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On Anti-Regular Ternary Algebras

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Abstract : A ternary algebra is an algebraic system consisting of a nonempty set together with two different ternary algebraic operations, called additive ternary and multiplicative ternary. The notions of anti- additive regular ternary algebras and anti- multiplicative regular ternary algebras are defined. Some properties of anti- additive regular ternary algebras and anti- multiplicative regular ternary algebras are given. Moreover, we define the notions of anti-additive and anti - multiplicative equivalence relations on ternary algebras. Furthermore, we introduce the notion of quotient ternary algebras by using anti-additive and anti-multiplicative regular equivalence relations.

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1 Introduction

In 1971, W.G. Lister [10] introduced the notion of a ternary ring and provided some types of representations of ternary ring. According to Lister [10], a ternary ring is an algebraic system consisting of a nonempty set R together with a binary operation, called addition and a ternary multiplication, which forms an abelian

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group relative to addition, a ternary semigoup relative to ternary multiplication and left, right and lateral distributive laws hold. The notion of ternary semiring was first introduced by Dutta and Kar [1] as a generalization of ternary ring. Some earlier works on ternary semirings may be found in [2-7] Recently, Pradchaya Piajan and Utsanee Leerawat [8] introduced a new algebraic system, which is called ternary algebra. A ternary algebra is a nonempty set with a ternary addition and a ternary multiplication. They generalized the notions of regular and completely regular in ternary algebras and provided some interesting properties of completely regular ternary algebras. In 2017, N. Sheela and A. Rajeswari [9] studied the structure of an anti-regular semirings and investigated some of its properties.

In this paper, we define the notion of anti-regular ternary algebra by using the concept of anti-regular semiring. We also study some interesting properties of anti - regular ternary algebras. Moreover, we define the notions of anti-additive and anti - multiplicative equivalence relations on ternary algebras. As a consequence of anti-additive and anti - multiplicative regular equivalence relations on a ternary algebra, we obtain quotient ternary algebras.

2 Preliminaries

In this section, we recall some known definitions and examples [8].

Definition 2.1. [8] *Let T be a nonempty set. A ternary operation on T is a function from $T \times T \times T$ to T. If the ternary operation is denoted $*$, then we use the notation $a * b * c = d$ if $(a, b, c) \in T \times T \times T$ is mapped to $d \in T$ under the ternary operation.*

Remark. We will consider both additive and multiplicative ternary operations. When considering an additive ternary operation, we denote the image of (a, b, c) as $a + b + c$. When using multiplicative notation, we denote the image of (a, b, c) as abc (called the product of a, b and c).

Definition 2.2. [8] *A nonempty set T together with two ternary operations, called addition and multiplication is said to be a ternary algebra if T satisfies the following conditions:*

(i) $(a + b + c) + d + e = a + (b + c + d) + e = a + b + (c + d + e)$,

(ii) $(abc)de = a(bcd)e = ab(cde)$,

(iii) $ab(c + d + e) = abc + abd + abe$,
$a(b + c + d)e = abe + ace + ade$,
$(a + b + c)de = ade + bde + cde$,

for all $a, b, c, d, e \in T$.

Example 2.3. [8] $(\mathbb{N}, +, \times), (\mathbb{Z}, +, \times), (\mathbb{R}, +, \times)$ *and* $(\mathbb{C}, +, \times)$ *are ternary algebras under the usual addition and multiplication as the ternary operations.*

Example 2.4. [8] *Let* $T = \{\ldots, -5, -3, -1, 1, 3, 5, \ldots\}$. *Then* T *is a ternary algebra under the usual addition and multiplication as the ternary operations while* T *is not a ternary semiring under the usual addition and multiplication because* $(-5) + (-3) = -8 \notin T$.

Example 2.5. [8] *Let* $\mathbb{Z}^+$ *be a set of positive integers. Then* $\mathbb{Z}^+$ *is a ternary algebra under the addition and multiplication as follows:*

$$\begin{aligned} a + b + c &= lcm(a, b, c) \\ a \cdot b \cdot c &= abc \end{aligned}$$

for all $a, b, c \in \mathbb{Z}^+$.

Definition 2.6. [8] *A nonempty subset* S *of a ternary algebra* T *is called a ternary subalgebra if*

(*i*) $a + b + c \in S$,

(*ii*) $abc \in S$,

for all $a, b, c \in S$.

Example 2.7. [8] *Let* $T = \{\cdots, -5, -3, -1, 1, 3, 5, \cdots\}$ *be the ternary algebra under the addition and multiplication as a ternary operation. Then* $S = \{3k | k \in T\}$ *is a ternary subalgebra of* T.

Definition 2.8. [8] *A ternary algebra* T *is said to be abelian if*

$$x_1 + x_2 + x_3 = x_{\sigma(1)} + x_{\sigma(2)} + x_{\sigma(3)},$$

for every permutation σ *of* $\{1, 2, 3\}$ *and* $x_1, x_2, x_3 \in T$.

Definition 2.9. [8] *A ternary algebra* T *is said to be commutative if*

$$x_1 x_2 x_3 = x_{\sigma(1)} x_{\sigma(2)} x_{\sigma(3)},$$

for every permutation σ *of* $\{1, 2, 3\}$ *and* $x_1, x_2, x_3 \in T$.

Definition 2.10. [8] *An element* e *in a ternary algebra* T *is called additive idempotent if* $e + e + e = e$.

Definition 2.11. [8] *An element* e *in a ternary algebra* T *is called multiplicative idempotent if* $eee = e$.

Definition 2.12. [8] *An element* e *in a ternary algebra* T *is called idempotent if* e *is additive and multiplicative idempotent.*

3 Main Theorem

In this section, we define an anti-additive regular and anti-multiplicative regular element in ternary algebra.

Definition 3.1. *An element a in a ternary algebra T is called anti-additive regular if there exists an element x in T such that $a = x + a + x + a + x$ and $x = a + x + a + x + a$. A ternary algebra T is called anti-additive regular if all of its elements are anti-additive regular.*

Definition 3.2. *An element a in a ternary algebra T is called anti-multiplicative regular if there exists an element x in T such that $a = xaxax$ and $x = axaxa$. A ternary algebra T is called anti-multiplicative regular if all of its elements are anti-multiplicative regular.*

Theorem 3.3. *Let T be an idempotent abelian ternary algebra. If T is an anti-additive regular, then T is an anti-multiplicative regular.*

Proof. Since T is an anti-additive regular and $a \in T$, there exists an element $x \in T$ such that $a = x + a + x + a + x$ and $x = a + x + a + x + a$.
We get

$$\begin{aligned}
xaxax &= (a+x+a+x+a)a(a+x+a+x+a)a(a+x+a+x+a) \\
&= (a+(a+x+a+x+a)+a+x+a) \\
&\quad a(a+(a+x+a+x+a)+a+x+a) \\
&\quad a(a+(a+x+a+x+a)+a+x+a) \\
&= (a+a+a+a+a)a(a+a+a+a+a)a(a+a+a+a+a) \\
&= a,
\end{aligned}$$

and

$$\begin{aligned}
axaxa &= (x+a+x+a+x)x(x+a+x+a+x)x(x+a+x+a+x) \\
&= (x+(x+a+x+a+x)+x+a+x) \\
&\quad x(x+(x+a+x+a+x)+x+a+x) \\
&\quad x(x+(x+a+x+a+x)+x+a+x) \\
&= (x+x+x+x+x)x(x+x+x+x+x)x(x+x+x+x+x) \\
&= x.
\end{aligned}$$

Hence a is an anti-multiplicative regular.
Therefore T is a anti-multiplicative regular ternary algebra. □

Theorem 3.4. *Let T be an idempotent commutative ternary algebra. If T is anti-multiplicative regular, then T is anti-additive regular.*

Proof. Similar to the proof of Theorem 3.3 □

Definition 3.5. *Let T be a ternary algebra and let $a \in T$. Define*

$$\overline{A}(a) = \{x \in T \mid a = x + a + x + a + x \text{ and } x = a + x + a + x + a\ \}.$$

Theorem 3.6. *Let T be a ternary algebra and $a \in T$. Then*

1. *if T is an additive idempotent, then $a \in \overline{A}(a)$,*
2. *$b \in \overline{A}(a)$ if and only if $a \in \overline{A}(b)$ for all $b \in T$,*
3. *if T is an abelian ternary algebra, then $\overline{A}(x) \subseteq \overline{A}(a)$ for all $x \in \overline{A}(a)$.*

Proof. (i) Since T is an additive idempotent and $a \in T$,
then $a + a + a + a + a = a$. Hence $a \in \overline{A}(a)$.
(ii) Let $a, b \in T$ and $b \in \overline{A}(a)$.
We have

$$\begin{aligned} b \in \overline{A}(a) &\Leftrightarrow b = a + b + a + b + a \\ &\quad\ \ a = b + a + b + a + b \\ &\Leftrightarrow a \in \overline{A}(b). \end{aligned}$$

(iii) Let T be an abelian ternary algebra and $a, x \in T$ such that $x \in \overline{A}(a)$.
Suppose that $y \in \overline{A}(x)$.
We have $y = x + y + x + y + x$ and $x = y + x + y + x + y$.
Then

$$\begin{aligned} y &= x + y + x + y + x \\ &= (a + x + a + x + a) + (x + y + x + y + x) + (a + x + a + x + a) + \\ &\quad (x + y + x + y + x) + (a + x + a + x + a) \\ &= a + (x + a + x + a + x) + y + x + y + (x + a + x + a + x) + a + x + \\ &\quad y + x + y + (x + a + x + a + x) + a \\ &= a + a + y + x + y + a + a + x + y + x + y + a + a \\ &= a + a + y + a + a + (x + y + x + y + x) + y + a + a \\ &= a + a + y + a + a + y + y + a + a \\ &= a + (a + y + a + y + a) + y + a + a \\ &= a + y + y + a + a \\ &= a + y + a + y + a, \end{aligned}$$

Similarly, $a = y + a + y + a + y$. So, $y \in \overline{A}(a)$.
Therefore $\overline{A}(x) \subseteq \overline{A}(a)$ for all $x \in \overline{A}(a)$. □

Definition 3.7. *Let T be a ternary algebra. The relation ρ_A on T is defined as follows:*

$$a \rho_A b \Leftrightarrow a \in \overline{A}(b)$$

for all $a, b \in T$.

Theorem 3.8. *Let* T *be an additive idempotent abelian ternary algebra. Then* ρ_A *is an equivalence relation on* T.

Proof. Let $a, b, c \in T$.
1. Since T is an additive idempotent, then $a + a + a + a + a = a$. Hence $a\rho_A a$.
2. Let $a\rho_A b$.
Then

$$\begin{aligned} a &= b + a + b + a + b \\ b &= a + b + a + b + a. \end{aligned}$$

So, $b\rho_A a$.
3. Let $a\rho_A b$ and $b\rho_A c$.
Then

$$\begin{aligned} a = b + a + b + a + b \text{ and } b = a + b + a + b + a \\ b = c + b + c + b + c \text{ and } c = b + c + b + c + b. \end{aligned}$$

Now

$$\begin{aligned} a &= b + a + b + a + b \\ &= b + a + a \\ &= (c + b + c + b + c) + a + a \\ &= (c + b + b) + (a + a + a) + (a + a + a) \\ &= c + (b + a + a) + (b + a + a) + a + a \\ &= c + a + a + a + a \\ &= c + a + a \\ &= c + a + c + a + c, \end{aligned}$$

and

$$\begin{aligned} c &= b + c + b + c + b \\ &= b + c + c \\ &= (a + b + a + b + a) + c + c \\ &= (a + b + b) + (c + c + c) + (c + c + c) \\ &= a + (b + c + c) + (b + c + c) + c + c \\ &= a + c + c + c + c \\ &= a + c + c \\ &= a + c + a + c + a. \end{aligned}$$

Therefore ρ_A is an equivalence relation on T. □

Definition 3.9. *An equivalence relation ρ_A on a ternary algebra T is called anti-additive regular equivalence relation on T. We define the equivanlence class of $a \in T$ to be the set*

$$a\rho_A = \{x \in T | x\rho_A a\}.$$

The quotient set T/ρ_A is the set of all equivalence classes of T with repect to the relation ρ_A.
That is

$$T/\rho_A = \{a\rho_A | a \in T\}.$$

Theorem 3.10. *Let T be an additive idempotent abelian ternary algebra and ρ_A be an anti-additive regular equivalence relation on T. Then T/ρ_A is a ternary algebra under the addition and multiplication as follows:*

$$(a\rho_A) \oplus (b\rho_A) \oplus (c\rho_A) = (a+b+c)\rho_A$$
$$(a\rho_A) \odot (b\rho_A) \odot (c\rho_A) = (abc)\rho_A$$

for all $a, b, c \in T$.

Proof. Let $a\rho_A, \acute{a}\rho_A, b\rho_A, \acute{b}\rho_A, c\rho_A, \acute{c}\rho_A \in T/\rho_A$ be such that $a\rho_A = \acute{a}\rho_A, b\rho_A = \acute{b}\rho_A$,
$c\rho_A = \acute{c}\rho_A$.
We have

$$\begin{aligned}
a &= \acute{a} + a + \acute{a} + a + \acute{a} \\
\acute{a} &= a + \acute{a} + a + \acute{a} + a \\
b &= \acute{b} + b + \acute{b} + b + \acute{b} \\
\acute{b} &= b + \acute{b} + b + \acute{b} + b \\
c &= \acute{c} + c + \acute{c} + c + \acute{c} \\
\acute{c} &= c + \acute{c} + c + \acute{c} + c.
\end{aligned}$$

Now

$$\begin{aligned}
a + b + c = & (\acute{a} + a + \acute{a} + a + \acute{a}) + (\acute{b} + b + \acute{b} + b + \acute{b}) + \\
& (\acute{c} + c + \acute{c} + c + \acute{c}) \\
= & (\acute{a} + \acute{b} + \acute{c}) + (a + b + c) + (\acute{a} + \acute{b} + \acute{c}) + \\
& (a + b + c) + (\acute{a} + \acute{b} + \acute{c}),
\end{aligned}$$

and

$$\begin{aligned}
\acute{a} + \acute{b} + \acute{c} = & (a + \acute{a} + a + \acute{a} + a) + (b + \acute{b} + b + \acute{b} + b) + \\
& (c + \acute{c} + c + \acute{c} + c) \\
= & (a + b + c) + (\acute{a} + \acute{b} + \acute{c}) + (a + b + c) + \\
& (\acute{a} + \acute{b} + \acute{c}) + (a + b + c).
\end{aligned}$$

Similarly, we have

$$abc = \acute{a}\acute{b}\acute{c} + abc + \acute{a}\acute{b}\acute{c} + abc + \acute{a}\acute{b}\acute{c},$$

and

$$\acute{a}\acute{b}\acute{c} = abc + \acute{a}\acute{b}\acute{c} + abc + \acute{a}\acute{b}\acute{c} + abc.$$

So, $\oplus$ and $\odot$ are well-defined.
Next, we will show that T/ρ_A is a ternary algebra.
Let $a\rho_A, b\rho_A, c\rho_A, d\rho_A, e\rho_A \in T/\rho_A$ where $a, b, c, d, e \in T$
We have

$$\begin{gathered}(a\rho_A) \oplus (b\rho_A) \oplus (c\rho_A) = (a+b+c)\rho_A \in T/\rho_A \\ (a\rho_A) \odot (b\rho_A) \odot (c\rho_A) = (abc)\rho_A \in T/\rho_A.\end{gathered}$$

We will show that T/ρ_A satisfies $(i), (ii)$ and (iii) in Definition 2.2.

$$\begin{aligned}&((a\rho_A) \oplus (b\rho_A) \oplus (c\rho_A)) \oplus (d\rho_A) \oplus (e\rho_A) \\ &\qquad = (a+b+c)\rho_A \oplus (d\rho_A) \oplus (e\rho_A) \\ &\qquad = ((a+b+c)+d+e)\rho_A \\ &\qquad = (a+(b+c+d)+e)\rho_A \\ &\qquad = (a\rho_A) \oplus ((b+c+d)\rho_A) \oplus (e\rho_A) \\ &\qquad = (a\rho_A) \oplus ((b\rho_A) \oplus (c\rho_A) \oplus (d\rho_A)) \oplus (e\rho_A),\end{aligned}$$

$$\begin{aligned}&(a\rho_A) \oplus ((b\rho_A) \oplus (c\rho_A) \oplus (d\rho_A)) \oplus (e\rho_A) \\ &\qquad = (a\rho_A) \oplus ((b+c+d)\rho_A) \oplus (e\rho_A) \\ &\qquad = (a+(b+c+d)+e)\rho_A \\ &\qquad = (a+b+(c+d+e))\rho_A \\ &\qquad = (a\rho_A) \oplus (b\rho_A) \oplus ((c+d+e)\rho_A) \\ &\qquad = (a\rho_A) \oplus (b\rho_A) \oplus ((c\rho_A) \oplus (d\rho_A) \oplus (e\rho_A)).\end{aligned}$$

So,

$$\begin{aligned}&((a\rho_A) \oplus (b\rho_A) \oplus (c\rho_A)) \oplus (d\rho_A) \oplus (e\rho_A) \\ &\qquad = (a\rho_A) \oplus ((b\rho_A) \oplus (c\rho_A) \oplus (d\rho_A)) \oplus (e\rho_A) \\ &\qquad = (a\rho_A) \oplus (b\rho_A) \oplus ((c\rho_A) \oplus (d\rho_A) \oplus (e\rho_A)).\end{aligned}$$

Next,

$$\begin{aligned}
&((a\rho_A) \odot (b\rho_A) \odot (c\rho_A)) \odot (d\rho_A) \odot (e\rho_A)\\
&\qquad = (abc)\rho_A \odot (d\rho_A) \odot (e\rho_A)\\
&\qquad = ((abc)de)\rho_A\\
&\qquad = (a(bcd)e)\rho_A\\
&\qquad = (a\rho_A) \odot ((bcd)\rho_A) \odot (e\rho_A)\\
&\qquad = (a\rho_A) \odot ((b\rho_A) \odot (c\rho_A) \odot (d\rho_A)) \odot (e\rho_A)
\end{aligned}$$

$$\begin{aligned}
&(a\rho_A) \odot ((b\rho_A) \odot (c\rho_A) \odot (d\rho_A)) \odot (e\rho_A)\\
&\qquad = (a\rho_A) \odot ((bcd)\rho_A) \odot (e\rho_A)\\
&\qquad = (a(bcd)e)\rho_A\\
&\qquad = (ab(cde))\rho_A\\
&\qquad = (a\rho_A) \odot (b\rho_A) \odot ((cde)\rho_A)\\
&\qquad = (a\rho_A) \odot (b\rho_A) \odot ((c\rho_A) \odot (d\rho_A) \odot (e\rho_A)).
\end{aligned}$$

So,

$$\begin{aligned}
&((a\rho_A) \odot (b\rho_A) \odot (c\rho_A)) \odot (d\rho_A) \odot (e\rho_A)\\
&\qquad = (a\rho_A) \odot ((b\rho_A) \odot (c\rho_A) \odot (d\rho_A)) \odot (e\rho_A)\\
&\qquad = (a\rho_A) \odot (b\rho_A) \odot ((c\rho_A) \odot (d\rho_A) \odot (e\rho_A)).
\end{aligned}$$

Finally,

$$\begin{aligned}
&(a\rho_A)\odot(b\rho_A) \odot ((c\rho_A) \oplus (d\rho_A) \oplus (e\rho_A))\\
&\qquad = (a\rho_A) \odot (b\rho_A) \odot ((c + d + e)\rho_A)\\
&\qquad = (ab(c + d + e))\rho_A\\
&\qquad = (abc + abd + abe)\rho_A\\
&\qquad = (abc)\rho_A \oplus (abd)\rho_A \oplus (abe)\rho_A\\
&\qquad = ((a\rho_A) \odot (b\rho_A) \odot (c\rho_A)) \oplus ((a\rho_A) \odot (b\rho_A) \odot (d\rho_A))\\
&\qquad\quad \oplus ((a\rho_A) \odot (b\rho_A) \odot (e\rho_A)),
\end{aligned}$$

$$\begin{aligned}(a\rho_A)\odot&((b\rho_A)\oplus(c\rho_A)\oplus(d\rho_A))\odot(e\rho_A)\\&=(a\rho_A)\odot((b+c+d)\rho_A)\odot(e\rho_A)\\&=(a(b+c+d)e)\rho_A\\&=(abe+ace+ade)\rho_A\\&=(abe)\rho_A\oplus(ace)\rho_A\oplus(ade)\rho_A\\&=((a\rho_A)\odot(b\rho_A)\odot(e\rho_A))\oplus((a\rho_A)\odot(c\rho_A)\odot(e\rho_A))\\&\quad\oplus((a\rho_A)\odot(d\rho_A)\odot(e\rho_A)),\end{aligned}$$

and

$$\begin{aligned}((a\rho_A)\oplus&(b\rho_A)\oplus(c\rho_A))\odot(d\rho_A)\odot(e\rho_A)\\&=((a+b+c)\rho_A)\odot(d\rho_A)\odot(e\rho_A)\\&=((a+b+c)de)\rho_A\\&=(ade+bde+cde)\rho_A\\&=(ade\rho_A\oplus(bde)\rho_A\oplus(cde)\rho_A\\&=((a\rho_A)\odot(d\rho_A)\odot(e\rho_A))\oplus((b\rho_A)\odot(d\rho_A)\odot(e\rho_A))\\&\quad\oplus((c\rho_A)\odot(d\rho_A)\odot(e\rho_A)).\end{aligned}$$

Therefore T/ρ_A is a ternary algebra.

□

Definition 3.11. *Let T be a ternary algebra and let $a \in T$. Define*

$$\overline{M}(a) = \{b \in T \mid a = xaxax \text{ and } x = axaxa\ \}.$$

Theorem 3.12. *Let T be a ternary algebra and $a \in T$. Then*

1. *if T is multiplicative idempotent, then $a \in \overline{M}(a)$,*
2. *$b \in \overline{M}(a)$ if and only if $a \in \overline{M}(b)$ for all $b \in T$,*
3. *if T is commutative ternary algebra, then $\overline{M}(x) \subseteq \overline{M}(a)$ for all $x \in \overline{M}(a)$.*

Proof. Similar to the proof of Theorem 3.6 □

Definition 3.13. *Let T be a ternary algebra. The relation ρ_M is defined as follow:*

$$a\rho_M b \Leftrightarrow a \in \overline{M}(a)$$

for all $a, b \in T$.

Theorem 3.14. *Let T be an multiplicative idempotent abelian ternary algebra. Then ρ_M is an equivalence relation on T.*

Proof. Similar to the proof of Theorem 3.8 □

Definition 3.15. *An equivalence relation ρ_M on a ternary algebra T is called anti-multiplicative regular equivalence relation on T. We define the equivalence class of $a \in T$ to be the set*

$$a\rho_M = \{x \in T | x\rho_M a\}.$$

The quotient set T/ρ_M is the set of all equivalence classes of T with respect to the relation ρ_M.
That is

$$T/\rho_M = \{a\rho_M | a \in T\}.$$

Theorem 3.16. *Let T be an multiplicative idempotent commutative ternary algebra and ρ_M be an anti-multiplicative regular equivalence relation on T. Then T/ρ_M is a ternary algebra under the addition and multiplication as follows:*

$$(a\rho_M) \oplus (b\rho_M) \oplus (c\rho_M) = (a + b + c)\rho_M$$
$$(a\rho_M) \odot (b\rho_M) \odot (c\rho_M) = (abc)\rho_M$$

for all $a, b, c \in T$.

Proof. Similar to the proof of Theorem 3.10. □

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