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Modeling Dependence of Agricultural Commodity Futures through Markov Switching Copula with Mixture Distribution Regimes

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Abstract : Proposed is a Markov Switching copula with mixture distribution regimes for modeling the dependence of agricultural commodity futures. This model involves different dependence structures that can characterize the dependence behaviors in different regimes as the copula function in each regime can be different from that in another regime. By permitting different copula structure, this model is able to capture more complex dynamic patterns of daily movement of agricultural commodity futures (sugar, coffee, corn, wheat and soybean). The criteria as Akaike Information Criterion(AIC), Bayesian Information Criterion (BIC) and Log-Likelihood (LL) are based in-sample statistical performance have suggested that our model is superior to the single regime copula and two-regime Markov Switching copula in 9 out of 10 cases. This result reveals that the high and the low dependence of agricultural commodity futures exhibit a different dependence structure.

Keywords : Markov switching copula; mixture distribution regimes; agriculture commodity futures; dependence.

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1 Introduction

Recently, agricultural commodity futures have experienced somewhat synchronizing sequences of price trends and large price fluctuations. The rapid adjustments of agriculture commodities were in the spotlight following the food crises in late 2007 and early 2008, and again more recently in summer and fall of 2010 which have brought the issue about the price volatility into greater attention among researchers and investors in agriculture commodity future [1]. The investigation of how agricultural commodity futures interact with one another is of utmost importance, but the conclusions from previous studies regarding this issue are not unanimous. There are those who advocate that agricultural commodity futures exist in positive correlation, while others state just the opposite [2]. Thus, appropriately quantifying this dependence is of great importance in portfolio and risk management, option pricing and hedging.

Up until now, many empirical studies have shown the importance of several econometric approaches when investigating the links across commodity prices, especially GARCH and copula-based techniques. These models together are useful for capturing dependence or contagion effects related to volatility observations. The GARCH models can incorporate stylized facts of commodity returns, such as conditional heteroscedasticity and excess kurtosis, mostly by considering elliptical distributions. The copula-based models are a powerful and suitable framework, especially for dealing with nonlinear and tail dependence between random variables [3]. However, some recent studies raise the issue of the unsuitability of the time-invariant copula. Two main approaches have been proposed to deal with this issue. The first allows the parameters in a copula function to change over time such as dynamic copula of Patton [3] while the second allows the structure of copula function to shift across states or regimes of the economy such as Rodriguez [4] and Chollete et al. [5].

Numerous studies found that the estimated correlation coefficient between random variables will likely suggest a structural change (see, [6], [7], [8]). They found that there is an increase in correlation during the economic downtrend, while the correlation is low during economic uptrend. Therefore, the single regime copula approach fails to address this issue. Hence, we aim to deal with structural shifts in dependence through the Markov switching copula model. The characteristic of the model is that the copula function is subject to structural change according to a Hidden Markov process [6]. This approach permits for variability in the dependence structure, and it assumes that the copula parameter shifts across economic regimes [5].

However, the Markov switching copula model, as mentioned in the above paragraph, which has been limited to analyze the case of mixture distribution regimes. That is this model generally assumes the same copula function across two or more regimes which may not true in the reality. Maneejuk et al. [9] suggested that the financial time series could exhibit a mixture of distributions or populations, as a result of different characteristics of the data associated with different regimes, for example, distinct economic behaviors during economic upturn and downturn. So, we also expect that there might be a different copula function for different regimes.

Therefore, this study aims to develop the Markov switching of Rodriguez [4] and Chollete et al. [5] by allowing the different regime to have different copula function. The model is thus used to uncover the dependence and tail dependence among agricultural commodity futures where the dependence. may change according to the copula function. This model can well capture the potential dependence structure changes regime between positive and negative correlation across the agricultural commodity futures. Also, it provides complement for research framework to the prior studies that have relied on one copula regime over the whole sample period. In other words, we will allow the model to have different copula function across regimes, $\{1,...,h\}$, so, the copula density function, $c(u,v)$ for regime 1 can be either the same or different for other regimes. To simplify our analysis, we focus on two-regime Markov Switching model which is mostly implemented in the literature.

In our approach, in the first step we estimate the marginal distributions, while the dependence parameter is estimated in a second step through our proposed model. The GARCH functions as a filter to remove the serial dependence in the conditional means and conditional variances. Then, several combinations of two copula functions for two-regime model are estimated. To the best of our knowledge, this is the first attempt to extend flexibility the Markov Switching copula model. This is to say we allow the possibility of having two copulas from different copula classes. We then apply it to investigate the dependence among agricultural commodity futures.

The organization of the paper is the following. Section 2 describes methodology. Section 3 presents the data description. Section 4 presents estimated results. Section 5 provides some concluding remarks.

2 Methodology

In this section, the dependences between agricultural commodity futures pairs are modeled using a Markov Switching copula with mixture distribution regimes. In this study, we use two stage estimation method to estimate our proposed model, this method is also named as inference for the margins (IFM) [10]. Therefore, we can separate our estimation into two parts, namely, marginal and copula estimations. In this study, we employ a simple generalized autoregressive conditional heteroscedasticity (GARCH) with normal distribution to construct the marginal distributions and our model is used in the second step. We note that we consider the case of bivariate copula model in order to simplify our model and reduce the complexity of the model.

2.1 GARCH Model

The margins of agriculture futures series are modelled using GARCH model of Bollerslev [11]. This enables us to gain some important stylized facts of agriculture returns such as volatility clustering and also obtain approximate i.i.d. (independent and identically distributed) residuals that are suitable for further copula approach. Referring to most of the previous literature ([6], [7], [8]) we conduct a simple GARCH (1,1) specification which can be written as

$$y_t = c + \varepsilon_t = c + \sigma_t z_t \tag{2.1}$$

$$\sigma_t^2 = \omega + \alpha \varepsilon_{t-1} + \beta \sigma_{t-1}^2, \tag{2.2}$$

where y_t is the futures return, ω is intercept term and σ^2 is conditional variance of futures returns obtained from the GARCH process in Eq. (2.2). c is the constant term of mean Eq. (2.1). It is quite obvious the structure of GARCH (1, 1) consisting of parameter α and β which are assumed to be greater than zero and their summation must be less than 1. $\varepsilon_t = \sigma_t z_t$ is error term where z_t is a sequence of i.i.d. random variables with zero mean and unit variance and assumed to have normally distributed.

2.2 Dependence Modeling through Copula Function

Copula is the function that joins multivariate distribution functions of their uniform marginal distribution function [12]. Sklar [13] stated that a joint distribution can be factored into margins and a dependence function called a copula. In the bivariate case, if we assume x_1 and x_2 to have bivariate function F and univariate marginal distribution functions $F(x_1)$ and $F(x_2)$, then exists a bivariate copula C, such that

$$F(x_1, x_2) = C(F(x_1), F(x_2)) = C(u, v) \tag{2.3}$$

The copula function $C(u, v)$ is used to capture the dependence structure between uniform variables $u = F(x_1)$ and $v = F(x_2)$. If the marginals are continuous, then C is uniquely determined on $Ran(F(x_1)) \times Ran(F(x_2))$. Then, to obtain the function (called copula) density, Eq.(2.3) is differentiated. So, the bivariate copula density function c can be written by

$$c(u, v) = \frac{\partial C(F_1(x_1), F_2(x_2))}{\partial c(u, v)}. \tag{2.4}$$

2.3 Families of Copulas

In this subsection, we briefly present the bivariate copula density function used in this study. Here, we consider two classes, namely Elliptical copulas and Archimedean copulas.

2.3.1 Elliptical Copulas

Elliptical copulas are suitable for modeling the dependence structure in symmetric data. This class includes a Gaussian copula and t-copula.

1) Gaussian copula

Consider a case of two-dimensional copula, the density of the Gaussian copula is given by [14]

$$c_G(u,v|\theta_G) = \int_{-\infty}^{\Phi^{-1}(u\,\Phi^{-1}(v))} \int_{-\infty} \frac{1}{2\pi\sqrt{1-\theta_G}} \exp\left(\frac{x_1^2+x_2^2-2\theta_G x_1 x_2}{2(1-\theta_G^2)}\right) dx_1 x_2, \quad (2.5)$$

where Φ is bivariate standard normal cumulative distribution and θ_G is the correlation parameter of copula lying in the interval $[-1,1]$.

2) Students t copula

Students t copula is described as a copula function with fat tail. θ_T is the parameter of the copula lying in the interval $[-1,1]$. $t_v^{-1}()$ denotes the inverse distribution function of a Student's t random variable with degree of freedom v. In bivariate case, we can define the probability density function of the students t copula as

$$c_T(u,v|\theta_T) = \int_{-\infty}^{t_v^{-1}(u)} \int_{-\infty}^{t_v^{-1}(v)} \frac{1}{2\pi\sqrt{1-\theta_T}} \exp\left(\frac{x_1^2+x_2^2-2\theta_T x_1 x_2}{v(1-\theta_T^2)}\right)^{-(v+2)/2} dx_1 x_2, \quad (2.6)$$

where $t_d(v,0,\theta_T)$ is the standard univariate student-t distribution with v degree of freedom, mean 0 and variance $(v+2)/2$.

2.3.2 Archimedean Copulas

In order to capture asymmetric dependency in the tails, we must employ a copula which separately parameterizes either left or right tail. In the estimation, this class of copula allows modeling the dependence with only one parameter. The probability function of the bivariate Archimedean copulas, consisting density of the Clayton, Gumbel, Ali-Mikhail-Haq (AMH) and Frank. Clayton copula exhibits left-tailed asymmetry and Gumbel copula exhibits right-tailed asymmetry. Ali-Mikhail-Haq (AMH) copulas exhibits both negative and positive tail dependence. Frank copula is the only symmetric Archimedean copula.

1) Clayton copula

The Clayton copula is one of the asymmetric copulas exhibiting greater dependence in a negative tail than the one in a positive. This copula is given by

$$c_C(u,v|\theta_C) = \left(1+(u^{-\theta_C}-v^{-\theta_C}-1)\right)^{-1/\theta_C}, \quad (2.7)$$

where θ_C is the degree of dependence on the value of $0<\theta_C<\infty$. If $\theta_C \to \infty$, the Clayton copulas will have a positive dependence. But if $\theta_C = 0$, it will correspond to independence.

2) Gumbel copula

The density function of Gumbel copula is defined by

$$c_G(u,v|\theta_G) = \exp\left(-\left[(-\ln u)^{\theta_G}+(-\ln v)^{\theta_G}\right]\right)^{1/\theta_G}, \quad (2.8)$$

where the parameter θ_G is the degree of dependence on the value $1<\theta_G<\infty$.

3) The Ali-Mikhail-Haq (AMH) copulas

The Ali-Mikhail-Haq copula according to Ali et al. (1978) is defined by

$$c_A(u,v\,|\,\theta_A) = \frac{uv}{1-\theta(1-u)(1-v)}, \tag{2.9}$$

where $\theta_A \in [-1,1]$. It may be noted that AMH copula is one of the Archimedean copulas which has parameter lies on a closed interval between -1 and 1 and describes both, positive and negative, dependence.

4) Frank copula

The density function of Frank copula can be defined by

$$c_F(u,v\,|\,\theta_F) = -\theta_F^{-1}\log\left(1+\frac{(e^{-\theta_F(u)}-1)(e^{-\theta_F(v)}-1)}{e^{-\theta_F}-1}\right), \tag{2.10}$$

where θ_F is the degree of dependence $-\infty < \theta_F < \infty$.

2.4 Mixture Regimes Switching Copula Model

As we mentioned in the introduction section, this study proposed Markov Switching copula with mixture regimes model; therefore, we introduced the model properties and characteristics in the section. The idea behind this model is like the conventional one but it is generalized by allowing the copula function to be different across two regimes. That is the model allows the dependence copula parameter to reflect the real behavior of the data differently between two regimes. The regime switching is managed by a hidden variable at time t (S_t). Let S_t be the state variable, which is believed to have two states $(k=2)$, namely low dependence regime and high dependence regime. The joint distribution of $F(x_1,x_2)$ conditional on S_t, is defined as

$$F(x_1,x_2) = \Pr(S_t=1)\cdot C_1(u,v,\theta_{S_t=1}) + \Pr(S_t=2)\cdot C_2(u,v,\theta_{S_t=2}), \tag{2.11}$$

where $\theta_{S_t=1}$ and $\theta_{S_t=2}$ are the dependence parameter in regime 1 and 2, respectively. $C_1(\cdot)$ and $C_2(\cdot)$are the copula functions which can be different across two regimes. We expect that this generalized model could provide more flexibility to the Markov Switching copula model. $\Pr(S_t=1)$ is the probabilities of regime 1 at time t. $\Pr(S_t=2) = (1-\Pr(S_t=1))$ is the probabilities of regime 2 at time t. The unobservable regime S_t is regulated by the first order Markov chain, which is featured by the following transition probabilities P:

$$P_{ij} = \Pr(S_t=j\,|\,S_t=i) \text{ and } \sum_{j=1}^{2} p_{ij} = 1,\ i,j=1,2, \tag{2.12}$$

where p_{ij} is the probability of switching from regime i to regime j, and these transition probabilities can be written in the form of a transition matrix as follows:

$$P = \begin{bmatrix} p_{11} & p_{12}=1-p_{11} \\ p_{21}=1-p_{22} & p_{22} \end{bmatrix} \tag{2.13}$$

As the S_t is latent variable and $\Pr(S_t = 1)$ and $\Pr(S_t = 2)$ are not observable, thus, we apply Hamilton's (1989) filter. Accordingly, the transition probability matrix drives the regime probabilities which in turn define the density function of the complete dataset. Explicitly, the filtered process for 2 regimes is defined as

$$\Pr(S_t = 1\,|w_t) = \frac{c_t(u,v\,|S_t = 1, w_{t-1})\Pr[S_t = 1\,|w_{t-1}\,]\cdot p_{11}}{\sum_{k=1}^{2} c_t(u,v\,|S_t = k, w_{t-1})\Pr[S_t = k\,|w_{t-1}\,]} \tag{2.14}$$

$$\Pr(S_t = 2\,|w_t) = (1 - \Pr(S_t = 1\,|w_t))\,p_{22}, \tag{2.15}$$

where w_t and w_{t-1} is all parameters and data of our model at time t and $t-1$, respectively. This regime probabilities $\Pr(S_t = 1\,|w_t)$ at time t, conditional on information until time t are captured by the transition probability matrix P. With this recursive procedure it is straightforward to forecast the regime probabilities for time $t = \{1, ..., T\}$

2.5 Estimation Procedure

Estimation of the model parameters is done by maximum likelihood. The joint conditional density function is obtained by differentiating Eq. (2.11), and thus the full-sample log-likelihood takes the form

$$\begin{aligned} \log L = \textstyle\sum_{t=1}^{T} \log f_1(\varphi_1) + \log f_2(\varphi_2) + \\ \log\left[(S_t = 1)\cdot c_1(u,v,\theta_{S_t=1}) + (S_t = 2)\cdot c_2(u,v,\theta_{S_t=2})\right], \end{aligned} \tag{2.16}$$

where $f_1(\varphi_1)$ and $f_2(\varphi_2)$ are the conditional marginal density of GARCH (1,1) model in Eqs. (2.1-2.2), where $\varphi = \{c, \omega, \alpha, \beta\}$. $c_1(u,v,\theta_{S_t=1})$ and $c_2(u,v,\theta_{S_t=2})$ are the copula densities for regime 1 and 2, respectively. We note that the two-stage Maximum Likelihood estimation is used in this study as the full log-likelihood in this model may be difficult to maximize and thus we decompose the log-likelihood of this model into two parts: GARCH (1,1) and Markov Switching copula. Thus, we first restricted the GARCH (1,1) part to be regime independent process and estimate the GARCH parameter in the first step estimation. Then, the obtained parameters in the first step are plug in the full likelihood Eq. (2.16) as a fixed parameter in the second estimation step.

$$\widehat{\varphi}_i = \arg\max_{\varphi_i} \sum_{t=1}^{T} \log f_i(\varphi_i), \quad i = 1,2 \tag{2.17}$$

$$\widehat{\theta} = \arg\max \sum_{t=1}^{T} \log\left[\Pr(S_t = 1)\cdot c_1(u,v,\theta_{S_t=1}) + \Pr(S_t = 2)\cdot c_2(u,v,\theta_{S_t=2})\right] \tag{2.18}$$

3 Data Descriptive Statistics

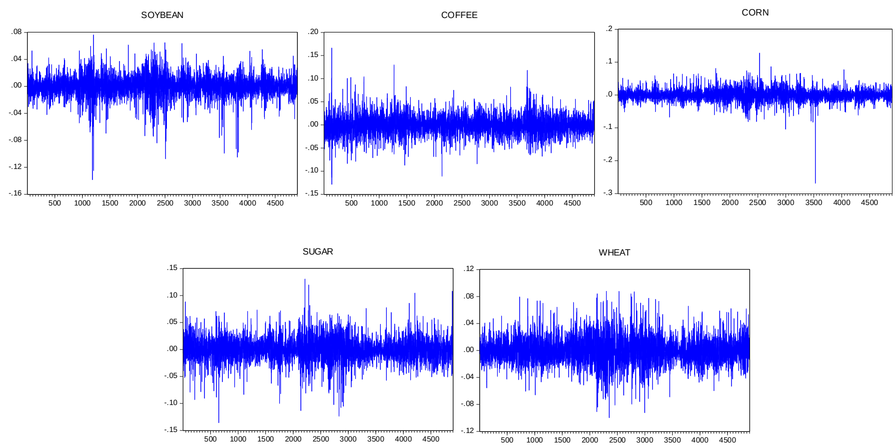

Figure 1: Plot of agricultural commodity futures returns

In this paper, our data consist of five agriculture futures prices. Sugar, coffee, corn, wheat and soybean are selected as the representative for agriculture market. The main reasons of our data sample selection is that these futures prices are well known and normally considered as the asset in the portfolios. We use daily data from January 4, 2000 to October 17, 2018. The data were collected from Bloomberg. The descriptive statistics for agricultural commodity futures returns are reported in Table 1. The skewness statistics show that the returns exhibit either negative or positive skewness. Additionally, the kurtosis is higher than normal distribution kurtosis (kurtosis=3). This indicates that that our futures returns may not have a normal distribution. Thus, Jarque-Bera test is used to confirm our hypothesis. In this study, we use Minimum Bayes factor (MBF) as the tool for checking the significant result. This MBF can be considered as an alternative of p-value [16]. The result shows that the MBF values for all series are 0.0000, indicting a decisive evidence supporting a non-normality of our series. Furthermore, the unit root test is performed to check whether our data are stationary. As shown by MBF values, we have a strong evidence for supporting the stationarity of our data. Figure 1 depicts the historical evolution of return trends for commodities. All the agricultural commodities have a similar behavioral trend over time.

Table 1: Data Descrition

	SOYBEAN	COFFEE	CORN	SUGAR	WHEAT
Mean	0.000	0.000	0.000	0.000	0.000
Median	0.000	0.000	0.000	0.000	0.000
Maximum	0.076	0.166	0.128	0.131	0.088
Minimum	−0.138	−0.129	−0.269	−0.136	−0.100
Std. Dev.	0.016	0.021	0.018	0.021	0.020
Skewness	−0.824	0.230	−0.618	−0.156	0.156
Kurtosis	8.723	6.653	15.969	6.084	4.919
Jarque-Bera	7242.835	2768.232	34659.890	1962.341	771.837
MBF Jarque-Bera	0.000	0.000	0.000	0.000	0.000
Probability	0.000	0.000	0.000	0.000	0.000
Unit Root	−69.672	−71.635	−68.869	−70.289	−69.931
MBF Unit Root	0.000	0.000	0.000	0.000	0.000

Note: MBF computed by $MBF_{01}(p) = \begin{cases} -\exp(1)p\log p & \text{for } p < 1/\exp(1) \\ 1 & \text{for } p \geq 1/\exp(1) \end{cases}$, where p is $p-value$ (see [17] and [18])

4 Empirical Results

4.1 Univariate Results

Table 2: GARCH (1,1) estimation results

Parameter	Sugar	Coffee	Corn	Wheat	Soybean
σ^2	−0.000047 (0.9853)	−0.000107 (0.9263)	0.00014 (0.8129)	0.000089 (0.9391)	0.000239 (0.4429)
ϖ	0.000003 (0.0012)	0.000006 (0.0000)	0.000003 (0.3155)	0.000002 (0.0031)	0.000003 (0.0566)
α	0.03485 (0.0000)	0.032562 (0.0000)	0.047929 (0.0000)	0.032961 (0.0000)	0.060813 (0.0000)
β	0.958884 (0.0000)	0.954137 (0.0000)	0.946197 (0.0000)	0.961327 (0.0000)	0.927468 (0.0000)

Note: () is MBF, MBF computed by $MBF_{01}(p) = \begin{cases} -\exp(1)p\log p & \text{for } p < 1/\exp(1) \\ 1 & \text{for } p \geq 1/\exp(1) \end{cases}$, where p is $p-value$ (see [17] and [18])

Table 2 reports the estimates for the conditional marginal distributions of the daily log returns of agriculture commodity futures. The results of GARCH (1,1) show that $\alpha+\beta$ of each data are quite close to unity indicating a high volatility persistence. These results indicate that our returns have long-run persistence and there are volatility shock effects in our returns. In this study, we use Minimum Bayes Factor (MBF) as the tool for checking the significant result. This MBF can be considered as an alternative of p-value (Held and Ott, 2016). If 1¡MBF ¡1/3, 1/3¡MBF¡1/10, 1/10¡MBF¡1/30, 1/30¡MBF¡1/100, 1/100¡MBF¡1/300 and MBF¡1/300, there are a chance that the MBF favors the weak evidence, moderate evidence, substantial evidence, strong evidence, very strong evidence and decisive evidence for $H_1 : \beta \neq 0$ respectively.

Table 3: Model Selection

Agriculture pair	Single regime Copula	Criteria		
		AIC	BIC	LL
Sugar-Coffee	Frank	−254.36	−247.87	128.18
Sugar-Corn	AMH	−111.3	−104.81	56.65
Sugar-Wheat	AMH	−105.24	−98.74	53.62
Sugar-Bean	AMH	−127.41	−120.92	64.71
Coffee-Corn	Frank	−116.31	−109.81	59.15
Coffee-Wheat	Gaussian	−120.33	−113.83	61.17
Coffee-Bean	Student-t	−141.52	−128.53	72.76
Corn-Wheat	Student-t	−2588.9	−2575.9	1296.45
Corn-Bean	Student-t	−2086.96	−2073.97	1045.48
Wheat-Bean	Student-t	−960.06	−947.07	482.03

Table 4: Model Selection (continue)

Agriculture pair	Markov Switching copula		Criteria		
	regime 1	regime 2	AIC	BIC	LL
Sugar-Coffee	Frank	Frank	−789.81	−784.48	398.9
Sugar-Corn	Clayton	Clayton	−256.51	−251.19	132.26
Sugar-Wheat	Clayton	Clayton	−342.01	−336.68	175
Sugar-Bean	Clayton	Clayton	−205.46	−200.14	106.73
Coffee-Corn	Student-t	Student-t	−262.32	−267.65	127.16
Coffee-Wheat	Student-t	Student-t	−205.44	−210.77	98.72
Coffee-Bean	Gaussian	Gaussian	−192.88	−187.55	100.44
Corn-Wheat	Gaussian	Gaussian	−2697.41	−2692.08	1352.7
Corn-Bean	Gaussian	Gaussian	−2219.3	−2213.97	1113.65
Wheat-Bean	Student-t	Student-t	−1989.16	−1983.83	998.58

Table 5: Model Selection (continue)

Agriculture pair	Markov Switching copula with mixture regimes		Criteria		
	regime 1	regime 2	AIC	BIC	LL
Sugar-Coffee	Gumbel	Frank	−792.79	−787.46	400.39
Sugar-Corn	Frank	AMH	−343.97	−338.64	175.98
Sugar-Wheat	Frank	AMH	−399.86	−394.53	203.93
Sugar-Bean	Clayton	Gaussian	−238.18	−232.86	123.09
Coffee-Corn	Clayton	Frank	−257.91	−252.58	132.95
Coffee-Wheat	Gaussian	Frank	−355.04	−349.71	181.52
Coffee-Bean	Gaussian	Clayton	−196.72	−191.39	102.36
Corn-Wheat	Clayton	Frank	−2782.9	−2777.58	1395.45
Corn-Bean	Frank	Clayton	−2232.14	−2226.82	1120.07
Wheat-Bean	AMH	Student-t	−2103.84	−2098.51	1055.92

Table 3-5 report model selection by AIC, BIC and Log-Likelihood (LL) criteria for the comparison of various copula specifications. Ten pairs of agriculture futures margins are investigated in our analysis (see, Column 1 of Table 3). In this study, six single regime copulas, six conventional Markov Switching copula, and thirty Markov Switching copula with mixture regimes are compared. All copula models are estimated using the same residuals which result from the filtering with univariate GARCH models. As we need to deal with many model specifications copula models and our page space is limited, thus, in this section, the best fit model of single regime copula, conventional Markov Switching copula and Markov Switching copula with mixture regimes classes. The best fit models for each pair are presented in each row. As shown in Table 3-5, we find that our proposed model is superior to the two conventional models with respect to AIC, BIC and LL in the most pairs, except for Coffee-Corn pair. This result indicates that the dependence structure between two regimes can be different. This is to say; the dependence structure of each regime could be explained by different copula functions. This result confirms the higher performance of our proposed model in these applications.

Table 6 presents the estimated results for the best fit models for each pair of agricultural commodities. Columns 2 and 3, present the copula parameter estimates for both two regimes, while Columns 4-5, provide the theoretical Kendalls tau value corresponding to the bivariate copula parameter value. Columns 6-9 provide the theoretical tail dependence coefficients corresponding to the bivariate copula parameter value. The computation of these Kendalls tau value and tail dependence coefficient are referred to Joe [19]. The results show that all the copula parameters show a positive correlation for both two regimes, however, the degree of dependence between regime 1 and 2 are different. We observe that the positive values of regime 1 are larger than regime 2. This indicates that regime 1 can be viewed as high dependence regime, while regime 2 can be viewed as low dependence regime. However, it is difficult to interpret the results from the copula parameter, so we can explain the correlation for each pair based on the Kendalls tau value and tail dependence coefficient. Corresponding to copula parameter results, we also obtain the positive correlation for all agriculture commodity pairs.

To gain more insight regarding the optimal copulas listed in Table 6, we consider next their implied tail dependence. The tail dependence measures the probability of simultaneous large losses or profits in both assets, which is a good indicator of systemic risk under extreme market conditions, say turmoil and peak markets. In general, the values of either upper or lower tail dependence are positive. It means the degree of co-movement when agriculture commodities futures are under the extreme movements in the same direction.

Table 6: Dependence and Tail dependence of 5 Agriculture futures pairs

<table>
<tr><th rowspan="3">Pairs of agriculture</th><th colspan="2">Copula parameter</th><th colspan="2">Dependence</th><th colspan="4">Tail dependence</th></tr>
<tr><th rowspan="2">regime 1</th><th rowspan="2">regime 2</th><th rowspan="2">regime 1</th><th rowspan="2">regime 2</th><th colspan="2">regime 1</th><th colspan="2">regime 2</th></tr>
<tr><th>lower</th><th>upper</th><th>lower</th><th>upper</th></tr>
<tr><td>Sugar-Coffee (Gumbel-Frank)</td><td>88.217</td><td>1.495</td><td>0.989</td><td>0.055</td><td>0.000</td><td>0.992</td><td>0.000</td><td>0.000</td></tr>
<tr><td>Sugar-Corn (Frank-AMH)</td><td>50.780</td><td>0.404</td><td>0.226</td><td>0.101</td><td>0.000</td><td>0.000</td><td>0.000</td><td>0.000</td></tr>
<tr><td>Sugar-Wheat (Frank-AMH)</td><td>73.392</td><td>0.384</td><td>0.097</td><td>0.095</td><td>0.000</td><td>0.000</td><td>0.000</td><td>0.000</td></tr>
<tr><td>Sugar-Soybean (Clayton-Gaussian)</td><td>69.967</td><td>0.132</td><td>0.972</td><td>0.085</td><td>0.990</td><td>0.000</td><td>0.000</td><td>0.000</td></tr>
<tr><td>Coffee-Corn (Clayton - Frank)</td><td>10.313</td><td>0.965</td><td>0.838</td><td>0.106</td><td>0.764</td><td>0.764</td><td>0.083</td><td>0.083</td></tr>
<tr><td>Coffee-Wheat (Gaussian-Frank)</td><td>0.137</td><td>99.488</td><td>0.087</td><td>0.960</td><td>0.000</td><td>0.000</td><td>0.000</td><td>0.000</td></tr>
<tr><td>Coffee-Soybean (Gaussian-Clayton)</td><td>0.304</td><td>0.017</td><td>0.197</td><td>0.009</td><td>0.000</td><td>0.000</td><td>0.000</td><td>0.000</td></tr>
<tr><td>Corn-Wheat (Clayton-Frank)</td><td>14.304</td><td>5.074</td><td>0.877</td><td>0.461</td><td>0.952</td><td>0.000</td><td>0.000</td><td>0.000</td></tr>
<tr><td>Corn-Soybean (Frank-Clayton)</td><td>4.615</td><td>82.916</td><td>0.432</td><td>0.314</td><td>0.468</td><td>0.000</td><td>0.000</td><td>0.000</td></tr>
<tr><td>Wheat-Soybean (AMH- Student-t)</td><td>0.723</td><td>0.583</td><td>0.203</td><td>0.155</td><td>0.000</td><td>0.000</td><td>0.103</td><td>0.103</td></tr>
</table>

() presents the copula function

Table 7: The transition probability parameters

Pair of agriculture	regime 1	regime 2	p_{11}	p_{22}
Sugar-Coffee	Frank	Gumbel	0.1066	0.9617
Sugar-Corn	Frank	AMH	0.0145	0.9548
Sugar-Wheat	Frank	AMH	0.0001	0.956
Sugar-Soybean	Clayton	Gaussian	0.0001	0.9585
Coffee-Corn	Clayton	Frank	0.0001	0.9625
Coffee-Wheat	Gaussian	Frank	0.9587	0.0423
Coffee-Soybean	Gaussian	Clayton	0.9986	0.9983
Corn-Wheat	Frank	Clayton	0.9577	0.0191
Corn- Soybean	Clayton	Frank	0.0112	0.9559
Wheat- Soybean	Student-t	AMH	0.9824	0.9901

The probability of staying in regime 1 and 2 are provided in Table 7. p_{11} is the probability of being in the low dependence regime at time t that is conditional on being in the same regime at $t-1$ while p_{22} is defined as the probability of the high dependence regime. This table reports the transition probability parameters of 10 pairs of agriculture commodity futures. The estimated transition probabilities of p_{11} and p_{22} are mostly close to 1, except for p_{11} of Sugar-Corn, Sugar-Wheat, Sugar-Soybean, Sugar-Coffee , and Coffee-Corn and p_{22} of Corn-Wheat. These results reveal that the probability of staying in the same regime is, in the most cases, highly persistent. However, it is interesting to have a low regime persistency for Sugar-Corn, Sugar-Wheat, Sugar-Soybean, Sugar-Coffee, Coffee-Corn and Corn-Wheat pairs. So, we may say that the dependence of Sugar and other agricultural commodity futures may not be stable in the high dependence regime. Intuitively, the dependence between Sugar and the others are quite low and the duration of staying in the high dependence regime is lower than the low dependence regime.

Lastly, our results show the high performance of our model over the conventional models. Our study can confirm that there is a heterogenous dependence structure between high and low dependence regimes. Therefore, using the same copula structure for both regimes may lead to unreliable dependence results.

5 Conclusion

In this study, we allow the dependence structure of a random variables to be different across two regimes. We generalize the Markov Switching copula which assumes that the dependence function is the same for regime 1 and regime 2. To achieve this goal, we consider six copulas from two classes of copula function, namely Archimedean and Elliptical copulas. Several copula combinations are introduced to our proposed model. To show the performance of our proposed model, we apply the model to study the agricultural commodity futures: sugar, coffee, corn, wheat and soybean. From these data sets, we conduct ten pairs of futures in our analysis.

We compare the performance of our model with another two conventional models, namely single regime copula, two-regime Markov Switching copula models. The comparison criteria used in this study are log-likelihood, AIC and BIC. To choose the best model, we will look at the minimum AIC and BIC and maximum log-likelihood. The results have suggested that the Markov Switching copula with mixture distribution regimes model is superior to the single-regime copula and two-regime Markov Switching copula model. We find that 9 out of 10 cases prefer our proposed model.

Finally, we mention that the dependence between agriculture commodity futures have a specific feature in the sense that a different copula structure is detected across two regimes. There is however a time varying dependence for random variables along the sample period, and this study does not account for. In the future research, Markov Switching dynamic copula may be applied to the time-varying analysis.

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