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Markov Switching Dynamic Correlation: An Empirical Study of Hedging in Crude Oil and Natural Gas Markets

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Abstract : This article studies how to achieve risk-minimization through hedging strategy in crude oil and natural gas markets. In this study, the covariance of spot and futures returns is computed through the Markov Switching Dynamic Conditional Correlation GARCH model. The model is compared to single regime DCC-GARCH for both oil and gas spot/futures pairs in order to examine the presence of the structural change in the dynamic correlation. The result confirms the superiority of two-regime model in terms of log-likelihood, AIC, and BIC. Then, the obtained conditional volatility and correlation are further used to compute the hedge ratio and optimal portfolio weight for oil and gas spot/futures pairs. The results show that the risk-minimizing hedge ratio of oil and that of gas are averagely 0.844 and 0.32, respectively, and investors should hold only half of oil and gas futures contracts (52 and 48 percent) to lowest their risk. Investors should be careful about the situation in the markets before making investment or changing policy.

Keywords : hedging; Markov switching; crude oil; natural gas.

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1 Introduction

Energy is very important for country development as it is a vital factor in the production processes [1]. It is one of the most important inputs and consumption goods in the world. However, there are many factors that could affect the energy prices, for example, production technology, monetary policy [2], geopolitical event, natural disaster [3], etc. Thus, energy prices have moved up and down, and thereby fluctuating energy prices have posed a new threat to the global economy. Thus, the uncertainty in energy markets may trigger investments in other assets or in hedging instruments like precious metals [4]. Both academicians and energy market participants have focused on forecasting and modeling energy prices by quantifying and managing the risks inherent in their frequent volatilities [5].

It is widely accepted that the high variation of energy prices contributes a great risk to companies, especially the companies that get involved in trading and transportation. The stability of cash flow and income of companies are an important target that the policymakers always seek for. Nevertheless, the future income and cash flow might suffer from the devaluation of negative return as the actual cost of transaction is increased. Therefore, implementing a policy without hedging could affect the financial status of companies, not only benefit when the return is rising but also damage to companys cash flow when the return is low. To solve these severe risks, most companies aim to minimize their risk by using a hedging strategy. Conceptually, the companies or investors can reduce their risks of unfavorable price change through trading futures. For example, the airline companies often hedge future fuel consumption at a set price to avoid a profit squeeze if oil prices were to spike. Hedging can keep costs down and their fares competitive. The same strategy holds for natural gas consumers. Energy hedging can protect buyers against the risk of unexpected price surges, and producers can lock in prices for future output to help them meet the financial targets. We may say that hedging strategies are important for both energy producers and consumers to seek an optimal investment for lowering their risk exposure to price fluctuations in the underlying spot price.

The purpose of the paper is to construct hedge ratios for two energy prices, namely oil and gas futures derivatives using the Markov Switching Dynamic Conditional Correlation GARCH model (MS-DCC-GARCH). These two energy prices are the most influential resource of raw materials and primary energies as they have a strategic impact on economic development and social stability. Chang, McAleer and Tansuchat [6] mentioned that among the industries, oil and gas industry are the most used in hedging strategy. When an industry expects that the price of gas or oil will increase or decrease, they will use more or less futures, thereby increasing a futures prices higher or lower. Likewise, the hedging strategy used by crude oil producers normally involves selling the commodity futures in order to lock the futures selling prices or a price floor. Thus, they tend to take short positions in futures. At the same time, energy traders, investors or fuel oil users focusing to lock in a futures purchase price or price ceiling tend to long positions in futures.

In the literature, hedging involves the determination of the optimal hedge ratio (OHR). One of the most widely-used hedging strategies is based on the minimization of the variance of the portfolio, [7]. Basically the OHR is defined as $Cov\,(r_s, r_f)\,/Var\,(\Delta r_f)$, where r_s and r_f are the spot and futures returns respectively. Thus, the constant conditional correlation (CCC) GARCH model of Bollerslev (1990) is applied to estimate the variance-covariance between these two assets. This conventional covariance is derived from the joint variability of two random variables which show the similarity characteristic of variable. It is assumed that the relationship is static. Nevertheless, the idea of dynamic relationship between the variables, not only in spot and futures prices, has been popular over decade because it is more realistic than conventional static one which is based on the idea that the size of relationship has not been changed over time. In the same manner, the conditional covariance between spot and futures prices may have dynamic characteristic, which could be relavant to the hedging policy. For policymakers who have responsibility on risk reducing, avoiding the dynamic relationship might bring about wrong conclusion that can worsen investors decision and then lead to under-performed outcome. Thus, the dynamic conditional correlation (DCC) GARCH model of Engle [8] is considered as the ideal model for estimating time varying OHR. Several authors have used DCC-GARCH to investigate the dynamics between different markets and estimate the hedging ratios, for example Chang, McAleer and Tansuchat [6], Maghyereh, Awartani, Tziogkidis [9], and Bhatia, Das, and Mitra, [4]. However, Fong and See [10] and Alizadeh, Nomikos, and Pouliasis [11] revealed that the characteristic of uncertainty in energy futures and spot returns has been changed by the different contexts over time. Fong and See [10] reported significant regime shift in the conditional volatility of crude oil futures contracts and found that in a high variance regime a negative basis is more likely to increase the regime persistence than a positive basis and associate volatility regimes with specific market events. Alizadeh, Nomikos, and Pouliasis [11] mentioned that by allowing the hedge ratio to be regime dependent, we can obtain more efficient hedge ratios when compared to the simple DCC-GARCH models.

As we mentioned above, with the complexity in the characteristic of risk in the different state of economy, economic boom and recession might show the different characteristic of covariance that should be accounted for in the study. Thus, we employed the recent switching dynamic correlation model, namely MS-DCC-GARCH employed by Chodchuangnirun, Yamaka, and Khiewngamdee, [12] and Rakpho, Yamaka, and Sriboonchitta [13]. This model allows the GARCH and DCC parameters to switch across different regimes or to be state dependent according to the first order Markov process. They proved that this model is better than the conventional MS-DCC-GARCH model version of Billio and Caporin [14] and DCC-GARCH model.

This paper makes several important contributions to the literature. First, while many existing studies use DCC-GARCH models to estimate optimal hedge ratios, this current paper compares the optimal hedge ratios obtained from DCC type models with those obtained from MS-DCC-GARCH. This provides a more complete understanding of how optimal hedge ratios vary between different states of economy. Second, the model is used to construct dynamic hedge ratios and the hedging effectiveness and dynamic optimal weight in portfolio. The rest of this paper is structured as follows: Next section explains the methodology. Section 3 reports the data and preliminary analysis. Section 4 explains the empirical findings followed by conclusion in Section 5.

2 Methodology

2.1 DCC-GARCH

In univariate GARCH model, the classic model for studying volatility of variables is conducted by getting rid of autocorrelation and heteroscedasticity problem which decrease the performance of modeling, especially in the case that variables value highly depends on the information and standard deviation in the previous period, which could make the model become less efficient and biased. The univariate GARCH model is defined as

$$\begin{aligned} r_t &= \mu + \varepsilon_t\,, \varepsilon_t = \eta_t \sqrt{h_t} \\ h_t &= \omega_0 + \omega_1 \varepsilon_{t-1}^2 + \omega_2 h_{t-1} \end{aligned} \tag{2.1}$$

where r_t is the return of spot (or futures) which has constant mean with standard deviation (ε_t) being allowed to fluctuate around the mean. η_t is the standardized residual, h_t is the conditional variance of spot (or futures) price which is generated by using GARCH (1,1) model. The conditional returns have normal distribution and the conditional covariance could be shown as $H_t = E\left[r_t {r'}_t \,|\Psi_{t-1}\right]$, Ψ_{t-1} is the information set of the return in time $t-1$. [8] In dynamic conditional covariance model, the matrix of covariance is

$$H_t = D_t R_t D_t \tag{2.2}$$

where $D_t$is a diagonal matrix of time-varying standard variate from univariate GARCH-processes and R_t is the time-varying conditional correlation matrix of standardized disturbances. It is a correlation matrix is defined as a real, symmetric positive semi-definite matrix, with ones on the diagonal,

$$\varepsilon_t$$

$$\varepsilon_t = D_t^{-1} r_t \sim N(0, R_t).$$

We note that H_t is positive definite. Then, those variables can be shown as

$$D_t = \begin{bmatrix} \sqrt{h_{1t}} & 0 & 0 & \cdots & 0 \\ 0 & \sqrt{h_{2t}} & 0 & \cdots & 0 \\ 0 & 0 & \sqrt{h_{3t}} & & \vdots \\ \vdots & \vdots & & \ddots & 0 \\ 0 & 0 & \cdots & 0 & \sqrt{h_{nt}} \end{bmatrix}, \tag{2.3}$$

$$R_t = \begin{bmatrix} 1 & \rho_{12,t} & \rho_{13,t} & \cdots & \rho_{1n,t} \\ q_{21,t} & 1 & \rho_{23,t} & \cdots & \rho_{2n,t} \\ \rho_{31,t} & \rho_{32,t} & 1 & & \rho_{3n,t} \\ \vdots & \vdots & & \ddots & \vdots \\ \rho_{n1,t} & \rho_{n2,t} & \rho_{n3,t} & \cdots & 1 \end{bmatrix}, \tag{2.4}$$

Alternatively, $R_t$can be shortened into

$$R_t = Q_t^{*-1} Q_t Q_t^{*-1}, \tag{2.5}$$

where Q is a symmetric positive definite matrix which can be shown by

$$Q_t = (1 - \sum_{i=1}^{I} \alpha_i - \sum_{j=1}^{J} \beta_j)\bar{Q} + \sum_{i=1}^{I} \alpha_i \varepsilon_{t-i} {\varepsilon'}_{t-i} + \sum_{j=1}^{J} \beta_j Q_{t-j}, \tag{2.6}$$

where $\bar{Q}$ is the $n \times n$ is the unconditional covariance matrix of the standardized errors of the standardized residuals η_t. Thus,

$$Q_t^* = \begin{bmatrix} \sqrt{q_{11,t}} & 0 & \cdots & 0 \\ 0 & \sqrt{q_{22,t}} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & \sqrt{q_{nn,t}} \end{bmatrix}, \tag{2.7}$$

Bollerslev (1980) suggested applying the constraints $\sum\limits_{i=1}^{I} \alpha_i + \sum\limits_{j=1}^{J} \beta_j < 0$ and $\alpha_i, \beta_j \geq 0$ to guarantee $R_t = [-1, 1]$. The typical element of R_t will be of the form $\rho_{mn} = q_{mn,t}/\sqrt{q_{mm,t} q_{nn,t}}$. These parameters are associated with the exponential smoothing process that is used to construct the dynamic conditional correlations. The log-likelihood function of DCC part can be constructed as

$$L(\Theta) = -\frac{1}{2} \sum_{t=1}^{T} \left(n \log(2\pi) + \log |D_t| + \log |R_t| + {\varepsilon'}_t R_t^{-1} \varepsilon_t \right). \tag{2.8}$$

where Θ is the estimated parameters of Eq.6. We note that $|\,.\,|$ is a determinant. For the GARCH parameters, Engle [8] suggested to estimate GARCH and DCC parts separately, hence we firstly estimate the univariate GARCH model and the obtained standardized residuals η_t and the conditional variance h_t are used for the second estimation in DCC model. For more details of the general DCC(P,Q)-GARCH see [8].

2.2 Markov switching in GARCH model

The probability of the previous period causes the probability of the later period. Let the regime of each state be $\mathrm{S_t}$ which is unobserved and governed by first order Markov process. Following Chodchuangnirun, Yamaka, and Khiewngamdee [12] and Rakpho, Yamaka, and Sriboonchitta [13], the model allows both GARCH and DCC equations to be regime dependent, and hence the Markov-switching with 2 regimes can be written by the following equations

$$\begin{aligned} r_t &= \mu\left(\mathrm{S_t}\right) + \varepsilon_t\left(\mathrm{S_t}\right), \\ \varepsilon_t\left(\mathrm{S_t}\right) &= \eta_t\left(\mathrm{S_t}\right)\sqrt{h_t^S}\left(\mathrm{S_t}\right) \\ h_t\left(\mathrm{S_t}\right) &= \omega_0\left(\mathrm{S_t}\right) + \omega_1\left(\mathrm{S_t}\right)\varepsilon_{t-1}^2\left(\mathrm{S_t}\right) + \omega_2\left(\mathrm{S_t}\right)h_{t-1}\left(\mathrm{S_t}\right), \end{aligned} \tag{2.9}$$

$$Q_t\left(\mathrm{S_t}\right) = (1 - \sum_{i=1}^{P}\alpha_i\left(\mathrm{S_t}\right) - \sum_{j=1}^{Q}\beta_j\left(\mathrm{S_t}\right))\bar{Q} + \sum_{i=1}^{P}\alpha_i\left(\mathrm{S_t}\right)\varepsilon_{t-i}\varepsilon'_{t-i} + \sum_{j=1}^{Q}\beta_j\left(\mathrm{S_t}\right)Q_{t-j}, \tag{2.10}$$

$$R_t\left(\mathrm{S_t}\right) = Q_t^{*-1}\left(\mathrm{S_t}\right)Q_t\left(\mathrm{S_t}\right)Q_t^{*-1}\left(\mathrm{S_t}\right). \tag{2.11}$$

where $\omega_2\left(\mathrm{S_t}\right)$are the regime dependent parameters of the model. $R_t\left(\mathrm{S_t}\right)$ is the regime dependent correlation. So, we can write the log-likelihood function as

$$L\left(\Theta\right) = \sum_{s_t=1}^{2}\left[\begin{array}{l} \sum\limits_{m=1}^{M}\sum\limits_{t=1}^{T}\left\{\left(\frac{1}{(2\pi)^{m/2}}\exp\left(-\frac{1}{2}\frac{\varepsilon'_t\varepsilon_t}{h_t^2}\right)\left|\Pr(\mathrm{S_t}\left|\mathrm{S_{t-1}}\right.)\right.\right)\cdot\right. \\ \left.-\frac{1}{2}\left(n\log\left(2\pi\right) + \log\left|D_t\right| + \log\left|R_t\right| + \varepsilon'_t R_t^{-1}\varepsilon_t\left|\Pr(\mathrm{S_t}\left|\mathrm{S_{t-1}}\right.)\right.\right)\right\} \end{array}\right] \tag{2.12}$$

where$m = 1, ..., M$ is the number of GARCH equations. This study uses a one-step Maximum Likelihood estimation (MLE) to estimate the coefficients in the MS-DCC-GARCH. Thus, the log-likelihood function is maximized to obtain the optimal parameter estimates.

2.3 Hedging ratio and Portfolio Weights

The returns of spot and futures prices could be employed to find the optimal hedge ratio which is calculated from risk-minimizing method. In hedge ratio, covariance of the two returns and variance of futures derivatives return are employed. Supposing the covariance and variance have static property over time, the optimal hedge ratio could be defined like in the following equation.

$$H_t^{\mathrm{S_t}} = \frac{Cov\left(r_{s,t}, r_{f,t}\right)}{Var\left(\Delta r_{f,t}\right)} = \frac{R_{s,f,t}^{\mathrm{S_t}}\sqrt{h_{s,t}^{\mathrm{S_t}}}}{\sqrt{h_{f,t}}} \tag{2.13}$$

where $H_t^{\mathrm{S_t}}$ is the regime dependent optimal hedge ratio, the ratio of transaction which investors should know which provides the minimum-variance of transaction. r_s and r_f are returns of spot and futures, respectively. $h_{s,t}^{\mathrm{S_t}}$ and $h_{f,t}^{\mathrm{S_t}}$ are the regime dependent volatility of spot and futures prices returns which are the representatives of the derivatives returns, respectively, while $R_{s,f,t}^{\mathrm{S_t}}$ is the regime dependent correlation between r_s and r_f. To find the optimal portfolio weights, we follow Kroner and Ng [15] and the MS-DCC-GARCH is extended to optimal portfolio weights.

$$w_{sf,t}^{\mathrm{S_t}} = \frac{h_{f,t}^{\mathrm{S_t}} - h_{sf,t}^{\mathrm{S_t}}}{h_{s,t}^{\mathrm{S_t}} - 2h_{sf,t}^{\mathrm{S_t}} + h_{f,t}^{\mathrm{S_t}}}, \tag{2.14}$$

where $w_{sf}^{\mathrm{S_t}}\left(1 - w_{sf}^{\mathrm{S_t}}\right)$ is the regime dependent weight of the spot (futures) at time t. $h_{sf,t}^{\mathrm{S_t}}$ is the regime dependent conditional covariance between spot and futures returns. As we deal with spot and futures hedging strategy, thus the bivariate MS-DCC(1,1)-GARCH(1,1) is considered in this study.

3 Data

This paper employs the data of spot and futures of WTI crude oil and natural gas of Henry Hub between January 2, 2002 and July 26, 2018. Crude oil spot and futures prices are collected from Investing.com database while Henry Hub natural gas spot and futures are collected from U.S. Energy Information Administration. The continuously compounded daily returns are defined and calculated as the difference in the logarithms of daily spot and futures prices. Figure 1 presents daily returns for our spots and futures.

The descriptive statistics for each return series are provided in Table 1 on the important information of the data consisting of mean, standard deviation, maximum, minimum, skewness, and kurtosis. In addition, we conduct an Augmented Dickey-Fuller (ADF) test and Jarque-Bera test to check the stationarity and normality of the data, respectively. From this Table, the measures for skewness and excess kurtosis show that most return series are obviously skewed and highly leptokurtic with respect to the normal distribution. We observe that oil spot return has a negative skewness, while the others have a positive skewness. In addition, we find that all returns are higher than 3. Thus, the JarqueBera statistic is conducted and Maximum Bayes Factor (MBF) is used as a p-value calibration. The result shows a decisive evidence of non-normally distributed data. The result of ADF test with intercept and trend confirms that our returns have no unit root problem and stationary at level, thus these data can be used for modeling in the next step.

Table 1: Descriptive statistics of returns

	Oil_s	Oil_f	Gas_s	Gas_f
Mean	0.0001	0.0001	0.0006	0.0003
Median	0.0001	0.0002	0.0000	-0.0004
Maximum	0.0374	0.0466	0.5282	0.2041
Minimum	-0.0347	-0.0340	-0.3905	-0.1247
Std. Dev.	0.0059	0.0060	0.0351	0.0247
Skewness	-0.0321	0.1659	2.3857	0.8115
Kurtosis	7.1212	7.7970	38.1192	8.8133
Jarque-Bera	2,948.958^a	4,030.765^a	216,576.5^a	6,281.073^a
Unit root test (ADF)	-67.8240^a	-67.8415^a	-51.3128^a	-68.6031
Observations	4,166	4,166	4,138	4,138

Note: ^{a} denotes strongly support to reject the null hypothesis, according to Maximum Bayes Factor (MBF). Goodman [16].

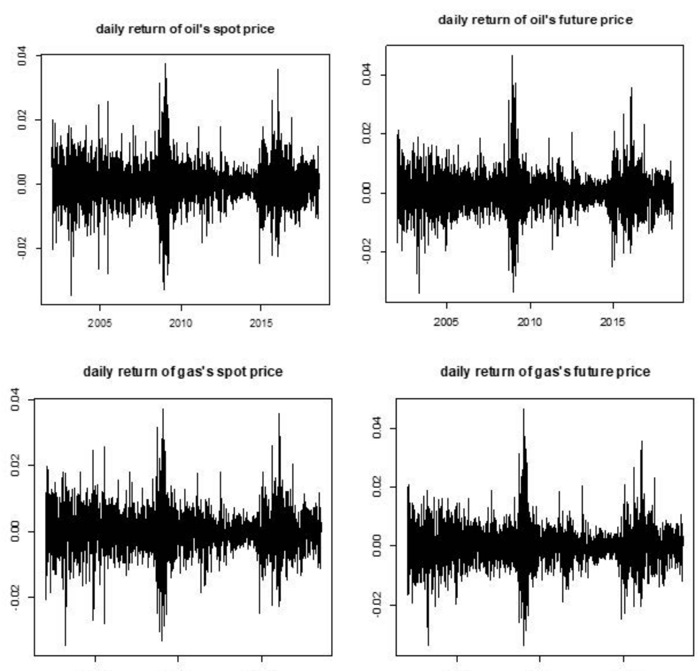


Figure 1: Spot and futures daily returns of crude oil and gas price during 2002 2018

4 Result

4.1 Model Selection

Prior to interpreting the result and investigating the hedging strategy, we check the existence of the regime switching in the correlation. To do that, we compare the performance of the MS-DCC-GARCH with the single-regime DCC-GARCH model. In this study, two-regime MS-DCC-GARCH is assumed. The choice of a two-regime model is motivated by the fact that this model captures the dynamics of the spot and futures returns in a more efficient way and is intuitively appealing since these two regimes can be associated with periods of low and high correlation. These two models are compared by Log Likelihood (LL), Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC), where the highest value of LL and the lowest value of AIC and BIC indicate the better performance. The result on model comparison is provided in Table 2. LL, AIC, and BIC show that MS-DCC-GARCH is better than the single-regime model for oil spot/futures and gas spot/futures pairs

Table 2: The model selection, comparison between DCC-GARCH (1,1) and MS-DCC-GARCH (1,1)

Distribution	DCC(1,1)-GARCH (1,1)		MS-DCC(1,1)-GARCH(1,1)	
	Crude oil	Gas	Crude oil	Gas
LL	32,605.64	15,420.33	32,535.33	16,806.18
AIC	-63,111.27	-30,822.65	-65,034.66	-33,576.35
BIC	-63,054.26	-30,765.70	-64,920.64	-33,462.45

4.2 MS-DCC-GARCH (1,1) model estimation

The results of MS-DCC(1,1)-GARCH(1,1) are shown in Tables 3-5. Firstly, we have to introduce about the volatility persistence in each regime for each pair. The persistence of volatility in MS-DCC-GARCH(1,1) can be calculated by $(\omega_1(S_t) + \omega_2(S_t))$ whereas the value should not be higher than unity, and the higher value indicates the more persistent volatility. Table 5 shows that all spot returns have higher volatility persistence than the futures returns, but there is not much different in volatility persistence between the two regimes. However, it can be seen that the unconditional volatility persistence within regime 1 is slightly lower than regime 2 for all GARCH models. So, the lower/higher uncertainty in spot/futures gives chance of higher/lower hedging ratio and weight to the practitioners. This shows the importance of regime switching models to model volatility.

Furthermore, we consider the results of the correlation persistence in Tables 3 and 4. Similar to the volatility persistence measure, the correlation persistence can be estimated by $\alpha\,(\mathrm{S}_t = i) + \beta\,(\mathrm{S}_t = i)$. The results show that the sum of $\alpha\,(\mathrm{S}_t = 1) + \beta\,(\mathrm{S}_t = 1)$ and that of $\alpha\,(\mathrm{S}_t = 2) + \beta\,(\mathrm{S}_t = 2)$are 0.9965 and 0.7972, respectively, for oil spot/futures pair, and 0.9487 and 0.5333 for gas spot/futures. Since these parameters control for the correlation persistence implied by the models, the findings suggest that the correlations are more persistent in regime 1 than in regime 2 in both pairs. As the manner of this Markov-switching model, the result can be distinguished into 2 regimes. Our findings reveal that the high correlation with low volatility persistency is found in regime 1, while regime 2 is found to have low correlation with high volatility in both oil and gas spot/futures pairs. Moreover, higher values of $\alpha\left(\mathrm{S}_t=i\right)+\beta\left(\mathrm{S}_t=i\right)$for the oil market compared to the gas market in both regimes imply that the correlation persistence is more pronounce in the oil spot and futures.

Furthermore, the regime persistence differences are also reflected in the transition probability estimates from the high to high P_{11} and low to low P_{22} correlation regime. We observe that the probability of staying in their own regime is high for both pairs. The probabilities of staying in regime 1 and 2 are respectively, 90% and 95% for oil spot/futures pair, and 0.9050 and 0.9431 for gas spot/futures pair. This result indicates that the probability of staying in each regime is high and persistent, thus there is only the extreme and severe events that switch the structure of spot and futures correlation. The smoothed probability estimates plotted in Fig 2 also significant features in both pairs. The result shows different characteristic of the regime shift between these two pairs. We observe the different market patterns over the sample period. These two plots confirm the existence of the structural change in the oil and gas markets.

The dynamic correlation of our model also illustrated in Figure 3. We make the comparison between the expected dynamic correlation between two-regime MS-DCC-GARCH(red) and single-regime (DCC-GARCH)(blue). We can observe that the variation of the dynamic correlation of the MS-DCC-GARCH is larger than DCC-GARCH and the structural change is clearly depicted by MS-DCC-GARCH.

The dynamic correlation of oil spot/futures in pre-2007 is lower than the post-2007, corresponding to the year of financial crisis in USA that brought about devastating effect to the world commodity market. The correlation before 2007 is obviously lower and more fluctuating than in the later period. The pattern of time varying correlation is inverted U shape during 2002 - 2007, the lowest correlation is in 2005, and becomes more stable after that. However, the result from natural gas price (Fig2), the dynamic correlation of MS-DCC-GARCH presents more fluctuation than DCC-GARCH and the correlation of the natural gas spot/futures is more stable than the crude oil prices.

The findings from this study may help investors understand more about the characteristic of the spot and futures price markets for crude oil and natural gas. The volatility persistence in each price and correlation between spot and futures prices can be determined for 2 regimes which investors should be careful about the situation in the markets before making investment or changing policy.

Finally, the obtained conditional volatility and the dynamic correlation obtained for MS-DCC-GARCH are further used to measure the optimal hedge ratio Eq.(2.12) and optimal portfolio weights Eq.(2.13). According to hedge ratio, it provides the information to minimize spot price buying risk by managing to buy futures derivative. The average of oil and gas hedge ratio is 0.844 and 0.32, respectively. The result can be interpreted that the risk in crude oil derivatives is higher than the natural gas. If investors want to minimize their risk in buying both spot and futures price contract of oil, and gas, they must long futures contracts for around 84% of the investment, while risk-minimizing finding for the natural gas suggests investors to long only 32% . Moreover, the results from optimal weights of portfolio indicate that investors should buy only half of futures contracts (52 and 48 percent for oil and gas) in their portfolio to minimize their risks in oil and gas markets.

Table 3: MS-DCC-GARCH estimation results of crude oil

Parameter	Oil-spot	Oil -futures
$\omega_0\,(\mathrm{S}_t = 1)$	0.0000^a (0.0000)	0.00001 (0.0000)
$\omega_1\,(\mathrm{S}_t = 1)$	0.2481^a (0.0029)	0.1985^a (0.1948)
$\omega_1\,(\mathrm{S}_t = 1)$	0.6672^a (0.0018)	0.5338^a (0.1001)
$\omega_0\,(\mathrm{S}_t = 2)$	0.0000^a (0.0000)	0.0000 (0.0000)
$\omega_1\,(\mathrm{S}_t = 2)$	0.2310^a (0.0078)	0.1848^a (0.1645)
$\omega_1\,(\mathrm{S}_t = 2)$	0.7011^a (0.00577)	0.5609^a (0.0844)
$\alpha\,(\mathrm{S}_t = 1)\beta\,(\mathrm{S}_t = 1)$	0.0808a (0.000699)	0.0647 (0.0579)
$\alpha\,(\mathrm{S}_t = 2)\beta\,(\mathrm{S}_t = 2)$	0.9157^a (0.0000)	0.7325^a (0.2899)
Mean of $R_t\,(\mathrm{S_t} = 1)$	0.84543	
Mean of $R_t\,(\mathrm{S_t} = 2)$	0.83393	
P_{11}	0.9000***(0.0183)	
P_{22}	0.9500***(0.0093)	

Note: In brackets is standard error. ^{a} denotes strongly support to reject the null hypothesis, according to Maximum Bayes Factor(MBF). Goodman [16].

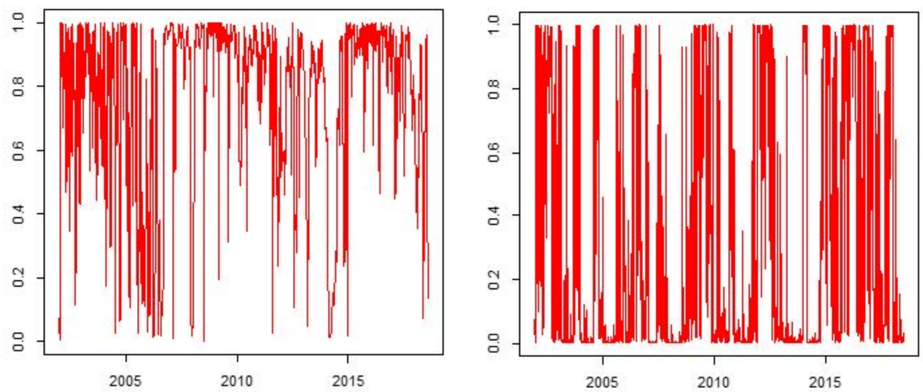
Table 4: MS-DCC-GARCH estimation results of natural gas

Parameter	Gas-spot	Gas-futures
$\omega_0\,(\mathrm{S}_t = 1)$	0.0003^a (0.00003)	0.0001 (0.00001)
$\omega_1\,(\mathrm{S}_t = 1)$	0.1012^a (0.00000)	0.1249^a (0.0162)
$\omega_1\,(\mathrm{S}_t = 1)$	0.8907^a (0.00001)	0.6899^a (0.1327)
$\omega_0\,(\mathrm{S}_t = 2)$	0.0001^a (0.00001)	0.0001^a (0.00003)
$\omega_1\,(\mathrm{S}_t = 2)$	0.0877^a (0.0006)	0.1257^a (0.0162)
$\omega_1\,(\mathrm{S}_t = 2)$	0.9046^a (0.00001)	0.7390^a (0.0657)
$\alpha\,(\mathrm{S}_t = 1)\beta\,(\mathrm{S}_t = 1)$	0.1387^a (0.0201)	0.0323 (0.0219)
$\alpha\,(\mathrm{S}_t = 2)\beta\,(\mathrm{S}_t = 2)$	0.8100^a (0.0010)	0.5012 (0. 54380)
Mean of $R_t\,(\mathrm{S_t} = 1)$	0.2816	
Mean of $R_t\,(\mathrm{S_t} = 2)$	0.2979	
P_{11}	0.9050***(0.0149)	
P_{22}	0.9431***(0.0182)	

Note: In brackets is standard error. ^{a} denotes strongly support to reject the null hypothesis, according to Maximum Bayes Factor(MBF). Goodman [16].

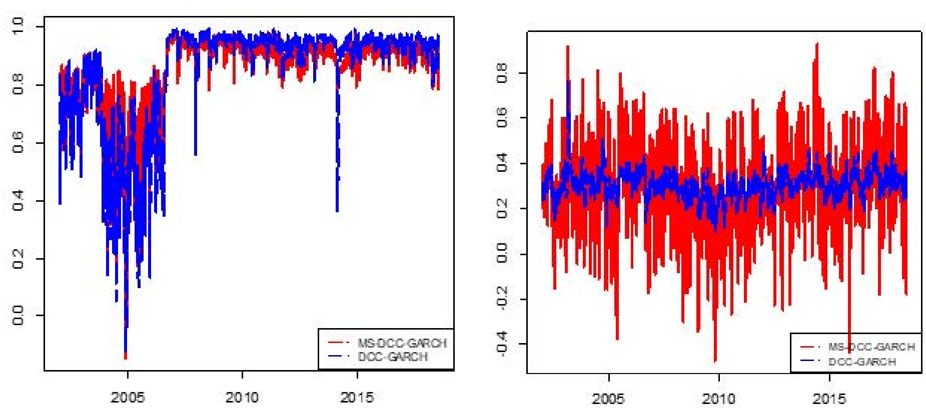
Table 5: The volatility persistence

regime	oil-spot	oil-futures	Gas-spot	Gas-futures
i=1	0.9153	0.7322	0.9919	0.8149
i=2	0.9321	0.7456	0.9924	0.8648.



(Oil) (Gas)

Figure 2: The smoothed probability of the pairs oil and gas spot/futures



(Oil) (Gas)

Figure 3: Dynamic correlation estimates from the MS-DCC-GARCH model and DCC-GARCH model

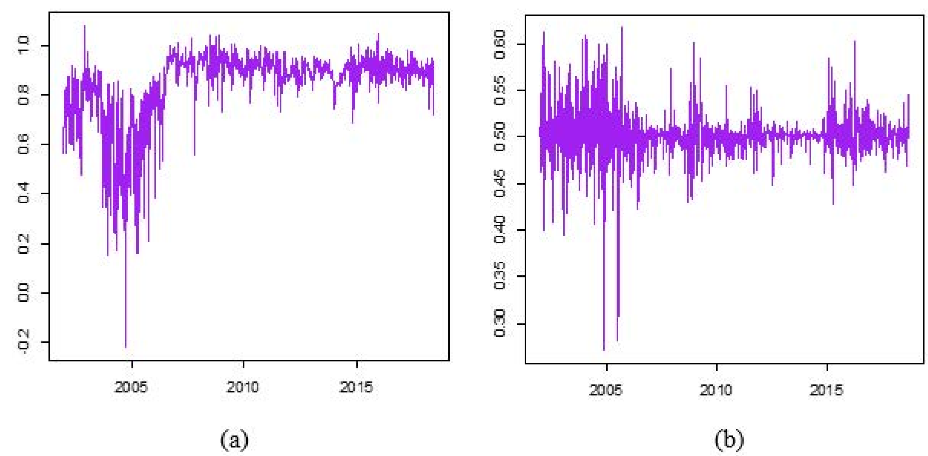


Figure 4: Time-varying hedge ratio (a) and optimal weight in portfolio (b) for oil spot/futures

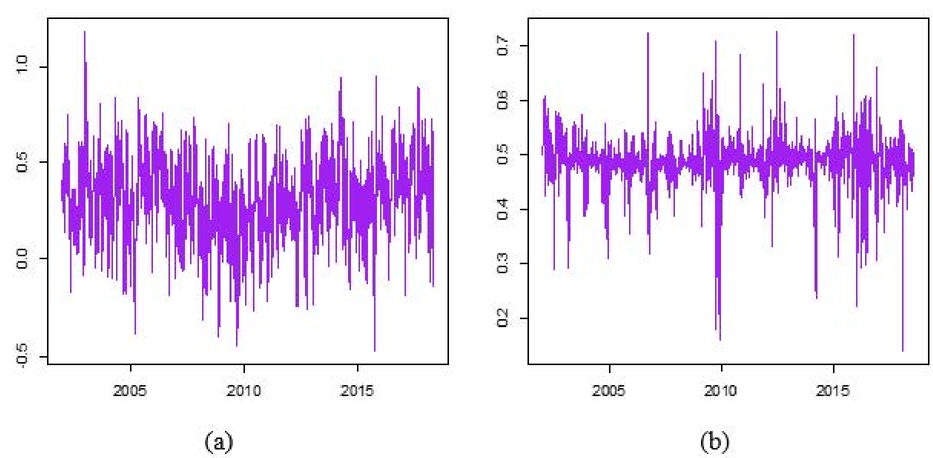


Figure 5: Time-varying hedge ratio (left) and optimal weight in portfolio (right) for gas spot/futures

5 Conclusion

This paper has an objective to study the risk-minimized hedge ratio. We assume that the relationship of spot and futures price has non-linear characteristic and should be modeled by two states model. The data employed in this study is from January 2, 2002 to July 26, 2018, including 4,182 observations. The data include WTI spot and futures prices which have been collected by Investing.com database, and Henry Hub Natural Gas spot and its futures prices provided by U.S. Energy Information Administration and Investing.com database. The model selection result indicates that the data will be more appropriate to be modeled with MS-DCC-GARCH(1,1) with 2 regimes when compared to the single regime DCC-GARCH. This confirms that the non-linear model performs better than the linear model in this study. The result of MS-DCC-GARCH shows that crude oil spot/futures pair has higher volatility and correlation than natural gas pair in both economic states.

We found that there exists structural change for both conditional volatility and the time-varying correlation for oil and gas spot/futures pairs. It is obvious that the dynamic volatility and correlation between pre-2007 and post-2007, which is corresponding to the Hamburger crisis, are different, especially in oil spot/futures pair. Our findings suggest that the investors should decide to invest their money with special concern in the multi states of the economies. Moreover, the investors should be more careful when the markets face high volatility of returns which may provide them high risk and also provide high returns or high losses. The risk-minimized hedge ratio could be employed in the situation that investors want to reduce their risks.

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