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Modeling Nonlinear Dependence Structure Using Logistic Smooth Transition Copula Model

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Abstract : This study introduces a new measure of dependence for financial studies in the context of nonlinear modelling, termed as the logistic smooth transition (LST) copula. The model is based on a bivariate copula incorporated with a threshold and smooth parameter. A Monte Carlo simulation shows that this dependence measure exhibits better performance than the classical copulas. Finally, this study applies the LST copula to measure the dependence structure between bond yields in advanced economies.

Keywords : nonlinear copula; logistic function; smooth transition; bond yields.
2010 Mathematics Subject Classification : 35K05; 91G20.

1 Introduction

Measures of dependence among random variables have been proposed and investigated empirically in various applications including economics and finance. The conventional method for measuring dependence was introduced by Karl Pearson in 1895, as Pearson correlation. However, this method came with a strong assumption of the sta-

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tistical linear dependence (Patton, [1]). This is unreliable and unrealistic for financial and economic applications that usually involve nonlinear correlation. Many dependency measures have been proposed after that to address this deficiency in which one of the well-known methods is Copula. Roughly, this method is simply a multivariate cumulative distribution function whose marginal distributions are uniform on the interval [0, 1]. Since copula is more flexible, there are many exiting studies in financial and economic pieces of literature that used it to measure the dependency among random variables. However, its use for capturing the nonlinear structure of dependence is still limited. Researchers have initiated proposals to incorporate the nonlinear structure and copula in modeling such as Rodriguez [2], Silvennoinen and Teräsvirta [3], Pastpipatkul, Yamaka and Sriboonchitta [4], and BenSada, Boubaker and Nguyen [5]. Most of them applied the Markov-switching (MS) approach of Hamilton [6] to the Copula to measure the dependence-switching phenomenon between financial variables. The switching in the dependence-switching copula model depends on an unobserved state variable governed by the Hidden Markov process.

The usual approach, however, has some limitations. First is the assumption of the switching regime governed by the first-order Markov chain, i.e. the current state variable depends on its immediate past value. However, this assumption is not likely to hold as the long memory of the regime change might exist. In addition, the model incorporates less prior information about the switching process. The switching process then may not reflect the real nonlinear structure dependence of the financial variables. Second is the transition function. According to the MS, the transition function relies on a filtered probability and does not incorporate the threshold variable. Thus, the information about the threshold point or break point in the MS-copula is concealed. Moreover, the Copula describing the dependence between the two variables may happen to change with time. Then the usual approach is to have two regimes, each with its own copula.

Therefore, this study proposes to modify the usual approach by assuming that there is a smooth transition between the regimes. This study applies the logistic transition function of Silvennoinen and Teräsvirta [3] to the Copula model and proposes a logistic smooth transition (LST) copula. The appealing feature of this model is an ability to capture gradual changes and sudden transition of dependence patterns. This method will allow the dependence structure between random variables to vary across different regimes. This dependence measure resembles the threshold (or change point) and smooth parameter. Moreover, in the estimation, Copula parameter can work separately from the parameters of a marginal model. That is, we first estimate the marginal distributions, while the dependence parameter is estimated in the second step through our proposed LST copula model. The asymptotic relative efficiency of the two-stage estimation maximum likelihood estimation was suggested and proved by Joe [7]. He mentioned that the two-stage estimation procedure was equivalent to the maximum likelihood estimation, and it was efficient enough to reach the global maximum and easy to be converged.

To assess the performance of the proposed dependence measure, the logistic smooth transition (LST) copula, we will perform two experiments: a Monte Carlo simulation and an empirical study. In the empirical study, we measure the relationship between bond yields of advanced economies and those of the United States. The economics literature shows that the interest rate in advanced countries that governments pay to borrowers in the long term depends on external factors, especially and importantly the yields of U.S. Treasury securities and monetary policy imposed by the Federal Reserve (Poghosyan, [8]). In addition, these correlations may contain a structural change as the yields are likely to covary when financial markets are heading down and up. Recently, for example, the world interest rates have been declined nearly the same in all advanced economies due to unprecedented forces. This finding suggests that the dependences among bond yields are not constant over time. Therefore, this study will estimate the dependency between the interest rates of six advanced economies (Canada, France, Germany, Italy, Japan, and the United Kingdom) and the United States using our smooth transition copula model.

The remainder of this paper proceeds as follows. In Section 2 we develop the logistic smooth transition copula model and describe its estimation strategy. In Section 3 we use a Monte Carlo simulation study to investigate the accuracy of our estimation. In Section 4 the empirical results are described. Conclusions are given in Section 5.

2 Methodology

This section is subdivided into: 2.1) a review of bivariate copula, which briefly explains the concept of bivariate copula and structure, 2.2) Logistic smooth transition copula, which is our proposed model, and 2.3) estimation technique

2.1 A Review of Bivariate Copula

According to Nelson [9], an n-dimensional copula $C(u_1,...,u_n)$ is a multivariate distribution function in [0, 1] whose marginal distributions are uniform in the interval [0, 1]. For the bivariate case, let x_1 and x_2 be continuous random variables that represents two bond returns with marginal distribution function $F_1(x_1)$ and $F_2(x_2)$; and let $H(x_1,x_2)$ be a joint distribution and $C()$ be a copula cumulative distribution function that completely describes the dependence structure between the two series

$$H(x_1,x_2) = C(F_1(x_1),F_2(x_2)). \tag{2.1}$$

If $F_1(x_1)$ and $F_2(x_2)$ are continuous, then the copula C associated with H is unique and may be obtained by

$$C(u_1,u_2) = H\left(F_1^{-1}(u_1),F_2^{-1}(u_2)\right), \tag{2.2}$$

where u_1 and u_2 are cumulative distribution function of standardized residuals which are all uniform on the interval [0,1]. Usually, we can measure the dependence of the random variable by θ^C. $F_1^{-1}(u_1)$ and $F_2^{-1}(u_2)$ are inverse cumulative distribution functions.

2.2 Logistic Smooth Transition Copula

We now present our nonlinear dependence structure copula model called Logistic smooth transition (LST) copula. Although our model can have more than two regimes, we only focus on the simple two-regime case throughout the study. This model combines the copula theory with Smooth Transition model of Silvennoinen and Teräsvirta [3]. Basically, the model is constructed in a similar way as introduced in the Markov Switching copula model, but the Hamiltons filtered probability (Hamilton, [6]) is replaced by logistic cumulative distribution function. This function is used as a smooth transition function in order to allow the copula parameter to switch across the regimes. We generalize the 1 regime copula dependence in Eq. (2.1) by introducing the logistic transition function $G(v_t,\gamma,\tau)$ which is assumed to be continuously differentiable with respect to the speed or smoothness (γ) and threshold (τ) parameters. Our model can be specified using the following model

$$\begin{array}{c} C\left(u_1,u_2\left|\theta_S^C,\gamma,\tau\right.\right)=G(v_t,\gamma,\tau)_t\cdot H\left(F_1^{-1}(u_1),F_2^{-1}(u_2)\left|\theta_{S=1}^C\right.\right)+ \\ (1-G(v_t,\gamma,\tau)_t)\cdot H\left(F_1^{-1}(u_1),F_2^{-1}(u_2)\left|\theta_{S=2}^C\right.\right) \end{array}, \tag{2.3}$$

where θ_S^C is the regime-dependent parameter with value assumed to be $\theta_{S=1}^C$ for high dependence regime and $\theta_{S=2}^C$ for low dependence regime. The logistic transition function is defined as

$$G(v_t,\gamma,\tau)_t=(1+[-\gamma(v_t-\tau)])^{-1} \quad , \tag{2.4}$$

where is the threshold variable, which is a generated sequential number between 0 and 1 with equal distance, say $v_t=[0,0.01,0.02,...,1]_T$., where the length of is equal to the number of observation T. We generate this threshold variable on the interval [0, 1] due to the marginal distributions u_1 and are uniform in the [0, 1]. Therefore, the threshold parameter can search the break point between 0 and 1. We note that when $\gamma\to\infty$, our model becomes the Sudden switch Threshold copula. When $\gamma\to 0$, our model becomes the one regime bivariate copula.

2.3 Estimation

In estimation, two step-estimation or the Inference Function of Margins (IFM) method (Joe and Xu, [10]) is used in this study as the one step estimation normally could be computationally intensive in the case of large parameter estimates as it requires the parameters of the margins and the parameters of the dependence and probability to be jointly estimated. Generally speaking, the method consists of estimating the parameters of the univariate marginal distributions in the first step and then using these estimates to estimate the dependence parameters in the second step. In this study, a AR(1)-GARCH(1,1) (this model will be explained later) is employed for estimating the marginal distributions in the first step, while Gaussian, Student-t, Gumbel, Clayton, Frank, or Joe copulas are used individually as a copula function in the second step.

We can write the likelihood for our problem as follows:

$$f(y_1,y_2)=f(\phi_1\left|y_1\right.)\cdot f(\phi_2\left|y_2\right.)\cdot\prod_{S=j}^{2}c\left(u_1,u_2\left|\theta_{S=j}^C,\gamma,\tau\right.\right), \tag{2.5}$$

where $c(u_1,u_2\left|\theta_{S=j}^C,\gamma,\tau\right.)=\partial C\left(u_1,u_2\left|\theta_S^C,\gamma,\tau\right.\right)/\partial u_1\partial u_2$ is the density function of the logistic smoothed transition bivariate copula in Eq. (2.3). $f(\phi_1\left|\mathrm{y}_1\right.)$ and $f(\phi_2\left|\mathrm{y}_2\right.)$ are the marginal density function of y_1 and y_2, respectively. Then, we can write the log-likelihood as

$$\log \mathrm{f}(\Theta\,|y_1, y_2) = \log f(\phi_1\,|y_1) + \log f(\phi_2\,|y_2) + \sum_{S=j}^{2} \log c(u_1, u_2\,\left|\theta^C_{S=j}, \gamma, \tau\right.), \quad (2.6)$$

where y_1 and y_2 are bond yields of US and another country. $\Theta_S = \{\phi_1, \phi_2, \theta^C_S, \gamma, \tau\}$ is the vector of all unknown parameters in the model. This estimation is straightforward, and we can estimate each AR(1)-GARCH(1,1) model in the first step:

$$\widehat{\phi}_m = \underset{\theta_m}{\arg\max} \sum_{t=1}^{T} \log l(\phi_m\,|y_m)\,, m = 1, 2. \quad (2.7)$$

We then collect the coefficients in a vector: $\widehat{\phi} = \left\{\widehat{\phi}_1, \widehat{\phi}_2\right\}$ and used these marginal parameters as the fixed parameters in our full likelihood Eq. 2.6. Thus, in the second step, we estimate only the two regime dependence parameters and or smoothness (γ) and threshold (τ) parameters jointly as

$$\widehat{\Theta}_{S_t} = \underset{\widehat{\Phi}}{\arg\max} \sum_{t=1}^{T} \left[\sum_{S=j}^{2} \log c(u_1, u_2\,\left|\theta^C_{S=j}, \gamma, \tau\right.) \log l(\widehat{\phi}_1\,|y_1) + \log l(\widehat{\phi}_2\,|y_2) \right] \quad (2.8)$$

Under certain regularity conditions, the IFM estimator verifies the property of asymptotic normality and can be seen as a highly efficient estimator compared to the one step estimation (Joe, [7]).

3 Simulation Study

In this section, we conduct a Monte Carlo simulation study to evaluate the performance of our proposed model. We exhibit different dependence structures that can be included by the LST Copula model and investigate the accuracy of the maximum likelihood estimation method using simulated data. Here, these data are drawn from six copula families namely Gaussian, Student-t, Gumbel, Clayton, Frank, or Joe copulas. Then, we simulate the data form the two-regime LST Copula model

$$\begin{aligned} C\left(u_1, u_2\,\middle|\theta^C_S, \gamma, \tau\right) = G\left(v_t, \gamma = 20, \tau = 0.4\right)_t \cdot H\left(F_1^{-1}(u_1), F_2^{-1}(u_2)\,\middle|\theta^C_{S=1}\right) + \\ (1 - G\left(v_t, \gamma = 20, \tau = 0.4\right)_t) \cdot H\left(F_1^{-1}(u_1), F_2^{-1}(u_2)\,\middle|\theta^C_{S=2}\right) \end{aligned}, \quad (3.1)$$

Table 1: Simulation sesults for Case 1, 2 and 3

T=100	Gaussian	Student-t	Gumbel	Clayton	Joe	Frank
$\theta^C_{S=1}$	0.387	0.350	2.797	0.599	3.069	3.984
	(0.099)	(0.117)	(0.313)	(0.090)	(0.345)	(0.890)
$\theta^C_{S=2}$	0.089	0.553	1.852	0.351	1.343	1.531
	(0.034)	(0.395)	(0.255)	(0.172)	(0.276)	(0.380)
γ	25.055	30.359	50.266	40.336	50.226	9.942
	(3.299)	(9.259)	(19.918)	(16.836)	(30.506)	(0.861)
τ	0.344	0.068	0.417	0.353	0.224	0.152
	(0.042)	(0.377)	(0.035)	(0.041)	(0.121)	(0.203)
T =500	Gaussian	Student-t	Gumbel	Clayton	Joe	Frank
$\theta^C_{S=1}$	0.635	0.560	2.631	0.515	2.511	3.405
	(0.049)	(0.044)	(0.124)	(0.046)	(0.146)	(0.391)
$\theta^C_{S=2}$	0.135	0.048	1.530	0.303	1.693	1.203
	(0.055)	(0.101)	(0.096)	(0.079)	(0.142)	(0.261)
γ	17.760	10.286	32.859	33.156	34.589	11.511
	(2.578)	(5.266)	(0.120)	(10.226)	(6.569)	(8.323)
τ	0.337	0.330	0.321	0.367	0.360	0.251
	(0.085)	(0.064)	(0.027)	(0.061)	(0.026)	(0.106)
T =1000	Gaussian	Student-t	Gumbel	Clayton	Joe	Frank
$\theta^C_{S=1}$	0.540	0.567	2.565	0.549	2.682	2.667
	(0.027)	(0.030)	(0.085)	(0.031)	(0.109)	(0.269)
$\theta^C_{S=2}$	0.107	0.090	1.579	0.329	1.505	1.261
	(0.055)	(0.081)	(0.082)	(0.058)	(0.125)	(0.163)
γ	20.216	10.026	23.277	20.006	26.088	15.126
	(0.022)	(4.392)	(0.099)	(0.002)	(0.637)	(2.556)
τ	0.353	0.365	0.298	0.357	0.361	0.312
	(0.015)	(0.031)	(0.026)	(0.052)	(0.019)	(0.022)

( ) denotes standard error

Consider three scenarios as follows:

1. Case 1: Gaussian and Student-t dependence parameters for regime 1 and 2 are, respectively, set to be $\theta^C_{S=1} = 0.4$ and $\theta^C_{S=2} = 0.1$. In the case of Student-t copula, we set degree of freedom of this copula to be 5 for both regimes
2. Case 2: Clayton dependence parameters for regime 1 and 2 are, respectively, set to be $\theta^C_{S=1} = 0.5$ and $\theta^C_{S=2} = 0.3$.
3. Case 3: Gumbel, Frank, and Joe dependence parameters for regime 1 and

2 are, respectively, set to be $\theta^{C}_{S=1} = 2.5$ and $\theta^{C}_{S=2} = 2$.

We simulated 100 bivariate data sets using the aforementioned parameters. Sample size T=100, 500, and 1,000 are used for all cases. We then fitted the simulated data sets and obtained the average of 100 parameter estimates and their standard deviations. Table 1 presents the simulated results where columns denote the averages of 100 parameter estimates and their standard deviations shown in the bracket for each copula family. It is found that the mean parameters are close to the true values and become closer when the sample size increases. The standard deviations are lower when the sample size increases. The two-regime copula parameters are estimated with a considerably high degree of accuracy. Only some estimated smooth parameters are found not close to the true values. This simulation study of the estimation method indicates that the two-stage estimation method tends to have a reliable and accurate prediction. Moreover, if a larger sample size is used, the higher degree of precision should be obtained.

4 Empirical Study

4.1 Data and Summary Statistics

In this study, we aim to examine the nonlinear dependence structure between the US bond yields and bond yields of six advanced countries. Therefore, our data consists of seven government bond returns of advanced countries. We collect monthly 10-year government bond returns of seven G7 member countries namely United States (US), Canada, France, United Kingdom (UK), Italy, Japan and Germany. The time period spans from the January 1991 to December 2017. with a total of 331 observations per country. The summary of descriptive statistics is illustrated in Table 2.

Table 2: Data description

	France	US	UK	Canada	Italy	Japan	Germany
Mean	-0.002	-0.077	-0.004	-0.003	-0.003	0.010	0.069
Median	-0.009	-0.012	-0.007	-0.006	-0.007	-0.019	-0.011
Maximum	1.043	6.000	0.308	0.270	0.340	5.000	18.000
Minimum	-0.561	-11.000	-0.270	-0.176	-0.169	-1.684	-1.154
Std. Dev.	0.118	0.976	0.059	0.056	0.061	0.369	1.085
Skewness	4.724	-5.656	0.422	0.840	1.574	7.601	14.572
Kurtosis	44.327	66.806	8.474	6.955	9.986	107.207	233.141
ADF-test	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]

[ ] is Maximum Bayes Factor (MBF)

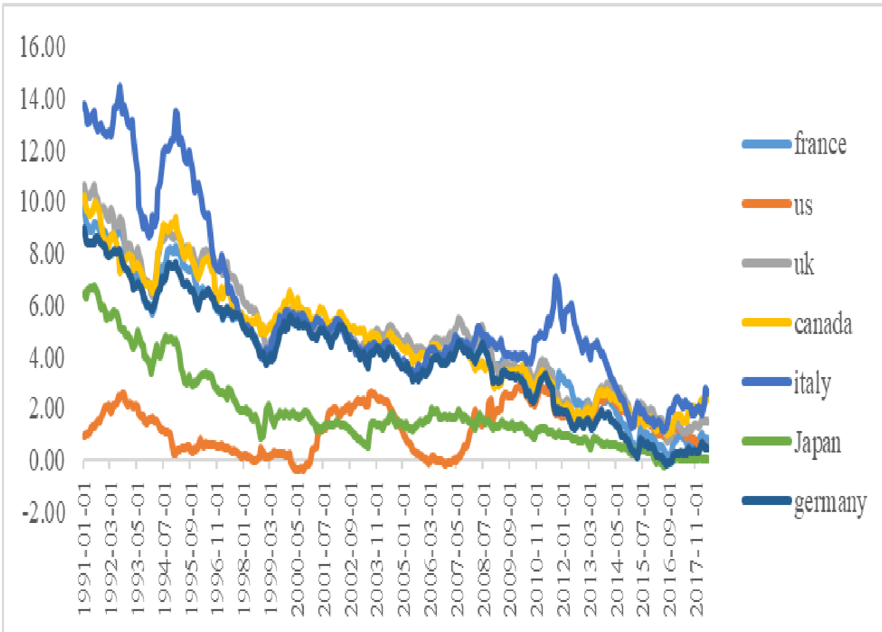

Figure 1: Nominal yields on long term government bonds over the past 30 years

Table 2 shows a statistical description and test including mean, median, maximum, minimum, standard deviation, skewness, kurtosis and Augmented Dickey Fuller (ADF) unit root test. The statistics points out that the average bond returns are smaller than their standard deviations, indicating relatively high risks in G7 bond markets. We observe that the means of bond returns of France, US, UK, Canada, Italy are positive. On the other hands, the means of Japan and Germany are negative. The volatility of bond returns of US and Germany are relatively high when compared to other markets. Furthermore, all returns have a distribution that is slightly right-tailed (positive skewness), except for US. Their kurtosis values are higher than the kurtosis value of the normal distribution (3). These characteristics indicate non-normality, asymmetry, and heavy tails for all series. Finally, stationary test is conducted, and the results show that our returns are strongly stationary, according to the zero Maximum Bayes Factor (MBF).

The modeling procedure adopted follows the steps described in Section 2. First the AR(1)-GARCH(1,1) with Student-t is estimated to obtain the standardized volatility for all bond returns. Then, the LST copula is conducted to measure the dependence between the volatility of US bond and those of other bond markets.

4.2 Marginal Distribution Modeling

Next, we discuss the specification of marginal models. In this study, we use AR(1)-GARCH(1,1) as the marginal model for the first step of two-stage maximum likelihood. Specifically, the model is expressed as

$$y_t = a_0 + a_1 y_{t-1} - a_2 \varepsilon_{t-1} + \varepsilon_t, \tag{4.1}$$

$$\varepsilon_t = \sigma_t z_t, \tag{4.2}$$

$$\sigma_t^2 = w + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2, \tag{4.3}$$

Where σ^2 is conditional variance obtained from the GARCH(1,1) process in Eq. (4.3). $\phi = \{a_0, a_1, a_2, w, \alpha, \beta, \nu\}$ is the vector of model parameters, where ν is the shape parameter of Student-t distribution. z_t is a standardized residual which is assumed to have Student-t distribution. Then, the standardized residuals are to be transformed into a uniform distribution in (0,1) using cumulative Student-t distribution. We note that these marginal parameters are plugged in the full likelihood of the LST copula and then estimate the dependence parameter in two regimes.

Table 3: Marginal estimation results

	a_0	a_1	w	α	β	ν
France	-0.016	0.241	0.006	0.648	0.351	2.760
	(0.007)	(0.055)	(0.002)	(0.149)	(0.078)	(0.244)
US	-0.009	0.303	0.000	0.328	0.671	4.708
	(0.003)	(0.058)	(0.000)	(0.105)	(0.082)	(1.200)
UK	-0.004	0.302	0.000	0.185	0.801	27.058
	(0.003)	(0.056)	(0.000)	(0.075)	(0.091)	(44.124)
Canada	-0.006	0.214	0.000	0.061	0.938	6.346
	(0.003)	(0.055)	(0.000)	(0.031)	(0.053)	(2.109)
Italy	-0.006	0.266	0.000	0.192	0.796	9.098
	(0.003)	(0.057)	(0.000)	(0.082)	(0.088)	(4.364)
Japan	-0.016	0.101	0.002	0.639	0.360	3.522
	(0.004)	(0.059)	(0.001)	(0.161)	(0.103)	(0.570)
Germany	-0.010	0.244	0.001	0.659	0.340	3.152
	(0.003)	(0.060)	(0.000)	(0.147)	(0.074)	(0.393)

( ) denotes standard error

The results of the marginals estimation are presented in Table 3. Apparently, the sum of the estimated parameters $\alpha + \beta$ is close to 1 for all cases. This means that the unconditional variance of the error terms is finite where the conditional variance evolves over time and is persistent. The estimated shape parameters of the Student-t distribution indicate the benefits of employing Student-t distribution to approximate the true shape of our returns. We then transform the standardized residuals generated from AR(1) - GARCH(1,1) model through the cumulative Student-t distribution. These uniform series are later used to model the dependence structure between the marginal distributions of each pair of US bond and an advanced bond market.

4.3 Model Comparison and the Estimation Results

Before discussing the empirical results, this study estimates the 1-regime copula model, including Gaussian, Student-t, Gumbel, Clayton, Frank, or Joe copulas. Then, we compare the performance of our proposed model to these single regime models by solely looking at the information criterion (AIC, BIC) approach as reported in Table 4. It is found that four out of six pairs (the US against France, UK, Canada, and Germany) are in favour of the 2-regime model as they have lower AIC and BIC compared to the single regime model. This result indicates that there exists a nonlinear dependence structure between bond markets and 1-regime copula which has been intensively employed in the literature may not be reliable in some financial markets. When considering the copula functions, we find the evidence that Frank copula yields the lowest AIC and BIC in US-UK, US-Italy and US-Germany pairs, Joe for US-Japan pair Student-t for US-France pair and Gaussian for US-Canada pair.

Table 5 presents the parameter estimates, Kendalls tau, tail dependence and robust standard errors. Details of how to compute from a theoretical Kendalls tau and tail dependence can be found in Joe [11]. Table 5 presents the estimated results of the best fitting models. We note that τ and γ are the threshold and speed or smooth parameter, respectively. We can see that the threshold parameter is 0.6852 for the pair of US-France, 0.6602 for US-UK, 0.6530 for US-Canada and 0.6562 for US-Germany. Interestingly, the threshold parameters of these four pairs are slightly different, though we expect that there might be a similar pattern of government bond yields in G7. The 2-regime estimated dependence parameters for US-France, US-UK, US-Canada, US-Germany, and the 1-regime estimated dependence parameters for US-Italy and US-Japan are reported in Table 5. First, let us consider US-Italy and US-Japan pairs, the dependence -measured by Kendalls tau- of US-Italy pair is higher than the US-Japan pair. It indicates the close relationship between the monetary policy of the US and Italy. For the other four pairs, the best fitting copula is LST copula. We can observe that the dependence between US bond yields and other bond yields in the 2-regimes model are different where the dependence of the 1-regime model is higher than that of the 2-regime model for all pairs. US-France is the only case in which we have an evidence of a tail dependence structure across regimes. This indicates a strong dependence between the US and France bond yields in the extreme event, such as economic boom and bust. Likewise, we also find that the US-Canada pair is the only case that exhibits a different sign of dependence between the two regimes. The theoretical Kendalls tau is 0.5086 for regime 1 switching to -0.0135 in the regime 2. This case provides strong evidence of switching parameters between regimes.

Figure 2 reports the estimated transition functions where the vertical axis is probability and the horizontal axis is time. This figure illustrates that the level of dependence between the interest rates of the four advanced countries and the US switch from the low dependence regime to the high dependence regime almost at the same time, i.e. during 2007-08 and later for the pair of the US-France. This period coincides with the financial crisis in 2007-2008 known as the global financial crisis. The plots of the estimated transition show that the level dependency is stronger during this period as confirmed by the greater magnitude of Kendalls tau values as presented in Table 5. However, the switching does not suddenly happen but gradually occurs as a smooth transition. This is likely to happen in most economic situations.

Table 4: Model Comparison

2-regime	Gaussian	Student-t	Clayton	Gumbel	Joe	Frank
US vs. France	-54.82	**-62.35**	-39.73	-54.88	-47.11	-36.57
	-39.65	**-47.18**	-24.56	-39.71	-31.93	-21.4
US vs. UK	-65.84	-42.34	-31.59	-58.88	-44.72	**-74.72**
	-50.67	-27.17	-16.42	-43.71	-29.55	**-59.55**
US vs. Canada	**-63.94**	-37.97	-43.24	-48.6	-32.55	-60.46
	-48.77	-22.8	-28.07	-33.43	-17.37	-45.29
US vs. Italy	5.13	35.35	4.8	4.17	4.35	14.95
	20.3	50.52	19.97	19.34	19.52	30.12
US vs. Japan	31.01	31.01	4.05	4.79	6.78	6.23
	46.18	46.18	19.23	19.96	21.95	21.4
US vs. Germany	-26.67	1.33	-10.1	-14.03	-6.26	**-32.35**
	-11.51	6.51	-5.06	-1.13	8.9	**-17.18**
1-regime	**Gaussian**	**Student-t**	**Clayton**	**Gumbel**	**Joe**	**Frank**
US vs. France	-10.46	-49.08	-11.29	-16.75	-17.56	-11.21
	-6.67	-41.5	-7.5	-12.96	-13.76	-7.42
US vs. UK	-12.53	-18.42	-4.38	-19.92	-16.21	-21.87
	-8.73	-10.84	-0.58	-16.13	-12.41	-18.08
US vs. Canada	-12.11	-17.09	-1.83	-18.76	-15.82	19.76
	-8.32	-9.51	1.97	-14.96	-12.03	-15.96
US vs. Italy	1.94	5.67	2	2.01	2.01	**1.94**
	5.74	13.26	5.8	5.8	5.8	**5.73**
US vs. Japan	1.23	3.68	2	1.27	**-0.67**	1.55
	5.02	11.26	5.79	5.07	**3.12**	5.34
US vs. Germany	-9.42	-10.54	-0.8	-11.01	-7.45	-14.56
	-5.63	-2.95	3	-7.22	-3.66	-10.76

Note: The first row is AIC value and the second row is BIC value of each model. Bold number indicates the lowest value.

Table 5: Estimation results

Model	1-regime Copula	2-regime Logistic Smooth Transition Copula			
	θ	$\theta_{S=1}$	$\theta_{S=2}$	γ	τ
US vs. France (Student-t)		0.822 (0.056) [0.614] {0.472,0.472}	0.569 (0.071) [0.385] {0.246,0.246}	59.189 (24.488)	0.685 (0.028)
US vs. UK (Frank)		7.192 (0.146) [0.571] {0.000,0.000}	0.146 (0.427) [0.016] {0.000,0.000}	100.000 (51.012)	0.660 (0.010)
US vs. Canada (Gaussian)		0.717 (0.039) [0.509] {0.000,0.000}	-0.014 (0.066) [-0.021] {0.000,0.000}	11.226 (0.259)	0.653 (0.011)
US vs. Italy (Frank)	0.086 (0.350) [0.010] {0.000,0.000}				
US vs. Japan (Joe)	1.0285 -0.04 [0.016] {0.000,0.038}				
US vs.Germany (Frank)		4.577 (0.684) [0.429] {0.000,0.000}	0.065 (0.528) [0.007] {0.000,0.000}	55.265 (6.490)	0.656 (0.011)

Note: Degree of freedom of Student-t copula are not provided. ( ) is standard error, [ ] is theoretical Kendalls tau associated with dependence copula parameter. are the lower and upper tail dependences respectively.

5 Conclusion

The presence of the structural shift in the dependence has been challenged by many empirical studies, however, the models used to deal with this issue are limited if not scarce. In this study, we introduce a new nonlinear dependence structure copula called smooth transition copula model. Unlike the conventional nonlinear dependence model, e.g. Markov Switching (MS) copula, the smooth transition copula is the regime switching model in which the transition probability function is governed by the logistic function with threshold and smooth parameters. The advantage feature of this model is an ability to capture gradual change and sudden transition of dependence patterns. Therefore, the model allows the dependence structure between random variables to vary across different regimes. The two-step maximum likelihood is used in this study to estimate the parameters in GARCH model and the smooth transition copula separately. The accuracy of the estimation is proved by the Monte Carlo simulation study and empirical study. The results show that the two-stage estimation method is accurate and reliable for our model and can be improved further if a larger data set is used. Finally, we apply our model to investigate the dependence between the US bond yields and the yields of six advanced countries in G7. Several copula families for both one-regime and two-regime copula models are compared using the AIC and BIC. It is found that four out of six pairs confirm the superiority of LST copula over the single regime model. According to the empirical results, this paper provides evidence that the dependence structure between the US bond yields and other bond yields in G7 changed after the occurrence of the financial crisis in 2007.

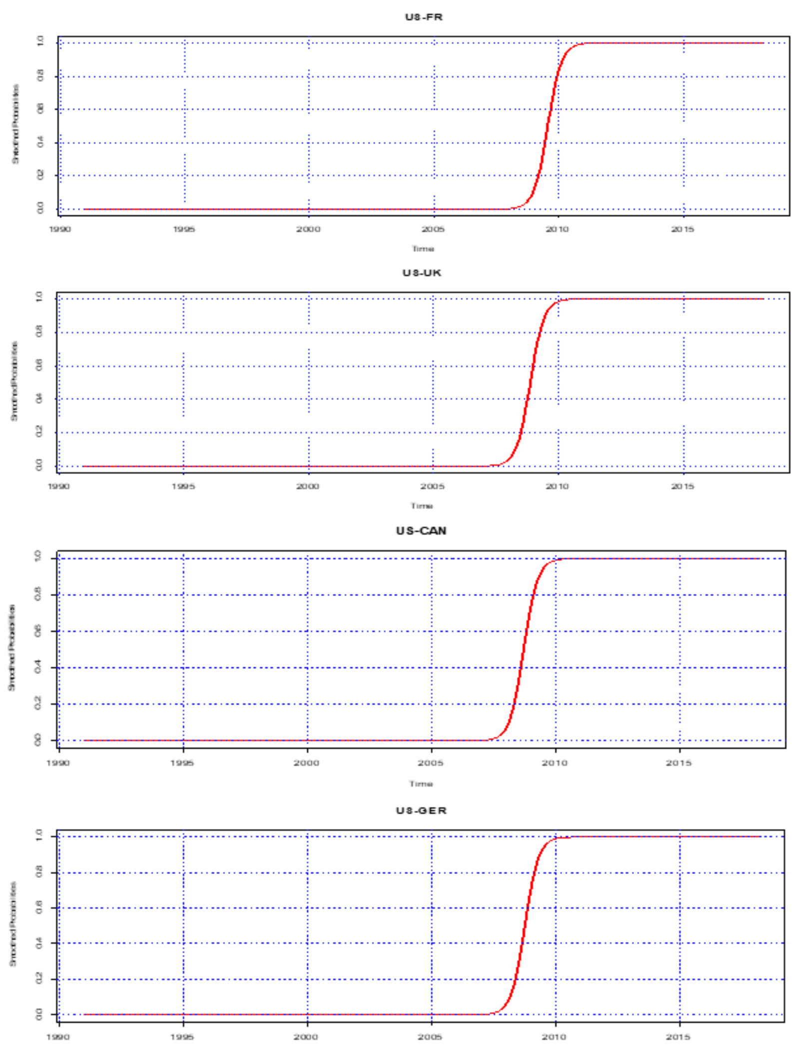

Figure 2: Estimated transition functions over time

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