp\left[-\frac{1}{2\sigma_1^2}\left\{\sum_{i=1}^{n}\sum_{t=1}^{T_B}\left(y_{it}-\rho_1 y_{i,t-1}-(1-\rho_1)\mu_{i1}\right)^2+2d_1\right.\right.$$
$$\left.+\frac{1}{\tau_1^2}\sum_{i=1}^{n}\left(\mu_{i1}-\vartheta_{i1}\right)^2\right\}-\frac{1}{2\sigma_2^2}\left\{\sum_{i=1}^{n}\sum_{t=T_B+1}^{T}\left(y_{it}-\rho_2 y_{i,t-1}-(1-\rho_2)\mu_{i2}\right)^2\right.$$
$$\left.\left.+\frac{1}{\tau_2^2}\sum_{i=1}^{n}\left(\mu_{i2}-\vartheta_{i2}\right)^2+2d_2\right\}\right] \tag{3.4}$$

From decision theory view point, in order to select the best estimator from the posterior distribution, a loss function must be specified in Bayesian inference. We have considered two loss functions: (1) A symmetric loss function which associate equal importance to the losses due to overestimation and underestimation in terms of equal magnitude and popularly known as squared error loss function (SELF) introduced by Gauss [25], (2) An asymmetric loss function known as entropy loss function (ELF) proposed by Calabria and Pulcini [26], stated that an over estimation may cause more seriousness than that of under estimation or vice versa. The notation of SELF and ELF loss functions are given as

$$L_S(\theta,\hat{\theta}) = \left(\hat{\theta}-\theta\right)^2$$
$$L_E(\theta,\hat{\theta}) = \left(\frac{\hat{\theta}}{\theta}\right) - \ln\left(\frac{\hat{\theta}}{\theta}\right) - 1$$

Bayes estimator of any parametric function under SELF and ELF is posterior mean and $(E(\theta^{-1}|y))^{-1}$ respectively. A major difficulty to carry out the Bayes procedure is that we do not get closed expression due to multiple integral in our posterior distribution.

4 Conditional Posterior Distribution

Using equation (3.4), conditional posterior distribution of each parameter is derived which may depend or not depend on other parameters conditionally. For computing the conditional posterior distribution of a particular parameter, one may integrate equation (3.4) with respect to other parameters and get the expression. Therefore, we have used Gibbs sampler technique to solve the integrals involve there in.

4.1 Conditional Posterior Distribution of Parameter at Pre-Break Point

$$\pi\left(\mu_{i1}|\rho_1, y, T_B\right) \propto \left[\tau_1^2 \sum_{i=1}^{n}\sum_{t=1}^{T_B}\left(y_{it}-\rho_1 y_{i,t-1}\right)^2 + 2d_1\tau_1^2 + \sum_{i=1}^{n}\vartheta_{i1}^2 - \sum_{i=1}^{n}\frac{\left(B(\rho_1)\right)^2}{A(\rho_1)} + A(\rho_1)\sum_{i=1}^{n}\left(\mu_{i1}-\frac{B(\rho_1)}{A(\rho_1)}\right)^2\right]^{-\left(\frac{nT_B+n}{2}+c_1\right)} \tag{4.1}$$

The shape of μ_{i1} follow a 3-parameter non-central Student t-distribution with $nT_B + 2c_1$ degree of freedom.

$$\pi\left(\sigma_1^2|\rho_1, y, T_B\right) \propto \left(\sigma_1^2\right)^{-\left(\frac{nT_B}{2}+c_1\right)} \exp\left[-\frac{1}{\sigma_1^2}\left\{d_1 + \frac{1}{2}\sum_{i=1}^{n}\left(\sum_{t=1}^{T_B}\left(y_{it}-\rho_1 y_{i,t-1}\right)^2 + \frac{\vartheta_{i1}^2}{\tau_1^2} - \frac{\left(B(\rho_1)\right)^2}{\tau_1^2 A(\rho_1)}\right)\right\}\right] \tag{4.2}$$

The conditional posterior distribution of σ_1^2 is inverse gamma distribution.

$$\pi\left(\rho_1|\mu_{i1}, \sigma_1^2, y, T_B\right) \propto \exp\left[-\sum_{i=1}^{n}\sum_{t=1}^{T_B}\frac{\left(y_{it}-\mu_{i1}\right)^2}{2\sigma_1^2}\left(\rho_1 - \frac{\left(y_{it}-\mu_{i1}\right)\left(y_{it-1}-\mu_{i1}\right)}{\left(y_{it}-\mu_{i1}\right)^2}\right)^2\right] \tag{4.3}$$

The form of ρ_1 is truncated normal distribution over the interval $(l_1, 1)$ where

$$A(\rho_1) = \tau_1^2 T_B (1-\rho_1)^2 + 1$$

$$B(\rho_1) = \tau_1^2(1-\rho_1)\sum_{t=1}^{T_B}(y_{it}-\rho_1 y_{i,t-1}) + \vartheta_{i1}$$

4.2 Conditional Posterior Distribution of Parameter at Post-Break Point

$$\pi\left(\mu_{i2}|\rho_2, y, T_B\right) \propto \left[\tau_2^2 \sum_{i=1}^{n} \sum_{t=T_B+1}^{T} \left(y_{it} - \rho_2 y_{i,t-1}\right)^2 + 2d_2\tau_2^2 + \sum_{i=1}^{n} \vartheta_{i2}^2 - \sum_{i=1}^{n} \frac{\left(D(\rho_2)\right)^2}{C(\rho_2)} + C(\rho_2)\sum_{i=1}^{n}\left(\mu_{i2} - \frac{D(\rho_2)}{C(\rho_2)}\right)^2\right]^{-\left(\frac{n(T-T_B)+n}{2}+c_2\right)} \tag{4.4}$$

The shape of μ_{i2} follow a 3-parameter non-central Student t-distribution with $n(T-T_B)+2c_2$ degree of freedom.

$$\pi\left(\sigma_2^2|\rho_2, y, T_B\right) \propto \left(\sigma_2^2\right)^{-\left(\frac{n(T-T_B)}{2}+c_2\right)} \exp\left[-\frac{1}{\sigma_2^2}\left\{d_2 + \frac{1}{2}\sum_{i=1}^{n}\left(\sum_{t=T_B+1}^{T}\left(y_{it} - \rho_2 y_{i,t-1}\right)^2 + \frac{\vartheta_{i2}^2}{\tau_2^2} - \frac{\left(D(\rho_2)\right)^2}{\tau_2^2 C(\rho_2)}\right)\right\}\right] \tag{4.5}$$

The conditional posterior distribution of σ_2^2 is inverse gamma distribution.

$$\pi\left(\rho_2|\mu_{i2}, \sigma_2^2, y, T_B\right) \propto \exp\left[-\sum_{i=1}^{n}\sum_{t=T_B+1}^{T}\frac{\left(y_{it}-\mu_{i2}\right)^2}{2\sigma_2^2}\left(\rho_2 - \frac{\left(y_{it}-\mu_{i2}\right)\left(y_{it-1}-\mu_{i2}\right)}{\left(y_{it}-\mu_{i2}\right)^2}\right)^2\right] \tag{4.6}$$

The form of ρ_2 is truncated normal distribution over the interval $(l_2, 1)$.
where

$$\begin{aligned} C(\rho_2) =& \tau_2^2(T-T_B)(1-\rho_2)^2 + 1 \\ D(\rho_2) =& \tau_2^2(1-\rho_2)\sum_{t=T_B+1}^{T}\left(y_{it} - \rho_2 y_{i,t-1}\right) + \vartheta_{i2} \end{aligned}$$

5 Simulation Study

Data can be analyzed properly in any field of science with the help of statistics. Statistics interpret the results in an approved manner with the help of simulation exercise. Simulation is a flexible methodology to analysis the behaviour of a proposed study and compare the performances. In simulation, generate a random sample random in such a way that generated series analyze the problem and summarize the results. It is one of the most widely used quantitative method because it is so flexible and can yield so many useful results.

Present section consist a simulation study to measure and examine the performance of proposed estimators. The performance of estimators are compared with other classical estimators using mean squared error based on simulated time series. Therefore, we have generated different size of series $T = \{120, 160, 200\}$ with single break point at different position $T_B = \{T/4, T/2, 3T/4\}$ under the assumption that break occurs in all panels is same. For series generation, initial observations are taken as $y_{i0} = \{10, 15, 20\}$ having three cross sectional units (n=3). The true value of parameters before and after the break is assumed to be different and prior distribution of these parameters contained hyper parameters which are also be known. For the autoregressive parameters, consider changes near the unit root, i.e. $\rho_1 = 0.90$ and $\rho_2 = 0.95$ with consideration of hyper parameter value of uniform prior is $l_1 = 0.8\rho_1$ and $l_2 = 0.8\rho_2$. The true value of intercept term at per-break point is $\mu_{i1} = \{13, 14, 15\}$ and post-break point is $\mu_{i2} = \{21, 22, 23\}$ while the hyper parameters are $\vartheta_{i1} = y_{i0}, \vartheta_{i2} = \bar{y}_{i.}, \tau_1^2 = 8$ and $\tau_2^2 = 16$. The initial value of error variance for making the simulation study is $\sigma_1^2 = 0.1$ and $\sigma_2^2 = 0.2$. The hyper parameter of error variance can be chosen such that prior may be known as non informative prior. For this we choose $c_1 = d_1 = c_2 = d_2 = 0.01$, i.e., variance nature is same, however short effect may change the variance value (see Meligkotsidou et al. [19]).

For complex and multiple integrals involved in posterior distribution, Markov Chain Monte Carlo (MCMC) technique is used which attempt to simulate a sample draws from posterior distribution. There are two popular MCMC methods, namely Gibbs sampler (Geman and Geman [27]) and Metropolis-Hastings (MH) algorithm (Hastings [28]). Here we used Gibbs sampler that provided the parameter estimate from the conditional posterior distributions using true values. Applying the actual value of the parameters and using the simulated series, estimate the parameter using conditional posterior distribution and calculated the mean squared error (MSE). The Gibbs algorithm simulation process consists the following steps:

1. Start with k=1 and initial value of $\mu_{i1}^0, \mu_{i2}^0, \sigma_1^{2,0}, \sigma_2^{2,0}, \rho_1^0, \rho_2^0$ at j^{th} stage.

2. Using Gibbs sampling, generate posterior samples from conditional posterior distribution.

3. Repeat steps 1-2 for all k= 1, 2,..., M and record the value of parameters after M iteration.

Table 1: AV and MSE of the estimators at pre-break point parameters with varying T and T_B

T		120			160			200		
TB		T/4	T/2	3T/4	T/4	T/2	3T/4	T/4	T/2	3T/4
μ_{11}	OLS	12.8073	12.9155	12.9896	12.8742	12.9635	12.9725	12.8824	12.9443	12.9796
		0.4053	0.1627	0.1125	0.2913	0.1507	0.0843	0.2651	0.1013	0.0737
	MLE	12.7682	12.8980	12.9780	12.8419	12.9512	12.9644	12.8592	12.9347	12.9744
		0.4053	0.1676	0.1127	0.2913	0.1522	0.0847	0.2651	0.1027	0.0737
	SELF	12.8794	12.9280	13.0033	12.8902	12.9734	12.9917	12.9014	12.9540	12.9932
		0.3637	0.1601	0.1064	0.2672	0.1454	0.0830	0.2526	0.1008	0.0728
	ELF	12.8794	12.9280	13.0033	12.8902	12.9734	12.9917	12.9014	12.9540	12.9932
		0.3814	0.1602	0.1064	0.2727	0.1454	0.0830	0.2602	0.1008	0.0731
μ_{12}	OLS	13.9521	14.0317	14.0171	14.0835	14.0306	13.9881	13.9871	14.0160	14.0179
		0.3733	0.1755	0.1014	0.2490	0.1192	0.0747	0.1830	0.0936	0.0625
	MLE	13.9521	14.0317	14.0171	14.0835	14.0306	13.9881	13.9871	14.0160	14.0129
		0.3733	0.1750	0.1010	0.2472	0.1186	0.0749	0.1830	0.0936	0.0625
	SELF	13.9723	14.0303	14.0134	14.0635	14.0305	13.9821	14.0011	14.0122	14.0100
		0.3590	0.1696	0.0984	0.2369	0.1185	0.0728	0.1801	0.0924	0.0602
	ELF	13.9703	14.0261	14.0136	14.0576	14.0232	13.9848	13.9907	14.0138	14.0099
		0.3538	0.1696	0.0984	0.2369	0.1185	0.0728	0.1812	0.0926	0.0602
μ_{13}	OLS	15.3226	15.1256	15.0788	15.2481	15.1066	15.0761	15.1563	15.0445	15.0421
		0.4887	0.1784	0.1300	0.3671	0.1284	0.0923	0.2465	0.0970	0.0699
	MLE	15.2829	15.1092	15.0684	15.2161	15.0948	15.0688	15.1326	15.0353	15.0370
		0.4887	0.1762	0.1300	0.3671	0.1283	0.0923	0.2465	0.0966	0.0697
	SELF	15.2040	15.1045	15.0510	15.1932	15.0760	15.0471	15.1191	15.0185	15.0217
		0.4346	0.1760	0.1288	0.3198	0.1297	0.0998	0.2317	0.0958	0.0697
	ELF	15.2040	15.1044	15.0510	15.1932	15.0760	15.0470	15.1191	15.0185	15.0217
		0.4170	0.1760	0.1283	0.3133	0.1278	0.0993	0.2281	0.0958	0.0696
σ_1^2	OLS	0.1468	0.1268	0.1201	0.1379	0.1206	0.1181	0.1289	0.1195	0.1181
		0.0039	0.0015	0.0007	0.0031	0.0008	0.0005	0.0018	0.0006	0.0005
	MLE	0.0945	0.0970	0.0986	0.0977	0.0973	0.0983	0.0964	0.0991	0.0989
		0.0002	0.0001	0.0001	0.0002	0.0001	0.0001	0.0002	0.0001	0.0001
	SELF	0.1044	0.1020	0.1020	0.1050	0.1011	0.1008	0.1023	0.1021	0.1009
		0.0002	0.0001	0.0001	0.0002	0.0001	0.0001	0.0001	0.0001	0.0001
	ELF	0.1021	0.1009	0.1012	0.1033	0.1002	0.1003	0.1010	0.1014	0.1005
		0.0002	0.0001	0.0001	0.0002	0.0001	0.0001	0.0002	0.0001	0.0001
ρ_1	OLS	0.8742	0.8790	0.8876	0.8770	0.8858	0.8907	0.8798	0.8869	0.8879
		0.0018	0.0011	0.0005	0.0016	0.0007	0.0005	0.0013	0.0006	0.0004
	MLE	0.8742	0.8790	0.8876	0.8770	0.8858	0.8907	0.8798	0.8869	0.8879
		0.0018	0.0011	0.0005	0.0016	0.0007	0.0005	0.0013	0.0006	0.0004
	SELF	0.8872	0.8911	0.8976	0.8907	0.8961	0.8992	0.8924	0.8962	0.8953
		0.0010	0.0007	0.0004	0.0010	0.0005	0.0004	0.0009	0.0004	0.0003
	ELF	0.8860	0.8904	0.8971	0.8897	0.8956	0.8988	0.8916	0.8958	0.8950
		0.0011	0.0008	0.0004	0.0010	0.0005	0.0004	0.0009	0.0004	0.0003

Table 2: AV and MSE of the estimators at post-break point parameters with varying T and T_B

T		120			160			200		
TB		T/4	T/2	3T/4	T/4	T/2	3T/4	T/4	T/2	3T/4
μ_{21}	OLS	20.7912	20.8184	20.2179	20.9388	20.6770	20.6801	20.8946	20.8229	20.6412
		1.1483	2.1233	7.8962	0.7813	1.4999	3.0693	0.6445	1.0334	2.5645
	MLE	20.7915	20.8182	20.0460	20.9387	20.6770	20.5629	20.8945	20.8226	20.6409
		1.1178	2.0601	7.8961	0.7745	1.4733	3.0570	0.6415	1.0133	2.5647
	SELF	21.0274	21.0881	20.6299	21.0614	20.9149	20.6793	21.0124	20.9710	20.7516
		1.0019	1.9363	3.8158	0.6827	1.3789	2.7385	0.6056	0.9274	2.2114
	ELF	20.9593	20.9861	20.6300	21.0256	20.8443	20.6829	20.9807	20.9219	20.7298
		1.0020	1.9364	4.1136	0.6827	1.3789	2.7490	0.6054	0.9273	2.1941
μ_{22}	OLS	21.8604	21.6798	21.1971	21.9628	21.8518	21.7840	21.8957	21.8320	21.7186
		0.9963	1.6893	6.0073	0.7942	1.3469	3.5373	0.6390	1.1022	2.5688
	MLE	21.8604	21.6796	21.0434	21.9626	21.8517	21.7885	21.8958	21.8316	21.7183
		0.9962	1.6890	6.0063	0.7745	1.3139	3.5440	0.6289	1.0907	2.5686
	SELF	22.1423	21.9663	21.5710	22.0287	22.0710	21.8535	22.0111	22.0129	21.8404
		1.0023	1.6992	3.2872	0.7223	1.1910	2.5980	0.6141	1.0282	2.1918
	ELF	22.0752	21.8709	21.5711	22.0061	22.0026	21.8186	21.9797	21.9660	21.7310
		0.9854	1.6620	3.5326	0.7224	1.1910	2.5339	0.6142	1.0282	2.1497
μ_{23}	OLS	22.7807	22.5848	22.3242	22.8338	22.7840	22.9362	22.9233	22.8721	22.6728
		1.3973	2.0027	6.9035	0.7889	1.3545	3.1224	0.7611	1.0031	2.7211
	MLE	22.7809	22.5846	22.1929	22.8337	22.7840	22.9395	22.9230	22.8720	22.6724
		1.3351	2.0027	6.9035	0.7692	1.3045	3.1137	0.7536	0.9946	2.7205
	SELF	23.0767	22.8766	22.6888	23.0110	23.0399	22.9726	23.0409	23.0280	22.7791
		1.1367	1.8177	3.9065	0.7002	1.0666	2.3822	0.6980	0.9644	2.0766
	ELF	23.0095	22.7821	22.6887	22.9694	22.9787	22.9588	23.0114	22.9842	22.6801
		1.1369	1.8004	4.1329	0.7001	1.0666	2.3477	0.6980	0.9642	2.0515
σ_2^2	OLS	0.2065	0.2047	0.2061	0.2039	0.2093	0.2076	0.2049	0.2048	0.2054
		0.0005	0.0006	0.0017	0.0003	0.0005	0.0009	0.0002	0.0003	0.0007
	MLE	0.1973	0.1940	0.1865	0.1972	0.1990	0.1921	0.1982	0.1959	0.1930
		0.0004	0.0005	0.0011	0.0002	0.0003	0.0006	0.0002	0.0003	0.0005
	SELF	0.2006	0.1995	0.1989	0.1995	0.2033	0.2021	0.1997	0.1991	0.2010
		0.0004	0.0005	0.0010	0.0002	0.0003	0.0006	0.0002	0.0003	0.0005
	ELF	0.1990	0.1972	0.1944	0.1984	0.2016	0.1986	0.1989	0.1978	0.1983
		0.0004	0.0005	0.0010	0.0002	0.0003	0.0006	0.0002	0.0003	0.0005
ρ_2	OLS	0.9418	0.9384	0.9293	0.9435	0.9393	0.9387	0.9437	0.9418	0.9383
		0.0002	0.0004	0.0011	0.0002	0.0003	0.0006	0.0001	0.0002	0.0004
	MLE	0.9418	0.9384	0.9293	0.9435	0.9393	0.9387	0.9437	0.9418	0.9383
		0.0002	0.0004	0.0011	0.0002	0.0003	0.0006	0.0001	0.0002	0.0004
	SELF	0.9489	0.9459	0.9348	0.9493	0.9461	0.9441	0.9489	0.9475	0.9446
		0.0001	0.0003	0.0006	0.0001	0.0002	0.0003	0.0001	0.0002	0.0003
	ELF	0.9488	0.9457	0.9344	0.9491	0.9460	0.9438	0.9488	0.9474	0.9443
		0.0001	0.0003	0.0006	0.0001	0.0002	0.0003	0.0001	0.0002	0.0003

For each generated series, classical and Bayes estimates have been calculated from the posterior samples. This process is repeated 5000 times, and average estimates (AV) and MSE of the estimators are obtained. The simulation results are summarized in Tables 1-2. As seen in Table 1, MSE for all estimators are decreasing as size of the series is increasing. When break point is considered from near to bottom of a series, MSE is decline. Both assumed classical method, ordinary least square (OLS) and maximum likelihood estimators (MLE) show approximately same magnitudes for their MSE. This shows that OLS is equally applicable as MLE for estimating the parameters exclude σ_1^2. For error variance, MSE of MLE is approximately same as Bayes estimator under SELF. Under Bayes estimator, ELF gives better estimates as compare to SELF in terms of smaller MSE value under ELF except ρ_1. Overall we concluded that Bayes estimator gives better estimates as compare to classical estimators because additional information is added about the parameters to reduce the MSE and improve the average estimates. It can also observed by considering the suitable hyper parameter values in the prior distribution.

One can seen from Table 2, increase the size of a series, MSE for all estimators decreases as similarly recorded in Table 1. Consider different break point position, MSE is increasing because size of the series is decreasing after the break point. In classical approach, MLE give better estimates as compare to OLS because magnitude of MSE of MLE is less than OLS estimates. In Bayes estimation, SELF and ELF having approximately equal performance in terms of MSE. For error variance, both Bayes estimators have similar value with MLE in terms of their MSE. Thus, simulated data provided that Bayes estimators are good approximation technique to estimate the parameters with reduces MSE because conditional posterior distribution are coming in conditional closed form distribution which shape is approximately similar as prior distribution.

6 Real Data Analysis

In this section, we target to study a real series and demonstrate the use of proposed model in financial, economical, agricultural and so many series. To justify our theoretical results, we have taken data from a yearly book "Agricultural Statistics at a Glance 2014 "which is piled up by Directorate of Economics & Statistics, Department of Agriculture & Cooperation, Government of India. This provides statistical data on a broad area of agricultural commodities. For analysis purpose, import series of different fertilizers namely Nitrogen (N), Phosphate (P), potash (K) are taken which covered the period from 1980-81 to 2013-14, and considered this fertilizers as a panel. First of all, identified break point, compared with different models and then estimate the parameters using classical and Bayesian estimators. In R language, a command "strucchange "developed by Kleiber et al.[29] which find out the break point from individual series. With the help of this, we identify the structural break which is recorded in the Table 3.

Table 3: Structural break present in different fertilizers

Break point	Fertilizers		
	N	P	K
T_1	24	25	16
T_2	28	-	24

From Table 3, two breaks are present in fertilizers series N and K whereas one break is present in fertilizer series P, to change the structure of the model. The lowest break point is 16 and highest break point is 28. Therefore, we identify a single break in between 16 to 28 where proposed model is well suitable. For this, we use Akaike information criterion (AIC) and Bayes information criterion (BIC) to fitting the model in the import fertilizers series and determine the break point where AIC and BIC have minimum value. For best fitted model, we have estimated the parameter using maximum likelihood estimate and evaluate the value of AIC and BIC. It is observed that minimum value of AIC and BIC at $T_B = 23$ which is shown in Table 4. So, we use $T_B = 23$ as a structural break for our proposed model.

Table 4: Fitting the proposed model in fertilizers series under different break point

Break Point	-LogL	AIC	BIC
16	292.447	604.8946	630.5381
17	291.663	603.3256	628.9691
18	291.701	603.4026	629.0460
19	291.124	602.2488	627.8923
20	347.515	715.0299	740.6734
21	290.117	600.2335	625.8769
22	287.157	594.3139	619.9574
23	284.208	588.4168	604.0602
24	286.827	593.6543	619.2977
25	290.221	600.4427	626.0861
26	299.427	618.8541	644.4976
27	304.539	629.0770	654.7205
28	312.459	644.9176	670.5611

After knowing the break point, next step is to compare the applicability of the proposed model with other models given in Section (2). In Table 5, models are (i) proposed model (ii) break in intercept and error variance (iii) break in autoregressive coefficient and error variance (iv) break in autoregressive coefficient and intercept (v) break in error variance (vi) break in intercept (vii) break in autoregressive coefficient (viii) no break. To check the adequacy of model based on real series, all models mentioned above are computed for AIC and BIC which is shown in Table 5. The smaller value of information criterion corresponding to better fit of fertilizers series. In this table, one can easily see that panel AR(1) model where break is present in intercept, error variance and autoregressive coefficient having minimum AIC and BIC value. Thus, proposed model fitted well to this data set as compare to other models, specially without break model. Then, the classical as well as Bayesian estimates of this series for panel AR(1) model which consider break in all parameters are summarized in Table 6.

Table 5: Comparing various model by using information criterion

Model	-LogL	AIC	BIC
$PAR(\rho_1, \rho_2, \mu_{i1}, \mu_{i2}, \sigma_1, \sigma_2)$	284.2084	588.4168	604.0602
$PAR(\rho, \mu_{i1}, \mu_{i2}, \sigma_1, \sigma_2)$	291.5556	601.1112	624.1903
$PAR(\rho_1, \rho_2, \mu_{i1}, \mu_{i2}, \sigma)$	291.6030	601.2060	624.2852
$PAR(\rho_1, \rho_2, \mu_i, \sigma_1, \sigma_2)$	292.4919	602.9838	626.0629
$PAR(\rho, \mu_i, \sigma_1, \sigma_2)$	290.8094	593.6189	609.0049
$PAR(\rho, \mu_{i1}, \mu_{i2}, \sigma)$	290.6285	597.2571	617.7719
$PAR(\rho_1, \rho_2, \mu_i, \sigma)$	294.9445	601.8890	617.2750
$PAR(\rho, \mu_i, \sigma)$	305.2156	620.4312	633.2530

Table 6: Classical and Bayes estimates of parameter based on import of fertilizers series at $T_B = 23$

Parameter	OLS	MLE	SELF	ELF
μ_{11}	7.8229	7.8229	8.0339	7.1495
μ_{12}	5.7765	5.7765	6.1084	5.2454
μ_{13}	12.8207	12.8207	13.5626	12.6645
σ_1^2	16.8789	14.2187	16.9125	16.4869
ρ_1	0.6087	0.6087	0.6951	0.6803
μ_{21}	42.8628	42.8627	41.6199	41.5698
μ_{22}	26.4481	26.448	25.6386	23.8211
μ_{23}	24.5149	24.5149	25.62	23.5058
σ_2^2	60.4658	52.3612	46.5135	43.7864
ρ_2	0.5898	0.5898	0.6571	0.6439

7 Realization of Empirical Data

Realization of a data is an idea to decide whether real series give appropriate results for this model or not under control condition. To check this, we have generated a series based on our proposed model using estimated values of real data. The generated series is compared with statistic value of standard panel AR(1) model and result is shown in Figure 1. At break point 23, AIC and BIC shows smaller value as compare to panel AR(1) model which gives similar result as recorded in real fertilizer series. Thus, proposed model is well fitted for this generated series, and its estimated values are given in Table 7.

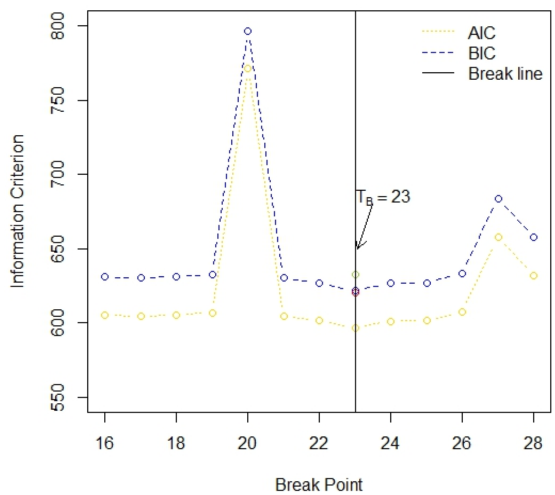

Figure 1: Break point identification under realization of the real series

Table 7: Classical and Bayes estimates under realization of the real series

Parameter	OLS	MLE	SELF	ELF
μ_{11}	6.7897	6.7897	6.9283	6.7489
μ_{12}	12.2024	12.2024	13.532	12.6326
μ_{13}	11.0318	11.0318	11.5135	10.3498
σ_1^2	32.6038	30.9623	30.1087	29.5276
ρ_1	0.4454	0.4454	0.5754	0.5495
μ_{21}	32.2276	32.2275	39.7964	35.3317
μ_{22}	23.3193	23.3193	24.3577	23.6828
μ_{23}	22.5701	22.5701	28.4783	24.9348
σ_2^2	106.7774	98.1452	114.2143	113.8955
ρ_2	0.5717	0.5717	0.8151	0.7933

8 Conclusion

In the proposed study, we examined the various types of parameter estimation in panel autoregressive time series model with structural break in intercept, autoregressive coefficient and error variance. A comparative study has been done to conclude that the Bayesian estimator gives better results in comparison with least square and maximum likelihood estimators. We conclude that our proposed model is well fitted in simulated and import fertilizers series. Realization of real data also analyzed that it correctly identified the break point. Thus, the Bayesian estimator is better than general classical estimator. The work may be extends for other models like model with time trend and non-normal error.

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