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Bayesian Markov Switching Quantile Regression with Unknown Quantile τ: Application to Stock Exchange of Thailand (SET)

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Abstract : This paper introduces a Bayesian Markov Switching quantile regression with unknown- quantile model that allows the quantile level to be an estimated parameter. This will enable the model to reflect the real behavior of the data series. In the conventional estimation, the maximum likelihood is employed for switching model. Nevertheless, there are some concerns that the conventional estimation may face the computation difficulties. Thus, we consider a Bayesian estimation as the alternative estimator for this model. The posterior distribution of the model is constructed from the Asymmetric Laplace Distribution and uninformative prior distribution. The Metropolis Hasting is employed as the sampling method for the posterior and the vector of parameters. Both simulation study and real data application are provided. The results confirm the accuracy of the Bayesian estimation in both simulation and real application study.

Keywords : Markov regime-switching quantile regression models; CAPM model; Stock Exchange of Thailand.
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1 Introduction

Although quantile regression has been found successful in finding the relationship between a set of regressors and the outcome variable as it was introduced as a robust regression technique when the typical assumption of normality of the error term might not be strictly satisfied (Koenker and Bassett [1]). Nevertheless, there are still many unsolved problems. First, it is often hard simply to find the significant relationship between dependent and its covariates when there exists structural change in the relationship. Second, the quantile level is somewhat difficult to specify.

Fortunately, the Markov regime-switching quantile regression model as employed by Liu [2] and Tungtrakul, Maneejuk, and Sriboonchitta [3] was introduced to deal with the issue of the structural change in the casual relationship. Later, this model was further extended by Rakpho, Yamaka and Sriboonchitta [4] by considering the quantile level of dependent variable as a parameter estimate. Thus, the model becomes more flexible to accommodate various relationship structures. That is, the model is governed by an unobserved state variable that follows a Markov process with unknown transition probabilities and unknown quantile level. Hence, in each economic state or regime, this model has an ability to examine the effect of explanatory variables on the dependent variable at appropriate quantiles of the dependent variables conditional distribution.

In the work of Rakpho, Yamaka and Sriboonchitta [4], the Markov Switching quantile regression with unknown quantile (MSQU) is estimated by the maximum likelihood method. As suggested by Hamilton [5], who is the pioneer of the Markov Switching model, this model may face the ill-behaved likelihood surface (multiple local Maxima) as the usual numerical maximization of regime switching likelihood functions is subject to computational difficulties. Thus, we also have a concern that it may be difficult to estimate the parameters in the likelihood of the Markov Switching regression with unknown quantile. Previous researchers resort to a Bayesian method using Markov-chain Monte Carlo (MCMC) technique to estimate the parameter of the Markov Switching model (see, [3], [6] and [7]) and hence the application of this method to the MSQU model is straightforward.

The Bayesian approach has many advantages over classical methods including providing the entire posterior distribution of the parameters of interest, allowing for parameters uncertainty when making predictions, and flexible handling of complex model situation. Thus, this study develops Bayesian Markov Chain Monte Carlo estimation procedure that is more informative, efficient, and flexible than a maximum likelihood-based approach to estimate MSQU model. Our Bayesian estimation of MSQU starts with specifying a likelihood, which can often be specified as the Asymmetric Laplace Distribution (ALD). The ALD has a nice hierarchical representation which facilitates the implementation of the maximum likelihood estimation. Snchez, Lachos, and Labra [8]. Tu, Wang, Sun [9] mentioned that the ALD is closely related to the quantile regression approach as the maximum ALD is statistically equivalent to the least absolute square estimation which is commonly used as the estimator of quantile regression model. As the entire posterior distribution needs a likelihood function, this interesting ALD motivates us to adopt the ALD as the likelihood function in the MSQU model. Then, the estimation procedure estimates regimes at each time point, regime transition probabilities, and vector process parameters comprising coefficients, variance, and quantile within each regime.

This study, thus, will contribute to the recent literature on Markov switching quantile regression by extending their works into the Bayesian approach. An efficient MetropolisHastings method is developed for sampling the parameters conditional on the posterior distribution of the MSQU model. To show the performance of the model, we fitted the model to the Capital Asset Pricing Model (CAPM) and applied to the Stock Exchange of Thailand (SET). We also conducted a simulation study to assess the performance and accuracy of the Bayesian estimation.

The rest of this article is organized as follows, Section 2 briefly describes the methodology used in this study: the Asymmetric Laplace Distribution, Markov switching quantile regression with unknown quantile, and its Bayesian estimation. Section 3 presents the model simulations under various scenarios. Section 4 describes CAPM model and data used in this study. Section 5 provides an application results. Conclusion is provided in Section 6.

2 Methodology

As we mentioned before, we consider using the ALD as the likelihood function, thus we first briefly explain the function

2.1 Asymmetric Laplace Distribution

The Asymmetric Laplace Distribution (ALD) is a continuous probability distribution which is a generalization of the Laplace Distribution. The likelihood function based on ALD of Koenker and Machado [10] can be shown

$$L(y|m,\sigma,\tau) = \frac{\tau(1-\tau)}{\sigma}\{-\rho_\tau\left(\frac{y-m}{\sigma}\right)\}, \tag{2.1}$$

where $0<\tau<1$. $\rho_\tau(A)=A(\tau-I(A<0))$, $A=\frac{y-m}{\sigma}$ is called the check function, with $I(\cdot)$ being the usual indicator function. where m is the mean parameter, $\sigma>0$ is considered as a scale parameter and skewness parameter or quantile level τ.

2.2 MS-Quantile Regression with Asymmetric Laplace Distribution

Following Tungtrakul, Maneejuk, and Sriboonchitta [3] and Rakpho, Yamaka and Sriboonchitta [4], let s_t be an unobserved discrete-valued indicator variable, such that at any time t the process will be in regime $s_t \in \{1,...,H\}$. The Markov switching and quantile regression model is given by

$$y_t = \beta^0_{s_t}(\tau) + \sum_{k=1}^{K}\beta^k_{s_t}(\tau)x^k_t + \varepsilon_{s_t,t} \quad ;t=1,...,T, \tag{2.2}$$

where y_t is a dependent variable and x_t is a matrix of independent variables, $\beta^0_{s_t}(\tau)$ and $\beta^k_{s_t}(\tau)$ are the regime dependent coefficients at the estimated quantile τ. $\beta^k_{s_t}(\tau)$ coefficients indicate a vector of unknown parameters which define a relationship between vector x_t and the conditional quantile function of y_t . $\varepsilon_{s_t,t}$ is regime dependent and identically distributed (i.i.d) random errors and is assumed to have ALD with variance $\sigma^2_{s_t}(\tau)$ and mean zero. As we mentioned before, the quantile level is unknown and treated as another parameter to be estimated and it is assumed to be a regime independent and restricted to have the value in $[0.1]$. In this study, the state variable s_t is an ergodic homogeneous Markov chain on a finite set, with a transition matrix

$$P = \begin{bmatrix} p_{11} & p_{21} & \cdots & p_{H1} \\ p_{12} & p_{22} & \cdots & p_{H2} \\ \vdots & \vdots & \ddots & \vdots \\ p_{1H} & p_{2H} & \cdots & p_{HH} \end{bmatrix}, \tag{2.3}$$

where p_{ij} denotes the probability of transition from regime i is followed by regime j, $i = 1,...,H$; $j = 1,...,H$ and $\sum p_{ij} = 1$.

The total parameter set to be estimated is $\Psi = (\beta_{s_t}(\tau), \varepsilon_{s_t,t}(\tau), \tau)$ can be estimated by the maximum likelihood, which is homologous to the case of the conventional Markov quantile regression. To simplify the estimation procedure, we assume two regime model ($s_t = 1,2$), thus, the sample conditional likelihood function of the MSQR with unknown quantile model with 2 regimes can be defined as

$$L(\Psi|y_t, x_t) = \sum_{s_t=1}^{2} \left\{ \frac{\tau(1-\tau)}{\sigma_{s_t}} \rho_\tau \cdot A \cdot Pr(s_t|\Theta_{t-1};\Psi) \right\}, \tag{2.4}$$

where $A = \left(\frac{y_t - \beta_{s_t,0}(\tau) - \sum_{k=1}^{K} \beta^k_{s_t}(\tau) x^k_t}{\sigma_{s_t}} \right)$. Θ_{t-1} is all available information set at time $t-1$ in model, and $(\Pr(s_t|\Theta_{t-1};\Psi)$ is weighted probabilities computed recursively from the Hamiltons filter algorithm Hamilton [5]. Thereby, filtered probabilities of each state computed recursively can be shown as follows:

$$Pr(s_t|\Theta_{t-1};\Psi) = \{p_{11} Pr(s_t = i|\Theta_{t-1};\Psi) + p_{22} Pr(s_t = j|\Theta_{t-1};\Psi)\} \tag{2.5}$$

$$Pr(s_t = i|\Theta_{t-1};\Psi) = \frac{f(y_t|(s_t = i|\Theta_{t-1};\Psi)(s_t = i|\Theta_{t-1};\Psi)}{\sum_{h=1}^{2} (f(y_t|(s_t = h|\Theta_{t-2};\Psi)(s_t = h|\Theta_{t-2};\Psi))}, \; i = 1,2. \tag{2.6}$$

2.3 Bayesian Markov Switching Quantile Regression with Unknown Tau

The Bayesian inference for Markov Switching quantile regression has many advantages over classical methods including providing the entire posterior distribution of the parameter of interest, allowing for parameters uncertainty when making predictions, and flexible handling of complex model situations.

According to the Bayes theorem, the sample of the posterior distribution of this model can be shown as follows

$$Pr(\Psi,P,s_t\,|y_t,x_t) \propto Pr(\Psi,P,s_t)\,L(y_t,x_t\,|\Psi,P,s_t) \tag{2.7}$$

where $L(y_t,x_t\,|\Psi,P,s_t)$ is the likelihood function is the Markov switching quantile regression model in Eq. (2.4). The rest of the function is the prior distribution $\Pr(\Psi,P,s_t)$, which can be formed as

$$\Pr(\Psi,P,s_t) = \Pr(\Psi)\Pr(P)\Pr(s_t\,|\Psi,P), \tag{2.8}$$

To draw the joint posterior distribution of the model and the parameters, given the sample data in Eq. (2.7), the Metropolis-Hasting (MH) sampler is employed The resulting simulated samples from the parameter space can be used to make inferences about the distribution of the process parameters and regimes. For more detail of MH, our study refers to Chib and Greenberg [11].

There are four parts in the prior distribution. The first part is the unknown parameters $\beta_{s_t}(\tau)$. The second part is variance of the model $\sigma_{s_t}(\tau)$, the third part is transition matrix (P). The final part is the quantile parameter τ. We assume the prior distribution for the unknown parameters to be uninformative priors are adopted, thus, the prior distributions for $\beta_{s_t}(\tau)$, $\sigma_{s_t}(\tau)$, P and τ are assumed to have normal distribution, Inverse gamma, Dirichlet distribution, and uniform distribution, respectively, Therefore we have

$$\begin{array}{ccc} \beta_{s_t}(\tau) & \sim & N(0,\Sigma), \\ \sigma_{s_t}(\tau) & \sim & IG(0.01,0.01), \\ P & \sim & Dirichlet(q), \\ \tau & \sim & uniform(0,1), \end{array} \tag{2.9}$$

where Σ is the diagonal variance matrix parameter $\beta_{s_t}(\tau)$. We select these three priors since the sign of the $\beta_{s_t}(\tau)$ can be either positive or negative, the sign of $\sigma_{s_t}(\tau)$ must be positive and P should be persistently staying in their own regime. The MH iterations for all parameters can be described as follows:

1. Starting at an initial parameter value, $\Omega^0 = \Psi^0, P^0$

2. Choosing a new parameter value close to the old value based on proposal function. The proposal function employed in the MH algorithm is a normal distribution with mean at Ω^0 and covariance (C_t), that is Proposal $= \left(\centerdot\,|\Omega^0,\ldots,\Omega^{j-1},C_t\right) = N\left(\Omega^{(j-1)},C_t\right)$. In MH algorithm, covariance of the proposal distribution, C_t is set as $C_t = \sigma_d \operatorname{cov}\left(\Omega^0,\ldots,\Omega^{j-1}\right) + \sigma_d \varepsilon I_d$ after initial period, where σ_d is a parameter that depends on dimension d and ε is a constant term which is very tiny when compared with the size of the likelihood function.

3. Computing the acceptance probability which is calculated by

$$\theta_j = \frac{L(\Omega^*\,|y_t,x_t)\left(\Omega^{j-1},C_t\right)}{L(\Omega^{j-1}\,|y_t,x_t)\,(\Omega^*\,|\Omega^{j-1}\,,C_t)} \tag{2.10}$$

If $\theta_j \geq 1$ then draw $\Omega^j = \Omega^{j-1}$. If $\theta_j < 1$ then draw trace Ω^j from a proposal distribution.

4. Repeat steps 2-3 for $j = 1,\ldots,N$ in order to obtain samples $\Omega^1,\ldots,\Omega^N$.

2.4 Outline of the Estimation Procedure

According to the MH algorithm, it is difficult to sample all the parameters together as the candidate parameters obtained from the proposal function may not ensure convergence to the desired target density. To deal with this problem, we separate the parameters into four parts and the algorithm will revolve the repeated generation of variates from their full conditional densities as follows:

$$\begin{array}{rcl} \beta_{s_t}(\tau)^{(j+1)} & \leftarrow & \beta_{s_t}(\tau)^{(j)}, \sigma_{s_t}(\tau)^{(j)}, P^{(j)}, \tau^{(j)} \\ \sigma_{s_t}(\tau)^{(j+1)} & \leftarrow & \beta_{s_t}(\tau)^{(j)}, \sigma_{s_t}(\tau)^{(j+1)}, P^{(j)}, \tau^{(j)} \\ P(\tau)^{(j+1)} & \leftarrow & \beta_{s_t}(\tau)^{(j)}, \sigma_{s_t}(\tau)^{(j)}, P^{(j+1)}, \tau^{(j)}, \\ \tau(\tau)^{(j+1)} & \leftarrow & \beta_{s_t}(\tau)^{(j)}, \sigma_{s_t}(\tau)^{(j)}, P^{(j)}, \tau^{(j+1)} \end{array} \tag{2.11}$$

3 Simulation Study

To test the performance of the proposed framework, we simulate MSQU model to assess the accuracy of Bayesian estimation. In this section we will describe a simulation study of the model. This study considers 2 regime $s_t = 1, 2$, we then simulate data from the model

$$y_t = \beta^0_{s_t}(\tau) + \beta^1_{s_t}(\tau) x_t + \varepsilon_{s_t,t}(\tau), \quad s_t = \{1, 2\}, \tag{3.1}$$

where $\beta^0_{s_t=1}(\tau) = 1$, $\beta^1_{s_t=1}(\tau) = 2$, $\beta^0_{s_t=2}(\tau) = 2$ and $\beta^1_{s_t=2}(\tau) = 3$. We simulated the independent variables x_t from $N(0,1)$. The simulated filter probabilities for two regime model are generated from $U[0,1]$, where the transition probabilities $p_{11} = p_{22} = 0.95$. For the error term, we remark that the asymmetric Laplace distribution quantile parameters τ are set to be invariant as Markov regime switches. We discuss two cases hereinafter: Case 1: N= 100, 500, 1000. Case 2 $\tau = 0.3, 0.5, 0.7$.

We evaluate the performance of our proposed model and compare the parameter estimates with the true values in each case. The MCMC estimation procedure described *m* the Appendix was used to generate 10,000 samples from the joint parameter density of the model. We choose the posterior mean of parameters and reported in Table 1. It can be seen that the estimated parameters are quite close to their true values with satisfactory standard derivations, meaning that we have an accurate and reliable estimation for the model. Moreover, the estimated parameters tend to be close to their true values when the sample size increases. According to this simulation study, we can summarize that the Bayesian estimation for the model is accurate.

Table 1: Simulation results

Parameter	True Value	N=100	N=500	N=1000
$\alpha_{1,s_t=1}(\tau)$	1	1.15 (0.01)	0.96 (0.000)	1.01 (0.002)
$\beta_{1,s_t=1}(\tau)$	2	2.25 (0.008)	2.09 (0.000)	1.93 (0.001)
$\alpha_{1,s_t=2}(\tau)$	2	1.44 (0.369)	2.30 (0.001)	2.08 (0.005)
$\beta_{1,s_t=2}(\tau)$	3	3.10 (0.276)	3.32 (0.000)	3.19 (0.002)
p_{11}	0.95	0.94 (0.013)	0.93 (0.000)	0.94 (0.000)
p_{22}	0.95	0.98 (0.009)	0.95 (0.000)	0.95 (0.000)
τ	0.3	0.24 (0.026)	0.26 (0.000)	0.28 (0.000)
$\alpha_{1,s_t=1}(\tau)$	1	1.36 (0.026)	0.99 (0.022)	0.89 (0.007)
$\beta_{1,s_t=1}(\tau)$	2	1.98 (0.032)	2.03 (0.023)	1.95 (0.007)
$\alpha_{1,s_t=2}(\tau)$	2	1.77 (0.029)	2.10 (0.189)	1.88 (0.018)
$\beta_{1,s_t=2}(\tau)$	3	3.14 (0.025)	2.77 (0.161)	3.15 (0.009)
p_{11}	0.95	0.97 (0.007)	0.90 (0.017)	0.94 (0.004)
p_{22}	0.95	0.98 (0.008)	0.95 (0.009)	0.96 (0.004)
τ	0.5	0.48 (0.012)	0.49 (0.010)	0.47 (0.004)
$\alpha_{1,s_t=1}(\tau)$	1	0.82 (0.000)	1.01 (0.026)	0.56 (0.000)
$\beta_{1,s_t=1}(\tau)$	2	1.97 (0.000)	2.00 (0.008)	1.09 (0.833)
$\alpha_{1,s_t=2}(\tau)$	2	1.30 (0.000)	2.01 (0.001)	1.16 (0.000)
$\beta_{1,s_t=2}(\tau)$	3	3.39 (0.000)	3.05 (0.000)	2.53 (0.000)
p_{11}	0.95	0.97 (0.000)	0.98 (0.000)	0.93 (0.007)
p_{22}	0.95	0.98 (0.000)	0.97 (0.000)	0.96 (0.007)
τ	0.7	0.67 (0.000)	0.72 (0.000)	0.69 (0.004)

Note: () denotes standard error

4 Data Description

Prior to explaining the data, let us briefly describe the basic concept of the CAPM. According to Sharpe [12], the CAMP is designed for examining the relationship between expected asset return and its risk. The idea is the trade-off compensation of risk and return while comparing to the market return, or how the asset return can be different from group. By setting expected asset return equal to risk-free asset return equal to risk-free asset return plus a multiplication of market risk premium and systematic risk [13] the model can be expressed as follows

$$r_t = \beta_0 + \beta_1 r_{m,t} + \varepsilon_{i,t} \tag{4.1}$$

where β_0 and β are parameters. $r_{it} = R_{it} - R_{ft}$ and $r_{mt} = R_{mt} - R_{ft}$ denote excess return on asset and on the market portfolio at time t, respectively. R_{it} is the return on asset i. R_{mt} is the market return. R_{ft} is the risk free asset return at time t.

We thus apply the MSQU model to investigate the CAPM. In this study, we use the daily return of Stock Exchange of Thailand (SET index) as a market return and consider seven companies stock returns consisting of PTT (PTT Public Company Limited), KBANK (Kasikornbank Public Company Limited), BBL (Bangkok Bank Public Company Limited), AOT (Airports of Thailand Public Company Limited), ADVANC (Advanced Wireless Network Company Limited), CK (CH. Karnchang Public Company Limited) and CPALL (C.P. All Public Company Limited) as individual asset return. In this paper, we assume risk free to be zero. All data were collected from Bloomberg database. The daily time series data cover the period from June 2005 to September 2018.

We then transform these time series variables into log-return rate and the movement of these returns are plotted in Figure 1. According to the Figure, the extremely high positive and negative returns are evident from September 2008 onward, and have continued well into 2009. These return series also show volatility under uncertainty situation particularly as occurred during the financial crisis of 2007-2009. The descriptive statistics are provided in Table 2. We observe that the returns of Stock Exchange of Thailand (SET index) and seven companies stocks are not normally distributed. Minimum Bayes factor (MBF) values indicate that the returns are decisive not normally distributed. Unit root test was also conducted to investigate the stationarity of the data and the result shows that all returns are stationary with decisively evidence. Consequently, these variables can be used to estimate in the Markov switching quantile with unknown quantile model in Bayesian approach in the next step. Note that the interpretation of MBF is referred to Vovk [14].

Table 2: Descriptive statistics

	SET50	PTT	KBANK	BBL	AOT	ADVANC	CK	CPALL
Mean	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.0008
Median	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Max	0.105	0.149	0.114	0.085	0.149	0.146	0.254	0.199
Min	-0.160	-0.185	-0.203	-0.177	-0.163	-0.233	-0.215	-0.1588
Std	0.011	0.019	0.018	0.017	0.020	0.019	0.025	0.019
Skewness	-1.065	-0.193	-0.312	-0.693	-0.209	-0.806	0.296	0.573
Kurtosis	21	9	9	13	11	17	11	13
JB-Test	5116	734	658	1561	1114	3078	999	1561
MBF- JB	0.000	0.000	0.000	0.0000	0.000	0.000	0.000	0.0000
MBF-UR	0.000	0.000	0.000	0.0000	0.000	0.000	0.000	0.000
Obs.	3597	3597	3597	3597	3597	3597	3597	3597

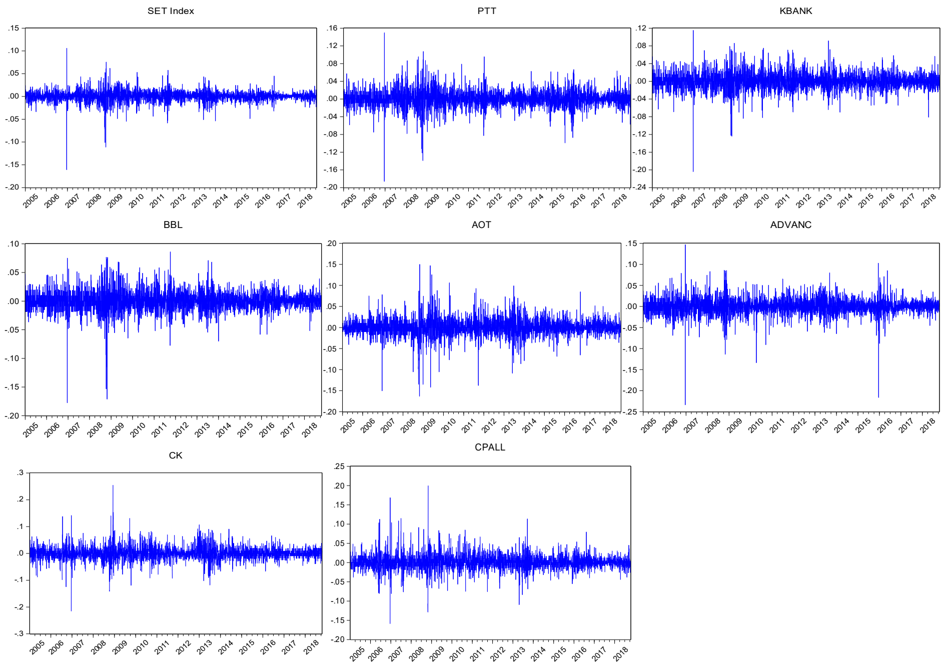

Figure 1: Daily returns of Stock Exchange of Thailand (SET index) and seven stocks from 2005 to 2018

5 Empirical Findings

5.1 Outline of the Estimation Procedure

The most important point in working with the MSQU model is to analyze whether a switch occurs or not, which is equivalent to comparing the MSQU model with the 1-regime quantile regression with unknown quantile (QU) model. In a Bayesian framework, the model comparison can be based on the deviance information criterion (DIC). DIC is a hierarchical modeling generalization of the Akaike information criterion (AIC) and Bayesian information criterion. DIC is particularly useful in Bayesian model selection problems where the posterior distributions of the models have been obtained by Markov chain Monte Carlo (MCMC) simulation. According to the results in Table 3, it is found that the DIC of MSQU model of PTT, KBANK, CK and CPALL are lower than those of the QU model. While, the DIC of MSQU model of BBL, AOT, and ADVANC are higher than 1-regime model. Therefore, we may conclude that MSQU is favorable over the QU model in 4 out of 7 cases and that a switch occurs in the mean of returns of these four stock returns. Also, these evidences confirm the variability of the beta risk across market conditions. In addition, we also compare our unknown quantile model with the conventional quantile model with $/tau = 0.5$, the results confirm the higher performance of the unknown quantile model compared to the conventional one. This indicates that our approach is indeed better.

Table 3: Model Selection based on DIC

Variable	unknown quantile		qunatile at 0.5	
	1 Regime	2 Regime	1 Regime	2 Regime
PTT	-25995.5	**-26008.27**	-25981.15	-25871.35
KBANK	-25389.1	**-25477.44**	-25378.69	-23214.64
BBL	**-25251.5**	-25231.6	-25243.32	-25176.02
AOT	**-23857**	-23529.39	-23853.1	-23632.1
ADVANC	**-23986.04**	-23529.39	-23983.25	-23305.97
CK	-24290.38	**-24379.67**	-24269.33	-23706.8
CPALL	-23392.52	**-23437.75**	-23390.04	-23305.97

5.2 Markov Switching Quantile with Unknown Quantile Using Bayesian Approach

Table 4: Estimation result of Markov switching quantile regression with unknown quantile

Parameter	PTT	KBANK	CK	CPALL
$\alpha_{1,s_t=1}(\tau)$	0.010 (0.000)	-0.001 (0.193)	0.050 (0.000)	0.001 (0.001)
$\beta_{1,s_t=1}(\tau)$	-0.507 (0.000)	0.410 (0.000)	0.130 (0.000)	0.146 (0.000)
$\alpha_{1,s_t=2}(\tau)$	0.0002 (0.000)	0.0001 (0.000)	0.001 (0.000)	-0.022 (0.000)
$\beta_{1,s_t=2}(\tau)$	0.456 (0.000)	0.813 (0.000)	0.257 (0.000)	0.293 (0.000)
σ_1	0.063 (0.000)	0.003 (0.001)	0.170 (0.000)	0.003 (0.000)
σ_2	0.002 (0.000)	0.005 (0.000)	0.003 (0.000)	0.097 (0.000)
P_{11}	0.999 (0.000)	0.930 (0.000)	0.999 (0.000)	0.924 (0.000)
P_{22}	0.999 (0.000)	0.950 (0.000)	0.942 (0.000)	0.999 (0.000)
τ	0.503 (0.000)	0.400 (0.000)	0.630 (0.000)	0.600 (0.000)

Note: () denotes Minimum Bayes factor computed by $e^{p\log p}$, where p is p-value (see [14] and [15])

Table 5: Estimation result of Markov switching quantile regression with unknown quantile

Parameter	BBL	AOT	ADVANC
$\alpha_1(\tau)$	0.0003 (0.005)	0.0001 (0.227)	0.0002 (0.052)
$\beta_1(\tau)$	0.454 (0.000)	0.277 (0.000)	0.285 (0.000)
σ_1	0.002 (0.000)	0.003 (0.000)	0.003 (0.000)
τ	0.503 (0.000)	0.503 (0.000)	0.490 (0.000)

Note: () denotes Minimum Bayes factor computed by $e^{p \log p}$, where p is p-value (see [14] and [15])

The best fit models from the model selection section are reported in Tables 4-5. Two-regime model for PTT, KBANK, CK, and CPALL equations are provided in Table 4 while one-regime model of BBL, AOT, and ADVANC are provided in Table 5. Considering Table 4, the beta risks $\beta_{1,s_t=1}(\tau)$ and $\beta_{1,s_t=2}(\tau)$ are calculated for two regimes, namely bull and bear economies. We observe that the beta risk in regime 1 is lower than regime 2. Hence, we can interpret first regime as a bull market or high-risk market, the second regime as bear market or low-risk market.

We note that the beta risk is less than one indicates that the stock return moves less than the stock market return (SET Index) while the beta risk greater than one, demonstrates that the stock return moves larger than the stock market return. The results show that beta risk of PTT is negative beta risk for bull economy but positive for bear economy. This means that the PTT stock has a negative correlation to the SET market. PTT move in the opposite direction to the SET market. Thus, PTT will decrease-increase in value by 0.507 % for each increase/decrease of 1% in the SET market, and vice versa. In the case of KBANK, CK and CPALL, we found that the beta risks are all positive and lower than unity for both regimes. Therefore, these three stock returns move less than SET market return in both bull and bear markets.

Table 4 also provides the transition probabilities of staying in regime 1 (P_{11}) and regime 2 (P_{22}). The results show that P_{11} and P_{22} are extremely high, indicating that the probability of staying in the same regime is substantially high whenever the current period is in either regime 1 or 2.

Table 5 shows the results of BBL, AOT and ADVANC fitted with 1-regime quantile regression model. The beta risks are all positive and less than one. Therefore, BBL, AOT and ADVANC stock return moves less than the SET market. Finally, let us consider the quantile parameter for all models, we can observe a heterogenous result of the optimal quantile estimates. We can observe that some of stock CAPM equations (KBANK, CK and CPALL)show the value of quantile parameter that deviates form 0.5. This confirms the usefulness of this model when the quantile level is generally unknown and difficult to specify in the model. In a nutshell, if we do not allow the model to split into 2 regimes, we may miss the true risk of the stock when there exists a structural change in the data series. Moreover, another challenge is that what is the best quantile or which quantile is the most informative to explain the behavior of the data. When the model contains many quantile levels, the best fit quantile could play an important role in the model building process to obtain a better interpretation and to improve the precision of model.

6 Conclusion

In this study, Bayesian Markov Chain Monte Carlo (MCMC) procedure was developed for estimating the joint parameter and regime density of Markov Switching quantile regression with unknown quantile (MSQU) model. Suppose that the true quantile level for the model is 0.70^{th}, the researchers may obtain unreliable parameter estimates, when they consider the model at 0.5^{th} quantile. Thus, this model becomes more flexible as it considers the quantile level as the parameter to be estimated. However, as the number of parameters increases, we are concerned that the model may face the computational difficulties. This motivates us to estimate the model with Bayesian approach. The simulation study is used to assess the accuracy of the model. The result shows the accuracy and reliability of Bayesian estimation for this model.

In the application study, we empirically study the CAPM and use the Stock Exchange of Thailand (SET index) as the market return and use PTT, KBANK, BBL, AOT, ADVANC, CK and CPALL as the individual asset return in the CAPM application. We conduct the DIC to examine the existence of the structural change in CAPM. The results reveal that MSQU model is chosen for 4 out of 7 models, confirming that the regime switching, in most cases, can describe the effect of SET market on individual stock return. In brief, PTT, KBANK, CK and CPALL CAPMs show a strong evidence of two regime model, while one regime model is the best fit model for BBL, AOT and ADVANC CAPMs.

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