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Impact of Population Aging on Consumption and Saving in Thailand : Structural Break Approach

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Abstract : This study aimed to investigate the impact of population aging on consumption and saving in Thailand over the period 1975-2016. Bai and Perron structural break test methodology was employed. Our analysis found that there are structure breaks in ratio of consumption to GDP and ratio of saving to GDP. Then, we employed least square with breakpoint to estimate consumption and saving equation. The results showed that impact of population aging are vary across regime. Moreover, there are negative impact of aging dependency on consumption and saving.

Keywords : structure break; population aging; consumption; saving. **2010 Mathematics Subject Classification :** 91B64; 91B84.

1 Introduction

The changes in population structure and its impact on income, consumption and saving have been a matter of controversial debate. In recent decade, Thailand is currently ranked the third most rapidly ageing population in the world [1]. The number of people aged 60 and over in Thailand now stands at about eight million,

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accounting for 13 percent of the population. Population ageing is a relatively new occurrence for Thailand; it was just in 2001 that Thailand became an ageing population with more than 7 percent of the population over 65. By 2040, Thailands aging population is expected to increase to 17 million, accounting for 25 percent of the population. This means that out of every four Thais, one will be a senior citizen. Moreover, Foundation of Thai Gerontology Research and Development institute [2] reported that fully one-third (34 percent) of elderly lived in families below the poverty line. The principal source (36.7 percent) of elderly income came from a child (including a foster or adopted child). The second most common source (33.9 percent) of elderly income was from work/employment. Nevertheless, the trend in elderly income from a child is on the decline, while work income is increasing.

Figure 1 showed that Thailand's consumption/GDP has been erratic with visible declines from 81 percent of GDP in 1975 to 66 percent of GDP in 2016, while saving/GDP and dependency ratio have risen steadily and more rapidly over the period 1975-2016.



Figure 1: Consumption, Saving and Old-Age dependency rate over the period 1975-2016

Figure 2 showed that the young-age dependency rate has continuously decreased since 1975. As the Figure 1 and 2, in Thailand, the elderly populations have increased, while and young populations have decreased slightly. This led to high dependency-ratio in Thailand.



Figure 2: Consumption, Saving and Young-Age Dependency rate over the period 1975-2016

Population aging is now becoming a new norm for Thailand. New norm brings paramount challenges. In this context, understanding the effects of population aging on consumption and saving in Thailand may provide valuable policy insights. If the old-age dependency ratio has a strong positive impact on consumption and negative effect on saving, then to some extent population aging will naturally and gradually promote domestic demand.

The purpose of this paper is to examine the impact of population aging on consumption and saving in Thailand for the period 1975-2016. In addition, since Perron [3] pointed out that the presence of structural breaks in a series can lead to misleading results; this paper takes into consideration the effect of structural breaks on the impact of population aging on consumption and saving in Thailand by using Bai and Perron [4] approach. This study examines the more general question of how serious a macroeconomic problem aging is and how policy should respond to it.

The remainder of the paper is organized as follows. Section 2 contains a brief literature review. In Section 3, we present a rigorous description of the methodology used in the analysis. Section 4 describes the data and presents the results of preliminary data analysis. The estimated models and empirical results are discussed in Section 5. Finally, Section 6 discusses the findings and draws conclusion.

2 Literature Reviews

Macroeconomic data such as income, consumption, and saving reflects different trends and pattern during different phases of an economic, social and political cycle. Structural break or unexpected shift in macroeconomic variable can lead to huge forecast error and unreliability of the model in general.

The existing literature on investigate impact of population aging on income consumption and saving is wide ranging both in terms of the different techniques employed and in terms of the different countries covered. Most of literature focus on no structural change relationship among them. For example, Dynan et al. [5] employed log-linear regression model to analyze effects of population aging on the relationship among aggregate consumption, saving, and income in US. Estrada et al. [6] use non-linear approach to examine the impact of the old-age dependency ratio on the share of consumption in the gross domestic products of 31 developing Asian economies and 122 from outside the region from 1998 to 2007. Erlandsen and Nymoen [7] investigated the empirical relationship between aggregate consumption and the age structure of the population in Norway.

However, there are few literatures investigated in the relationship among population aging, consumption and saving with structure change such as Attfield and Cannon [8] employ a vector error correction framework allowing for structural breaks.

3 Research Methodology: Least Squares with Breakpoints

The standard linear regression model assumes that the parameters of the model do not vary across observations. In spite of this assumption, structural change, which is the changing of parameters at dates in the sample period, plays an empirically relevant role in time series analysis. Hansen [9] and Perron [10] insist that estimations involving time series data should distinctly consider the effects of structural breaks or regime shifts. Therefore, this study employed least squares with breakpoints.

Tests for parameter instability and structural change in regression models have been an important part of applied econometric work dating back to Chow [11], who tested for regime change at a priori known dates using an F-statistic. To relax the requirement that the candidate breakdate be known, Quandt [12] modified the Chow framework to consider the F-statistic with the largest value over all possible breakdates. Andrews [13] and Andrews and Ploberger [14] derived the limiting distribution of the Quandt and related test statistics. More recently, Bai [15] and Bai and Perron ([16], [4]) provide theoretical and computational results that further extend the Quandt-Andrews framework by allowing for multiple unknown breakpoints.

The Bai and Perron ([16], [4]) approach to structural break considers a standard multiple linear regression model with T periods and m potential breaks (producing m + 1 regimes). Impact of Population Aging on Consumption ...

$$Y_{t} = X'_{t}\beta + Z'_{t}\gamma_{1} + \varepsilon_{t} \quad t = 1, ..., T_{1}$$

$$Y_{t} = X'_{t}\beta + Z'_{t}\gamma_{2} + \varepsilon_{t} \quad t = T_{1+1}, ..., T_{2}$$

$$\vdots$$

$$Y_{t} = X'_{t}\beta + Z'_{t}\gamma_{m} + \varepsilon_{t} \quad t = T_{m+1}, ..., T_{m}$$

$$(3.1)$$

where Y_t is observed dependent variable at time t. The regressors are divided into two groups. The X variables are those whose parameters do not vary across regimes, while the Z variables have coefficients that are regime-specific. β and $Y_j(j = 1, ..., m + 1)$ are the corresponding vectors of coefficients and ε is the disturbance term at time t. The break points (T_1, T_m) are treated as unknown. For the available data, the unknown regression coefficients and the break points are estimated. Once the number and identity of the breakpoints is determined, the model may be estimated using standard regression techniques. We may rewrite the equation specification as a standard regression equation.

$$Y_t = X'_t \beta + \bar{Z}'_t \bar{\gamma} + \varepsilon_t \tag{3.2}$$

with fixed parameter vectors β and $\bar{\gamma} = (\gamma_0, \gamma_1, ..., \gamma_m)$ where \bar{Z}'_t is an expanded set of regressors interacted with the set of dummy variables corresponding to each of the m + 1 regime segments. Bai and Perron[16] impose some restrictions on the possible values of the break dates. They define the following set for some arbitrary small positive number $\varepsilon \colon \Lambda = \{(\gamma_1, ..., \gamma_m); |\lambda_{i+1} - \lambda_i| \geq \varepsilon, \lambda_1 \geq \varepsilon, \lambda_m \leq 1 - \varepsilon\}$ to restrict each break date to be asymptotically distinct and bounded from the boundaries of the sample where the $\lambda_i (i = 1, ..., m)$ gives the break fraction $(\lambda_i = T_i/T_m)$.

The estimation method considers is based on the least squares principle proposed by Bai and Perron [16]. For each *m*-partition $(T_1, , T_m)$, denoted $\{T_j\}$, the associated least squares estimate of σ_j is obtained by minimizing the sum squared residuals $\sum_{t=1}^{m+1} \sum_{T_{i-1}+1}^{T_i} (Y_t - Z_t \gamma_i)^2$. Let $\hat{\gamma}(\{T_j\})$ denoted the resulting estimate. Substituting it in the objective function and denoting the resulting sum of squared residuals as $S_T(T_1, ..., T_m)$, the estimated break dates $(\hat{T}_1, ..., \hat{T}_m)$ are such that:

$$(\hat{T}_1, \dots \hat{T}_m) = \underset{(T_1, \dots, T_m)}{\arg\min} S_T(T_1, \dots, T_m)$$
(3.3)

where the minimization is taken over all partitions (T_1, T_m) such that $T_i - T_{i-1} \ge h$. Note that h is the minimal number of observations in each segment. Thus, the break point estimators are global minimizers of the objective function.

To test multiple structural changes in a linear model, there are four approaches:

- 1. A test of no break versus some fixed number of breaks. Requires a specification of the number of breaks under m.
- 2. A double maximum test, which tests a null hypothesis of no structural break against an alternative hypothesis of unknown number of breaks given some upper bound.

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 - 3. Tests of l versus l + 1 breaks. This considers a null hypothesis of l breaks against an alternative that additional break exists, the breaks obtained are obtained by global minimization of the sum of squared residuals.
 - 4. Sequential estimation of break points

In this study, we apply the sequential test which is a sequential test of l versus l+1 structural changes. The hypothesis is express as follows:

$$H_0: m = l$$

 $H_1: m = l + 1$ (3.4)

The strategy consists of applying tests for l+1 breaks against the null hypothesis of l breaks, for l = 0, 1, L-1, where L is the maximum number of breaks considered. The tests are applied for an increasing number l, but stop at $l = \hat{m}$ when the null hypothesis is not rejected for this l at the specified significance level. The F-test statistic is expressed as follows:

$$F_T(l+1|l) = \{S_T(\hat{T}_1, ..., \hat{T}_l) - \min_{1 \le i \le l} \inf_{r \in \wedge_{i,\rho}} S_T(\hat{T}_1, ..., \hat{T}_{i-1}, \tau, \hat{T}_i, ..., \hat{T}_l)\} / \hat{\sigma^2} \quad (3.5)$$

where $\wedge_{i,\rho} = \{\tau; \hat{T}_{i-1} + (\hat{T}_1 - \hat{T}_{i-1})\eta \le \tau \le \hat{T}_i - (\hat{T}_1 - \hat{T}_{i-1})\eta\},\$

 $S_T(\hat{T}_1, ..., \hat{T}_{i-1}, \tau, \hat{T}_i, ..., \hat{T}_l)$ is the sum of squared residuals resulting from the least squares estimation from each *m*-partition $(T_1, ..., T_m)$, and $\hat{\sigma}^2$ is a consistent estimation of σ^2 under the null hypothesis. η is a trimming parameter that sets the minimum length that a segment must be if it is further broken up. Following Bai and Perron [4] recommendation to achieve test with correct size of finite samples, we use a value of the trimming parameter of 0.15 and a maximum number of breaks m=5.

4 Framework and Data

There are two empirical models that will be used to test for the presence of structural break(s). The first one is consumption equation which is as follows:

$$\frac{C_t}{Y_t} = \beta_{0j} + \beta_{1j} DEPO_t + \beta_{2j} DEPY_t + \beta_3 lnGDPPC_t + \beta_4 lnPOP_t + u_{ct} \quad (4.1)$$

The second one is saving equation which is as follows:

$$\frac{S_t}{Y_t} = \alpha_{0j} + \alpha_{1j} DEPO_t + \alpha_{2j} DEPY_t + \alpha_3 lnGDPPC_t + \alpha_4 lnPOP_t + u_{st} \quad (4.2)$$

where $\frac{C_t}{Y_t}$ is the share of total consumption in GDP, $\frac{S_t}{Y_t}$ is the share of total saving in GDP, $DEPO_t$ is the ratio of people older than 64 to the working-age population, $DEPY_t$ is the ratio of people younger than 15 to the working-age population,

 $GDPPC_t$ is coord the log of gross domestic product per capita, and POP_t is the log of population. $\beta_{0j}, \beta_{1j}, \beta_{2j}, \alpha_{0j}, \alpha_{1j}$, and α_{2j} are allowed varying coefficients by regime while $\beta_3, \beta_4, \alpha_3$, and α_4 are coefficient that not vary over regime. u_{ct} and u_{st} are disturbance. bool To examine the link between age structure and consumption and saving more formally, we analyzed data from the World Bank's World Development Indicators [1] online database over the period 1975-2016.

Table 1 reports the basic descriptive statistics of all variables. The mean of the share of total consumption in GDP is 70.832 percent, while the mean of the share of total saving in GDP is 24.898 percent. Furthermore, the skewness and kurtosis of share of total consumption in GDP and share of total saving in GDP reveal the departure from normality in data, confirmed by the Jarque-Bera test.

Variables	Mean	Max	Min	S.D.	Skewness	Kurtosis	JB
$\frac{C_t}{Y_t}$	70.832	81.358	65.617	5.434	0.916	2.181	7.048
$\frac{S_t}{Y_t}$	24.898	44.340	5.700	12.023	-0.178	1.732	3.036
$DEPO_t$	9.117	15.342	6.508	2.732	0.745	2.286	4.773
$DEPY_t$	44.143	78.615	24.724	16.991	0.656	2.124	4.359
$lnGDPPC_t$	11.138	11.868	10.159	0.528	-0.355	1.767	3.545
$lnPOP_t$	17.875	18.048	17.561	0.145	-0.637	2.206	3.945

Table 1: Descriptive Statistics

5 Empirical Results

5.1 Impact of Population Aging on Thailands Consumption

Table 2 report the results of multiple break test or Bai and Perron [4] test . In order for the number of breaks to be determined, the null hypothesis of l breaks is tested versus the alternative of l+1 breaks, until it is no longer possible to reject the null hypothesis. Compare F-statistic with Bai and Perron [4] critical value, we found that the scaled F-statistic is significant at 5% level on 2 upward breaks in 1989 and 2007. This allows us to use the breakpoint regression as a next step.

Break Test	F-statistic	Scaled F-statistic	Bai and Perron critical value			
0 vs. 1 *	67.309	201.928	13.98			
1 vs. 2 *	49.498	148.494	15.72			
2 vs. 3	5.208	15.624	16.83			
Breaks : 1989, 2007						

Table 2: Multiple break test

Note: *denote significance at 5%.

We estimate a breakpoint model with $\frac{C_t}{Y_t}$ regressed on $DEPO_t$, $DEPY_t$, $lnGDPPC_t$, $lnPOP_t$, and a constant. The result is presented as Table 3.

	*		*	*
Variable	Coefficient	Std. Error	t-Statistic	MBF
Regime 1: 1975-1988				
Constant	997.114	288.530	-3.456	0.003
$DEPO_t$	-25.207	2.564	9.830	0.000
$DEPY_t$	-0.880	0.132	6.687	0.000
Regime 2: 1989-2006				
Constant	1169.098	276.835	-4.223	0.000
$DEPO_t$	-1.830	0.590	3.104	0.008
$DEPY_t$	-0.706	0.182	3.875	0.001
Regime 3: 2007-2016				
Constant	1058.319	278.550	-3.799	0.001
$DEPO_t$	-1.894	0.248	7.647	0.000
$DEPY_t$	-1.773	0.263	6.748	0.000
Non-Breaking Variables				
$lnGDPPC_t$	7.537	1.948	3.869	0.001
lnPOP _t	71.338	15.552	4.587	0.000

Table 3: Results of least squared with break: Consumption Equation

Note: This study calculated Minimum Bayes Factor(MBF) that proposed by Edwards, Lindman, and Savage [17]. A large MBF implies a total lack of evidence to reject the null. A small MBF implies possible rejection of the null.

Result from Table 3 revealed that the ratio of people older than 64 to the working-age population $(DEPO_t)$ exhibit a negative and significant effect on the share of total consumption in GDP (C_t/Y_t) over all the period 1975-2016. The estimated coefficient of $DEPO_t$ vary in subsequent periods between breaks. When we interpret the coefficient of the ratio of people older than 64 to the working-age population, the results mean that 1 percent increase of $DEPO_t$ lead to 25.207 percent, 1.83 percent, 1.894 percent decrease of C_t/Y_t in regime 1, regime 2, and regime 3, respectively.

Table 3 also showed that the ratio of people younger than 15 to the workingage population $(DEPY_t)$ has negative impact on the share of total consumption in GDP (C_t/Y_t) over all the period 1975-2016. 1 percent increase of $DEPY_t$ lead to 0.88 percent, 0.706 percent, and 1.773 percent decrease of C_t/Y_t in regime 1, regime 2, and regime 3, respectively.

Comparing coefficients, the results showed that population growth $(lnPOP_t)$ has a greater impact on the share of total consumption in GDP than the ratio of people older than 64 to the working-age population, the ratio of people younger than 15 to the working-age population, and gross domestic product per capita.

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5.2 Impact of Population Aging on Thailands Saving

Table 4 report the results of multiple break test or Bai and Perron [4] test. In order for the number of breaks to be determined, the null hypothesis of l breaks is tested versus the alternative of l + 1 breaks, until it is no longer possible to reject the null hypothesis. Compare F-statistic with Bai and Perron [4] critical value, we found that the scaled F-statistic is significant at 5% level on 1 upward breaks in 2001. This allows us to use the breakpoint regression as a next step.

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Break Test	F-statistic	Scaled F-statistic	Bai and Perron critical value		
0 vs. 1 *	7.677	23.032	13.98		
1 vs. 2	2.163	6.489	15.72		
Breaks: 2001					

 Table 4: Multiple break test

Note: *denote significance at 5%.

We estimate a breakpoint model with $\frac{S_t}{Y_t}$ regressed on $DEPO_t$, $DEPY_t$, $lnGDPPC_t$, $lnPOP_t$, and a constant. The result is presented as Table 5.

Variable	Coefficient	Std. Error	t-Statistic	MBF
Regime 1: 1975-2000				
Constant	-1119.161	648.244	-1.726	0.225
$DEPO_t$	-1.301	0.526	-2.472	0.047
$DEPY_t$	-0.629	0.319	-1.974	0.143
Regime 2: 1989-2006				
Constant	-1178.759	646.095	-1.824	0.189
$DEPO_t$	-0.789	0.639	-1.235	0.466
$DEPY_t$	-1.965	0.487	-4.034	0.000
Non-Breaking Variables				
$lnGDPPC_t$	18.610	3.391	5.488	0.000
lnPOP _t	-71.845	35.120	-2.046	0.123

Table 5: Results of least squared with break: Saving Equation

Note: This study calculated Minimum Bayes Factor(MBF) that proposed by Edwards, Lindman, and Savage [17]. A large MBF implies a total lack of evidence to reject the null. A small MBF implies possible rejection of the null.

Result from Table 5 revealed that the ratio of people older than 64 to the working-age population $(DEPO_t)$ exhibit a negative and significant effect on the share of total saving in GDP (S_t/Y_t) over all the period 1975-2016. When we interpret the coefficient of the ratio of people older than 64 to the working-age

population, the results mean that 1 percent increase of $DEPO_t$ lead to 1.301 percent, and 0.789 percent decrease of S_t/Y_t in regime 1, and regime 2, respectively. Table 5 also showed that the ratio of people younger than 15 to the working-age population $(DEPY_t)$ has negative impact on the share of total saving in GDP (S_t/Y_t) over all the period 1975-2016. 1 percent increase of $DEPY_t$ lead to 0.629 percent, and 1.965 percent decrease of S_t/Y_t in regime 1, and regime 2, respectively. Comparing coefficients, the results show that population growth has a greater impact on the share of total saving in GDP than the ratio of people older than 64 to the working-age population, the ratio of people younger than 15 to the working-age population, and gross domestic product per capita.

6 Conclusion

In this study we aimed to test impact of population aging on consumption and saving in Thailand over the period 1975-2016 by using the Bai and Perron ([16],[4]) structural break test methodology. The results of multiple break test or Bai and Perron [16] test showed that there are 2 upward breaks in 1989 and 2007 for Thailands consumption and there is 1 upward breaks in 2001 for Thailands saving.

Economic theories predict that age influences individuals saving and consumption behavior. In this study, we found that old-age dependency and young-age dependency have negative impact on consumption and saving. The negative impact imply that increase in the old-age dependency ratio and young age dependency ratio will reduce the disposable income of the three generations, child, working group and retiree, and this will mean a net decrease in the total consumption and saving of the family. This results similarly as Estrada et al. [6] who found evidence of a negative relationship, implying that their older populations reduce the share of consumption in national income.

The most straightforward policy implication of our results is that population aging in Thailand may not contribute significantly to robust consumption and domestic demand and hence to rebalancing, at least in economies in the early stages of the demographic transition. Policies may affect the relationship between population aging and aggregate consumption. For example, government should increase government expenditures on health care, education, pensions, and social protection. The underlying idea is that such spending will mitigate the need for savings and thus encourage households to spend more. Providing health care and pensions to the elderly will reduce the risks and uncertainties they face, so they will be likely to consume more.

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