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Some commutativity conditions for Γ-Generalized Boolean Semiring

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Abstract : In this paper, we investigate some properties of commutative Γ-ideals in Γ-generalized Boolean semiring. Also, we obtain commutativity of Γ-generalized Boolean semiring satisfying some suitable conditions. Furthermore, we investigate the commutativity of prime Γ-generalized Boolean semiring R possessing a non-identity commuting automorphism and satisfying right distribution: $(b+c)\alpha a = b\alpha a + c\alpha a$ for all $a, b, c \in R$ and $\alpha \in \Gamma$.

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1 Introduction

In 1934, H.S. Vandiver [8] gave the first formal definition of a semiring and developed the theory of a special class of semirings. A semiring R is given as an algebraic structure $(R, +, \cdot)$ such that $(R, +)$ and $(R, \cdot)$ are both semigroups satisfying $a(b+c) = ab + ac$ and $(b+c)a = ba + ca$ for all $a, b, c \in R$. M.K. Rao [7] introduced Γ-semiring as a generalization of semiring. The ideals, prime ideals, semiprime ideals, quasi-ideals, k-ideals and h-ideals of a Γ-semiring were studied

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and developed by S.K. Kyuno [3], T.K. Dutta and S.K. Sardar [2], R. Chinram [1] and S. Pianskool and P. Khachorncharoenkul [6].

In 2016, T. Makkala and U. Leerawat [4] established a Γ-generalized Boolean semiring (or simply Γ-GB-semiring), which was a generalization of Boolean semiring. Later, they introduced the notion of Γ-(f,g) derivations and Γ-(f,g) generalized derivations on Γ-generalized Boolean semirings, and investigated some related properties. Furthermore, they also investigated some commutativity results for Γ-generalized Boolean semiring involving Γ-(f,g) derivation and Γ-(f,g) generalized derivation (see [5]).

In this paper, we investigate some properties of commutative Γ-ideals in Γ-generalized Boolean semiring. Moreover, we study the commutativity of prime Γ-generalized Boolean semiring containing commutative Γ-ideal. Addtionally, we investigate the commutativity of prime Γ-generalized Boolean semiring R possessing a non-identity commuting automorphism and satisfying the right distribution: $(b+c)\alpha a = b\alpha a + c\alpha a$ for all a, b, c in R and $\alpha \in \Gamma$.

2 Preliminaries

We first recall some definitions, examples and lemmas (from T. Makkala and U. Leerawat [4], [5]) use in proving our main results.

Definition 2.1. *A Γ-generalized Boolean semiring (or simply Γ-GB-semiring) is a triple $(R,+,\Gamma)$, where*

(1) $(R,+)$ is an abelian group.

(2) Γ is a nonempty finite set of binary operations satisfying the following properties:

(i) $a\alpha b \in R$ for all $a, b \in R$ and $\alpha \in \Gamma$,

(ii) $a\alpha(b+c) = a\alpha b + a\alpha c$ for all $a, b, c \in R$ and $\alpha \in \Gamma$,

(iii) $a\alpha(b\beta c) = (a\alpha b)\beta c = (b\alpha a)\beta c$ for all $a, b, c \in R$ and $\alpha, \beta \in \Gamma$,

(iv) $a\alpha(b\beta c) = a\beta(b\alpha c)$ for all $a, b, c \in R$ and $\alpha, \beta \in \Gamma$.

A nonempty subset I of R is said to be a Γ-ideal of R if

(1) $(I,+)$ is a subgroup of $(R,+)$,

(2) $r\alpha a \in I$ for all $r \in R$, $a \in I$, and $\alpha \in \Gamma$, (i.e. $R\Gamma I \subseteq I$),

(3) $(r+a)\alpha s - r\alpha s \in I$ for all $r, s \in R$, $a \in I$, and $\alpha \in \Gamma$.

Example 2.2. *Let $R = \{0, a, b, c\}$ and let the addition $+$ and the multiplication $\cdot$ be defined as follows:*

$+$	0	a	b	c
0	0	a	b	c
a	a	b	c	0
b	b	c	0	a
c	c	0	a	b

$\cdot$	0	a	b	c
0	0	a	b	c
a	0	a	b	c
b	0	a	b	c
c	0	a	b	c

Then $(R,+,\Gamma)$ *is a* Γ*-GB-semiring where* Γ *contains only the multiplication* $(\cdot)$*. If* Γ *is a singleton, then we denote* $(R,+,\Gamma)$ *by* $(R,+,\cdot)$*.*

Example 2.3. *Let* $M_3(\mathbb{Z}) = \left\{ \begin{bmatrix} 0 & a & b \\ 0 & 0 & 0 \\ 0 & 0 & b \end{bmatrix} \middle| a, b \in \mathbb{Z} \right\}$*. Let* $+$ *be a usual matrix addition and* $\Gamma = \{\odot, \otimes\}$ *where* $\odot$ *is a Hadamard product (elementwise product) and* $\otimes$ *is a usual matrix multiplication. Then* $(M_3(\mathbb{Z}),+,\Gamma)$ *is a* Γ*-GB-semiring. Let* $I = \left\{ \begin{bmatrix} 0 & a & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \middle| a \in \mathbb{Z} \right\}$*. It is not hard to verify that* I *is a* Γ*-ideal of* $M_3(\mathbb{Z})$*.*

Definition 2.4. *A* Γ*-GB-semiring* R *is said to be zero-symmetric if* $0\alpha a = 0$ *for all* $\alpha \in \Gamma$ *and* $a \in R$*.*

Notice that if R is a zero-symmetric Γ-GB-semiring and I is a Γ-ideal of R, then we obtain that

$$R\Gamma I \subseteq I \quad \text{and} \quad I\Gamma R \subseteq I.$$

Example 2.5. *Let* $(R,+,\cdot)$ *be a zero-symmetric* Γ*-GB-semiring,* $M_2(R) = \left\{ \begin{bmatrix} 0 & a \\ b & c \end{bmatrix} \middle| a, b \in R \right\}$*, and* $\Gamma = \{\odot, \otimes, *, \cdot\}$ *a set of binary operations which are defined as follows: for any* $\begin{bmatrix} 0 & a \\ b & c \end{bmatrix}, \begin{bmatrix} 0 & x \\ y & z \end{bmatrix} \in M_2(R)$*,*

$$\begin{bmatrix} 0 & a \\ b & c \end{bmatrix} \odot \begin{bmatrix} 0 & x \\ y & z \end{bmatrix} = \begin{bmatrix} 0 & a\cdot x \\ b\cdot y & c\cdot z \end{bmatrix},$$

$$\begin{bmatrix} 0 & a \\ b & c \end{bmatrix} \otimes \begin{bmatrix} 0 & x \\ y & z \end{bmatrix} = \begin{bmatrix} 0 & a\cdot x \\ 0 & 0 \end{bmatrix},$$

$$\begin{bmatrix} 0 & a \\ b & c \end{bmatrix} * \begin{bmatrix} 0 & x \\ y & z \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ b\cdot y & 0 \end{bmatrix},$$

$$\begin{bmatrix} 0 & a \\ b & c \end{bmatrix} \cdot \begin{bmatrix} 0 & x \\ y & z \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & c\cdot z \end{bmatrix}.$$

Then $(M_2(R),+,\Gamma)$ *is a zero-symmetric* Γ*-GB-semiring.*

Definition 2.6. *Let* R *be a* Γ*-GB-semiring. Then*

1. *R is prime if $x\Gamma R\Gamma y = \{0\}$ for $x, y \in R$, then $x = 0$ or $y = 0$,*
2. *R is commutative if $a\alpha b = b\alpha a$ for all $a, b \in R$ and for all $\alpha \in \Gamma$.*

For examples, $(R,+,\cdot)$ as mentioned in Example 2.2. is prime but it is not commutative, and $(M_3(\mathbb{Z}),+,\Gamma)$ as mentioned in Example 2.3. is commutative but it is not prime since

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \Gamma M_3(\mathbb{Z}) \Gamma \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \left\{ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \right\}.$$

Here, we will indicate some basic properties on Γ-GB-semiring appearing in [4]. For any $a, b, c \in R$ and $\alpha \in \Gamma$, we have

(i) $-(-a) = a$,

(ii) $a\alpha 0 = 0$,

(iii) $a\alpha(-b) = -(a\alpha b)$,

(iv) $a\alpha(b - c) = (a\alpha b) - (a\alpha c)$,

(v) $-(a + b) = -a - b$,

(vi) $-(a - b) = -a + b$,

(vii) $-(a\alpha(b + c)) = -(a\alpha b) - (a\alpha c)$,

(viii) $-(a\alpha(b - c)) = -(a\alpha b) + (a\alpha c)$.

For any $x, y \in R$ and $\alpha \in \Gamma$, the symbol $[x, y]_\alpha$ will represent the commutator $x\alpha y - y\alpha x$ and the symbol $(x \circ y)_\alpha$ stands for the skew-commutator $x\alpha y + y\alpha x$.

Next, the following are some basic properties of commutator and skew-commutator. The proofs of these properties are straightforward and hence omitted.

(i) $[x\alpha y, z]_\beta = x\alpha[y, z]_\beta = y\alpha[x, z]_\beta + z\alpha[y, x]_\beta$,

(ii) $[x, y\alpha z]_\beta = y\alpha[x, z]_\beta = z\alpha[x, y]_\beta + x\alpha[y, z]_\beta$,

(iii) $(x \circ y\alpha z)_\beta = y\alpha(x \circ z)_\beta = z\alpha(x \circ y)_\beta + x\alpha[y, z]_\beta$,

(iv) $(x\alpha y \circ z)_\beta = x\alpha(y \circ z)_\beta = y\alpha(x \circ z)_\beta + z\alpha[x, y]_\beta$.

The center of R, written $Z(R)$, is defined to be the set

$$Z(R) = \{a \in R \mid a\alpha b = b\alpha a \text{ for all } b \in R, \text{ and } \alpha \in \Gamma\}.$$

Lemma 2.7. *Let R be a Γ-GB-semiring. If $x \in Z(R)$ then $y\alpha x \in Z(R)$ and $x\alpha y \in Z(R)$ for all $y \in R$ and $\alpha \in \Gamma$.*

3 Main Results

Theorem 3.1. *Let R be a prime Γ-GB-semiring. Suppose that $Z(R)$ is a nonempty subset of R and $Z(R) \neq \{0\}$. Then R is commutative.*

Proof Let $x, y, t \in R$ and $\alpha, \beta, \gamma \in \Gamma$. Since $Z(R) \neq \emptyset$ and $Z(R) \neq \{0\}$, there exists an element z in $Z(R)$ such that $z \neq 0$. By Lemma 2.7, $z\beta(t\gamma x) \in Z(R)$. Hence

$$z\beta t\gamma[x, y]_\alpha = [z\beta t\gamma x, y]_\alpha = 0.$$

This implies $z\gamma R\Gamma[x, y]_\alpha = \{0\}$. But $z \neq 0$. Thus, $[x, y]_\alpha$ must be zero. Therefore, R is commutative. □

Lemma 3.2. *Let R be a prime and zero-symmetric Γ-GB-semiring. If I is a commutative Γ-ideal of R, then $I \subseteq Z(R)$.*

Proof Let $a \in I$. We caim that $[a, b]_\alpha = 0$ for all $b \in R$, and $\alpha \in \Gamma$. If $a = 0$, then $[a, b]_\alpha = 0$ for all $\alpha \in \Gamma$. Assume that $a \neq 0$. Let $b \in R$, and $\alpha \in \Gamma$. Then

$$a\alpha b = (0 + a)\alpha b - 0\alpha b \in I.$$

Since I is commutative, $[a, a\alpha b]_\beta = 0$ for all $\beta \in \Gamma$.
That is,

$$a\alpha[a, b]_\beta = [a, a\alpha b]_\beta = 0,$$

for all $\alpha, \beta \in \Gamma$.

Now, we will show that $a\Gamma R\Gamma[a, b]_\alpha = \{0\}$ for all $\alpha \in \Gamma$. Clearly, $\{0\} \subseteq a\Gamma R\Gamma[a, b]_\alpha$. Conversely, let $c \in R$ and $\beta, \gamma \in \Gamma$. Then

$$a\beta c\gamma[a, b]_\alpha = a\beta[a, c\gamma b]_\alpha = 0.$$

Hence, $a\Gamma R\Gamma[a, b]_\alpha = \{0\}$. Since R is prime, and $a \neq 0$, we have $[a, b]_\alpha = 0$. It follows that $a \in Z(R)$. Therefore, $I \subseteq Z(R)$. □

Lemma 3.3. *Let R be a prime Γ-GB-semiring. If $I \neq \{0\}$ is a Γ-ideal of R such that $I \subseteq Z(R)$, then R is commutative.*

Proof Suppose that $I \neq \{0\}$ is a Γ-ideal of R with $I \subseteq Z(R)$. Let $x, y \in R$ and $\alpha \in \Gamma$. Since $I \neq \{0\}$, there exists an element a in I such that $a \neq 0$. We will show that $a\Gamma R\Gamma[x, y]_\alpha = \{0\}$. Let $r \in R$ and $\gamma, \beta \in \Gamma$. Then

$$\begin{aligned} a\beta r\gamma[x, y]_\alpha &= r\beta a\gamma[x, y]_\alpha \\ &= r\beta[a\gamma x, y]_\alpha = r\beta[x\gamma a, y]_\alpha = 0. \end{aligned}$$

That is, $a\Gamma R\Gamma[x, y]_\alpha \subseteq \{0\}$. Hence, $a\Gamma R\Gamma[x, y]_\alpha = \{0\}$. Since R is prime and $a \neq 0$, we have $[x, y]_\alpha = 0$. Therefore, R is commutative. □

The proof of the following theorem follows from Lemma 3.2. and Lemma 3.3.

Theorem 3.4. *Let R be a prime and zero-symmetric Γ-GB-semiring. If there exists a nonzero commutative Γ-ideal of R, then R is also commutative.*

Lemma 3.5. *Let R be a prime and zero-symmetric Γ-GB-semiring. Let $I \neq \{0\}$ be a Γ-ideal of R. If $(x \circ y)_\alpha = 0$ for all $x, y \in I$, and $\alpha \in \Gamma$, then I is commutative.*

Proof Let $x, y \in I, t \in R$, and $\alpha, \beta, \gamma \in \Gamma$. There exists an element $z \neq 0$ in I. Then

$$\begin{aligned} z\alpha t\gamma[x, y]_\beta &= y\gamma((z\alpha t) \circ x)_\beta + z\alpha t\gamma[x, y]_\beta \\ &= ((z\alpha t) \circ (x\gamma y))_\beta = 0. \end{aligned}$$

It follows that $z\Gamma R\Gamma[x, y]_\beta = \{0\}$ for all $\beta \in \Gamma$. Since R is prime and $z \neq 0$, we obtain $[x, y]_\beta = 0$. This yields that I is commutative. □

Corollary 3.6. *Let R be a prime and zero-symmetric Γ-GB-semiring. Let $I \neq \{0\}$ be a Γ-ideal of R. If $(x \circ y)_\alpha = 0$ for all $x, y \in I$, and $\alpha \in \Gamma$, then R is commutative.*

Theorem 3.7. *Let R be a prime and zero-symmetric Γ-GB-semiring. Let $I \neq \{0\}$ be a Γ-ideal of R. If, for all $x, y \in I$ and $\alpha \in \Gamma$, either $[x, y]_\alpha \in Z(R)$ or $(x \circ y)_\alpha \in Z(R)$, then I is commutative.*

Proof First, suppose that $[x, y]_\alpha \in Z(R)$ for all $x, y \in I$, and $\alpha \in \Gamma$. Let $x, y \in I, t \in R$, and $\alpha, \beta, \gamma \in \Gamma$. Then

$$\begin{aligned} [x,y]_\alpha\beta[x,y]_\alpha &= [x,y]_\alpha\beta(x\alpha y - y\alpha x) \\ &= [x,y]_\alpha\beta(x\alpha y) - [x,y]_\alpha\beta(y\alpha x) \\ &= [x,y]_\alpha\beta(x\alpha y) - (y\alpha x)\beta[x,y]_\alpha \\ &= [x,y]_\alpha\beta(x\alpha y) - (x\alpha y)\beta[x,y]_\alpha = 0. \end{aligned}$$

This implies

$$[x,y]_\alpha\gamma t\beta[x,y]_\alpha = t\gamma[x,y]_\alpha\beta[x,y]_\alpha = t\gamma 0 = 0.$$

That is, $[x, y]_\alpha \Gamma R \Gamma [x, y]_\alpha = \{0\}$. Since R is prime, we obtain $[x, y]_\alpha = 0$ for all $\alpha \in \Gamma$. This yields that I is commutative.

Now, we suppose that $(x \circ y)_\alpha \in Z(R)$ for all $x, y \in I$, and $\alpha \in \Gamma$. Let $x, y \in I, t \in R$, and $\alpha, \beta, \gamma \in \Gamma$. Then

$$\begin{aligned} (x\circ y)_\alpha\beta[x,y]_\alpha &= [x,y]_\alpha\beta(x\alpha y - y\alpha x) \\ &= (x\circ y)_\alpha\beta(x\alpha y) - (x\circ y)_\alpha\beta(y\alpha x) \\ &= (x\circ y)_\beta(x\alpha y) - (y\alpha x)\beta(x\circ y)_\alpha \\ &= (x\circ y)_\alpha\beta(x\alpha y) - (x\alpha y)\beta(x\circ y)_\alpha = 0. \end{aligned}$$

By the same argument as above, we can show that $(x \circ y)_\alpha \Gamma R \Gamma [x, y]_\alpha = \{0\}$. But R is prime. That means either $(x \circ y)_\alpha = 0$ or $[x, y]_\alpha = 0$. If $(x \circ y)_\alpha = 0$ for all $\alpha \in \Gamma$, by Lemma 3.5., we have I is commutative. If $[x \circ y]_\alpha = 0$ for all $\alpha \in \Gamma$, as above $[x \circ y]_\alpha \in Z(R)$, we obtain the result. □

Corollary 3.8. *Let R be a prime and zero-symmetric Γ-GB-semiring. Let $I \neq \{0\}$ be a Γ-ideal of R. If either $[x, y]_\alpha \in Z(R)$ or $(x \circ y)_\alpha \in Z(R)$ for all $x, y \in I$, and $\alpha \in \Gamma$, then R is commutative.*

Definition 3.9. *Let R and R' be Γ-GB semirings. A homomorphism of R into R' is a function from $f : R \to R'$ if*

1. *$f(a + b) = f(a) + f(b)$, and*
2. *$f(a\alpha b) = f(a)\alpha f(b)$,*

for all $a, b \in R$ and $\alpha \in \Gamma$.

A homomorphism from R on itself is called an endomorphism of R, and if it is bijective then it is called an automorphism.

An automorphism f of R is called a commuting automorphism if $[f(x), x]_\alpha = 0$ for all $x \in R$ and $\alpha \in \Gamma$.

Theorem 3.10. *Let R be a prime and zero-symmetric Γ-GB-semiring. Let $I \neq \{0\}$ be a Γ-ideal of R. If f is an automorphism on R satisfying either $I\Gamma f[x,y]_\alpha = \{0\}$ or $I\Gamma f(x \circ y)_\alpha = \{0\}$ for all $x, y \in I$, and $\alpha \in \Gamma$, then R is commutative.*

Proof Let $r \in R, x, y \in I$, and $\alpha \in \Gamma$ such that

$$I\Gamma f[x,y]_\alpha = \{0\} \quad \text{or} \quad I\Gamma f(x \circ y)_\alpha = \{0\}.$$

Case I: Suppose $I\Gamma f[x,y]_\alpha = \{0\}$. Since $I \neq \{0\}$, there exists a nonzero element z in I. Hence,

$$z\Gamma R\Gamma f[x,y]_\alpha \subseteq I\Gamma f[x,y]_\alpha = \{0\},$$

and then,

$$z\Gamma R\Gamma f[x,y]_\alpha = \{0\}.$$

Since R is prime and $z \neq 0$, we obtain $f[x,y]_\alpha = 0$. Since f is injective, $[x,y]_\alpha = 0$. It follows that I is commutative and by Theorem 3.4., R is also commutative.

Case II: Suppose $I\Gamma f(x \circ y)_\alpha = \{0\}$. By the same argument in the proof of the case I, we have $(x \circ y)_\alpha = 0$ for all $x, y \in I$ and $\alpha \in \Gamma$. By Corollary 3.6., R is commutative. □

Theorem 3.11. *Let R be a prime Γ-GB-semiring satisfying $(b+c)\alpha a = b\alpha a + c\alpha a$ for all $a, b, c \in R$ and $\alpha \in \Gamma$. If f is a non-identity commuting automorphism, then R is commutative.*

Proof Since f is a non-identity automorphism on R, there exists an element a in R such that $f(a) \neq a$. Since $[f(x), x]_\alpha = 0$ for all $x \in R$ and $\alpha \in \Gamma$, we have

$$[f(x), y]_\alpha = [x, f(y)]_\alpha$$

for all $x, y \in R$ and $\alpha \in \Gamma$. Let $x, y, t \in R$ and $\alpha, \beta, \gamma \in \Gamma$. Since

$$\begin{aligned} [t\beta x, f(a\gamma z)]_\alpha &= [f(t\beta x), a\gamma z]_\alpha \\ f(a)\gamma[t\beta x, f(z)]_\alpha &= a\gamma[f(t\beta x), z]_\alpha = a\gamma[t\beta x, f(z)]_\alpha, \end{aligned}$$

and so,

$$\begin{aligned} (f(a) - a)\gamma[t\beta x, y]_\alpha &= 0 \\ (f(a) - a)\gamma t\beta[x, y]_\alpha &= 0. \end{aligned}$$

It follows that

$$(f(a) - a)\Gamma R\Gamma[x, y]_\alpha = \{0\}.$$

for all $x, y \in R$ and $\alpha \in \Gamma$. Since R is prime and $f(a) - a \neq 0$, $[x, y]_\alpha = 0$ for all $x, y \in R$ and $\alpha \in \Gamma$. Therefore, R is commutative. □

Theorem 3.12. *Let R be a prime Γ-GB-semiring. Let f be an endomorphism of R. If $f(R) \subseteq Z(R)$, then R is commutative or f vanishes.*

Proof Suppose that f does not vanish and $f(R) \subseteq Z(R)$. There exists an element z in R such that $0 \neq f(z) \subseteq Z(R)$. Let $x, y \in R$, and $\alpha, \beta \in \Gamma$. Then

$$\begin{aligned} f(z)\beta[x,y]_\alpha &= [f(z)\beta x, y]_\alpha = [x\beta f(z), y]_\alpha \\ &= x\beta[f(z).y]_\alpha = 0. \end{aligned}$$

Hence, $f(z)\beta[x,y]_\alpha = 0$ for all $x, y \in R$, and $\alpha, \beta \in \Gamma$. This gives

$$f(z)\beta t\gamma[x,y]_\alpha = 0$$

for all $x, y, t \in R$ and $\alpha, \beta, \gamma \in \Gamma$. Since R is prime and $f(z) \neq 0$, we have $[x,y]_\alpha = 0$ for all $x, y \in R$, and $\alpha \in \Gamma$. Therefore, R is commutative. ☐

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