Thai Journal of Mathematics
Volume 5(2007) Number 2 : 183–190

www.math.science.cmu.ac.th/thaijournal

Hybrid Iteration Method for Fixed Points of Nonexpansive Mappings

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Abstract : In this paper, a hybrid iteration method is studied and the strong convergence of the iteration scheme to a fixed point of nonexpansive mapping is obtained in Hilbert spaces.

Keywords : Nonexpansive mapping; Fixed point; Hilbert space; Hybrid iteration method.
2002 Mathematics Subject Classification : 47H09, 47J25

1. Introduction

Let H be a Hilbert space with inner product $\langle \cdot, \cdot \rangle$ and norm $\| \cdot \|$. A mapping $T : H \to H$ is said to be nonexpansive if $\|Tx - Ty\| \leq \|x - y\|$ for any $x, y \in H$. A mapping $F : H \to H$ is said to be $\eta-$strongly monotone if there exists constant $\eta > 0$ such that $\langle Fx - Fy, x - y \rangle \geq \eta \|x - y\|^2$ for any $x, y \in H$. $F : H \to H$ is said to be $k-$Lipschitzian if there exists constant $k > 0$ such that $\|Fx - Fy\| \leq k\|x - y\|$ for any $x, y \in H$.

The interest and importance of construction of fixed points of nonexpansive mappings stem mainly from the fact that it may be applied in many areas, such as imagine recovery and signal processing (see, e.g., [1,3,11]). Iterative techniques for approximating fixed points of nonexpansive mappings have been studied by various authors (see, e.g.,[1,5-10,12-23, etc.]), using famous Mann iteration method, Ishikawa iteration method and Halpern iteration method [5]. But Genel and Lindenstrass [4] proved that Mann iteration sequence just converges weakly to a fixed point of a nonexpansive mapping, even in Hilbert space. Since then, some authors introduced some iteration method, such as viscosity approximation method [8,14,21], CQ method [10], to approximate fixed points of nonexpansive mappings by modifying Mann iteration method.

Let K be a closed convex subset of a Hilbert space H, and T be a nonexpansive mapping on K. For any given $u \in K$, $x_0 \in K$, Halpern [5] introduced the following iteration method which is now called Halpern iteration:

$$x_{n+1} = \alpha_n u + (1 - \alpha_n) T x_n \quad n \geq 0, \tag{1.1}$$

where $\{\alpha_n\}$ is a sequence in [0,1], and also proved that the sequence $\{x_n\}$ converges weakly to a fixed point of T. After that, Lions [9], Wittmann [18] and Xu [20] extended Halpern's result, respectively.

For approximating the fixed points of nonexpansive mappings, Zeng and Yao [23] introduced the following implicit hybrid iteration method.

For an arbitrary given $x_0 \in H$, the sequence $\{x_n\}$ is generated as follows:

$$x_n = \alpha_n x_{n-1} + (1-\alpha_n)[T_n x_n - \lambda_n \mu F(T_n x_n)], \quad n \geq 1, \tag{1.2}$$

where $T_n = T_{nmodN}$, $\{\alpha_n\}$ is a sequence in (0,1). By using the iteration scheme (1.2), Zeng and Yao obtained the following results.

Theorem 1.1.[23] Let H be a real Hilbert space and let $F : H \to H$ be a mapping such that for some constant $k, \eta > 0$, F is $k - Lipschitzian$ and $\eta - strongly$ montone. Let $\{T_i\}_{i=1}^N$ be N nonexpansive self-maps of H such that $C = \bigcap_{n=1}^N F(T_i) \neq \phi$. Let $\mu \in (0, 2\eta/k^2)$, $\{\lambda_n\} \subset [0,1)$ and $\{\alpha_n\} \subset (0,1)$ satisfying the conditions: $\sum_{n=1}^{\infty} \lambda_n < \infty$ and $\alpha \leq \alpha_n \leq \beta$, $n \geq 1$, for some $\alpha, \beta \in (0,1)$. Then the sequence $\{x_n\}$ generated by (1.2) converges weakly to a common fixed point of the mappings $\{T_i\}_{i=1}^N$.

Theorem 1.2.[23] Let H be a real Hilbert space and let $F : H \to H$ be a mapping such that for some constant $k, \eta > 0$, F is $k - Lipschitzian$ and $\eta - strongly$ montone. Let $\{T_i\}_{i=1}^N$ be N nonexpansive self-maps of H such that $C = \bigcap_{n=1}^N F(T_i) \neq \phi$. Let $\mu \in (0, 2\eta/k^2)$, $\{\lambda_n\} \subset [0,1)$ and $\{\alpha_n\} \subset (0,1)$ satisfying the conditions: $\sum_{n=1}^{\infty} \lambda_n < \infty$ and $\alpha \leq \alpha_n \leq \beta$, $n \geq 1$, for some $\alpha, \beta \in (0,1)$. Then the sequence $\{x_n\}$ generated by (1.2) converges strongly to a common fixed point of the mappings $\{T_i\}_{i=1}^N$ if and only if $\liminf_{n\to\infty} d(x_n, \bigcap_{n=1}^N F(T_i)) = 0$.

Very recently, motivated by above work and earlier results of Yamada [24], Wang [19] introduced an explicit hybrid iteration method for nonexpansive mappings and obtained the following convergence theorem.

Theorem 1.3.[19] Let H be a Hilbert space, $T : H \to H$ be a nonexpansive mapping with $F(T) \neq \phi$, and $F : H \to H$ be a $\eta - strongly$ monotone and $k - Lipschizian$ mapping. For any given $x_0 \in H$, $\{x_n\}$ is defined by

$$x_{n+1} = \alpha_n x_n + (1-\alpha_n) T^{\lambda_{n+1}} x_n, \quad n \geq 0 \tag{1.3}$$

where $T^{\lambda_{n+1}} x_n = T x_n - \lambda_{n+1} \mu F(T x_n)$. If $\{\alpha_n\} \subset [0,1)$ and $\{\lambda_n\} \subset [0,1)$ satisfy the following conditions: (1) $\alpha \leq \alpha_n \leq \beta$ for some $\alpha, \beta \in (0,1)$; (2) $\sum_{n=1}^{\infty} \lambda_n < \infty$; (3) $0 < \mu < 2\eta/k^2$, then,

(1) $\{x_n\}$ converges weakly to a fixed point of T.

(2) $\{x_n\}$ converges strongly to a fixed point of T if only if $\liminf_{n\to\infty} d(x_n, F(T)) = 0$.

In this paper, we propose another explicit hybrid iteration method for nonexpansive mapping T in Hilbert space: for arbitrary $u \in H$ and $x_0 \in H$

$$x_{n+1} = \alpha_n u + \beta_n x_n + \gamma_n T^{\lambda_{n+1}} x_n, \quad n \geq 0 \tag{1.4}$$

where $T^{\lambda_{n+1}}x_n = Tx_n - \lambda_{n+1}\mu F(Tx_n)$, $\{\alpha_n\}$, $\{\beta_n\}$, $\{\gamma_n\}$ and $\{\lambda_n\}$ are real sequences in [0,1) and $\alpha_n + \beta_n + \gamma_n = 1$ for all $n \geq 0$. At the same time, we show that $\{x_n\}$ generated by (1.4) converges strongly to a fixed point of T.

Remark. In (1.4), when $\alpha_n = 0$ for all nonnegative integer n, the iteration (1.4) reduces to the iteration (1.3). When $\beta_n = 0$ and $\lambda_n = 0$ for all nonnegative integer n, the iteration (1.4) reduces to Halpern iteration.

2. Preliminaries

We restate the following lemmas which play important roles in our proofs.

Lemma 2.1.[24] Let $T^\lambda x = Tx - \lambda\mu F(Tx)$, where $T : H \to H$ is a nonexpansive mapping from H into itself and F is a $\eta-$ strongly monotone and $k-$Lipschitzian mapping from H into itself. If $0 \leq \lambda < 1$ and $0 < \mu < 2\eta/k^2$, then T^λ is a contraction and satisfies

$$\|T^\lambda x - T^\lambda y\| \leq (1 - \lambda\tau)\|x - y\|, \quad \forall x, y \in H,$$

where $\tau = 1 - \sqrt{1 - \mu(2\eta - \mu k^2)}$.

Lemma 2.2.[15] Let $\{x_n\}$ and $\{y_n\}$ be bounded sequences in a Banach space E and let $\{\beta_n\}$ be a sequence in [0,1] with $0 < \liminf_{n\to\infty} \beta_n \leq \limsup_{n\to\infty} \beta_n < 1$. Suppose $x_{n+1} = (1-\beta_n)y_n + \beta_n x_n$, for all integer $n \geq 0$ and $\limsup_{n\to\infty}(\|y_{n+1} - y_n\| - \|x_{n+1} - x_n\|) \leq 0$. Then, $\lim_{n\to\infty} \|y_n - x_n\| = 0$.

The following Lemma 2.3 is a well-known subdifferential inequality.

Lemma 2.3. Let E be a Banach space, J be duality mapping from E to E^*. Then for any $x, y \in E$, any $j(x+y) \in J(x+y)$, and any $j(x) \in J(x)$, the following inequalities hold:

$$\|x\|^2 + 2\langle y, j(x)\rangle \leq \|x + y\|^2 \leq \|x\|^2 + 2\langle y, j(x + y)\rangle.$$

Lemma 2.4.[20] Let $\{s_n\}$ be a sequence of nonnegative real numbers satisfying

$$s_{n+1} = (1 - \alpha_n)s_n + \alpha_n\beta_n + \gamma_n, \quad n \geq 0,$$

where $\{\alpha_n\}$, $\{\beta_n\}$ and $\{\gamma_n\}$ satisfy the following conditions:

(1) $\{\alpha_n\} \subset [0, 1]$, $\sum_{n=0}^{\infty} \alpha_n = \infty$;

(2) $\limsup_{n\to\infty} \beta_n \leq 0$;

(3) $\gamma_n \geq 0$, $\sum_{n=0}^{\infty} \gamma_n < \infty$.

Then $\lim_{n\to\infty} s_n = 0$.

3. Main Results

Theorem 3.1. Let H be a Hilbert space, $T : H \to H$ be a nonexpansive mappping with $F(T) \neq \phi$, and let $F : H \to H$ be a $k-$Lipschitzian and $\eta-$strongly monotone. Suppose that the sequence $\{x_n\}$ is generated by (1.4), where $T^{\lambda_{n+1}}x_n =$

$Tx_n - \lambda_{n+1}\mu F(Tx_n)$, $\{\alpha_n\}$, $\{\beta_n\}$, $\{\gamma_n\}$ and $\{\lambda_n\}$ are real sequences in [0,1) and satisfy the following conditions:

(1) $\alpha_n + \beta_n + \gamma_n = 1$;

(2) $\lim_{n\to\infty} \alpha_n = 0, \sum_{n=0}^{\infty} \alpha_n = \infty$;

(3) $0 < \liminf_{n\to\infty} \beta_n \leq \limsup_{n\to\infty} \beta_n < 1$;

(4) $\sum_{n=1}^{\infty} \lambda_n < \infty$, $0 < \mu < 2\eta/k^2$;

then the sequence $\{x_n\}$ converges strongly to a fixed point q of T, which solves the following variational inequality

$$\langle u - q, p - q\rangle \leq 0, \quad \forall p \in F(T).$$

Proof. For any $p \in F(T)$, by (1.4), we have

$$\begin{aligned} \|x_{n+1} - p\| &= \|\alpha_n(u-p) + \beta_n(x_n - p) + \gamma_n(T^{\lambda_{n+1}}x_n - p)\| \\ &\leq \alpha_n\|u-p\| + \beta_n\|x_n - p\| + \gamma_n\|T^{\lambda_{n+1}}x_n - p\|, \end{aligned}$$

where (by Lemma 2.1),

$$\begin{aligned} \|T^{\lambda_{n+1}}x_n - p\| &= \|T^{\lambda_{n+1}}x_n - T^{\lambda_{n+1}}p + T^{\lambda_{n+1}}p - p\| \\ &\leq \|T^{\lambda_{n+1}}x_n - T^{\lambda_{n+1}}p\| + \|T^{\lambda_{n+1}}p - p\| \\ &\leq (1 - \lambda_{n+1}\tau)\|x_n - p\| + \lambda_{n+1}\mu\|F(p)\|. \end{aligned}$$

Thus,

$$\begin{aligned} \|x_{n+1} - p\| &\leq \alpha_n\|u-p\| + \beta_n\|x_n - p\| + \gamma_n\|x_n - p\| + \lambda_{n+1}\mu\|F(p)\| \\ &= \alpha_n\|u-p\| + (1-\alpha_n)\|x_n - p\| + \lambda_{n+1}\mu\|F(p)\|. \end{aligned}$$

By induction, for any positive integer n, we have,

$$\|x_n - p\| \leq max\{\|u-p\|, \|x_0 - p\|\} + \mu\|F(p)\|\sum_{n=1}^{\infty}\lambda_n.$$

Since $\sum_{n=1}^{\infty} \lambda_n < \infty$, $\{x_n\}$ is bounded. So are $\{Tx_n\}$, $\{T^{\lambda_{n+1}}x_n\}$ and $\{F(Tx_n)\}$.

Define $x_{n+1} = (1-\beta_n)y_n + \beta_n x_n$, then we have

$$\begin{aligned}
y_{n+1} - y_n &= \frac{x_{n+2} - \beta_{n+1}x_{n+1}}{1-\beta_{n+1}} - \frac{x_{n+1} - \beta_n x_n}{1-\beta_n} \\
&= \frac{\alpha_{n+1}u + \beta_{n+1}x_{n+1} + \gamma_{n+1}T^{\lambda_{n+2}}x_{n+1} - \beta_{n+1}x_{n+1}}{1-\beta_{n+1}} \\
&\quad - \frac{\alpha_n u + \beta_n x_n + \gamma_n T^{\lambda_{n+1}}x_n - \beta_n x_n}{1-\beta_n} \\
&= (\frac{\alpha_{n+1}}{1-\beta_{n+1}} - \frac{\alpha_n}{1-\beta_n})u + \frac{\gamma_{n+1}}{1-\beta_{n+1}}T^{\lambda_{n+2}}x_{n+1} - \frac{\gamma_n}{1-\beta_n}T^{\lambda_{n+1}}x_n \\
&= (\frac{\alpha_{n+1}}{1-\beta_{n+1}} - \frac{\alpha_n}{1-\beta_n})u + \frac{\gamma_{n+1}}{1-\beta_{n+1}}(Tx_{n+1} - \lambda_{n+2}\mu F(Tx_{n+1})) \\
&\quad - \frac{\gamma_n}{1-\beta_n}(Tx_n - \lambda_{n+1}\mu F(Tx_n)) \\
&= (\frac{\alpha_{n+1}}{1-\beta_{n+1}} - \frac{\alpha_n}{1-\beta_n})u + \frac{\gamma_{n+1}}{1-\beta_{n+1}}(Tx_{n+1} - Tx_n) \\
&\quad + (\frac{\gamma_{n+1}}{1-\beta_{n+1}} - \frac{\gamma_n}{1-\beta_n})Tx_n \\
&\quad - \frac{\gamma_{n+1}}{1-\beta_{n+1}}\lambda_{n+2}\mu F(Tx_{n+1}) + \frac{\gamma_n}{1-\beta_n}\lambda_{n+1}\mu F(Tx_n).
\end{aligned}$$

So,

$$\begin{aligned}
&\|y_{n+1} - y_n\| - \|x_{n+1} - x_n\| \\
&\quad \le |\tfrac{\alpha_{n+1}}{1-\beta_{n+1}} - \tfrac{\alpha_n}{1-\beta_n}|(\|u\| + \|Tx_n\|) + \tfrac{\gamma_{n+1}}{1-\beta_{n+1}}\lambda_{n+2}\mu\|F(Tx_{n+1})\| + \tfrac{\gamma_n}{1-\beta_n}\lambda_{n+1}\mu\|F(Tx_n)\|.
\end{aligned}$$

Since $\{T(x_n)\}$ and $\{F(Tx_n)\}$ are bounded, we have

$$\limsup_{n\to\infty}(\|y_{n+1} - y_n\| - \|x_{n+1} - x_n\|) \le 0.$$

Thus it follows from Lemma 2.2 that $\lim_{n\to\infty}\|y_n - x_n\| = 0$. In addition, since $\|x_{n+1} - x_n\| = (1-\beta_n)\|y_n - x_n\|$, we have $\lim_{n\to\infty}\|x_{n+1} - x_n\| = 0$.

It follows from (1.4) that

$$\begin{aligned}
\|x_n - T^{\lambda_{n+1}}x_n\| &\le \|x_n - x_{n+1}\| + \|x_{n+1} - T^{\lambda_{n+1}}x_n\| \\
&\le \|x_n - x_{n+1}\| + \alpha_n\|u - T^{\lambda_{n+1}}x_n\| + \beta_n\|x_n - T^{\lambda_{n+1}}x_n\|,
\end{aligned}$$

which implies that $\lim_{n\to\infty}\|x_n - T^{\lambda_{n+1}}x_n\| = 0$.

On the other hand,

$$\begin{aligned}
\|x_n - Tx_n\| &\le \|x_n - T^{\lambda_{n+1}}x_n\| + \|T^{\lambda_{n+1}}x_n - Tx_n\| \\
&= \|x_n - T^{\lambda_{n+1}}x_n\| + \lambda_{n+1}\mu\|F(Tx_n)\|,
\end{aligned}$$

hence, we have

$$\lim_{n\to\infty}\|x_n - Tx_n\| = 0. \tag{3.2}$$

Let z_t be the unique fixed point of the contraction S_t defined by $S_t x = tu + (1-t)Tx$, $t \in (0,1)$. In [2], Browder proved that z_t converges strongly to a fixed point q of T as $t \to 0$. We now show that $\limsup_{n\to\infty}\langle u - q, x_n - q\rangle \le 0$.

Since $z_t - x_n = t(u - x_n) + (1-t)(Tz_t - x_n)$, then, by Lemma 2.3, we have

$$\begin{aligned} \|z_t - x_n\|^2 &\leq (1-t)^2\|Tz_t - x_n\|^2 + 2t\langle u - x_n, z_t - x_n\rangle \\ &\leq (1-t)^2(\|Tz_t - Tx_n\| + \|Tx_n - x_n\|)^2 + 2t(\|z_t - x_n\|^2 \\ &\quad +\langle u - z_t, z_t - x_n\rangle) \\ &\leq (1+t^2)\|z_t - x_n\|^2 + \|Tx_n - x_n\|(2\|z_t - x_n\| + \|Tx_n - x_n\|) \\ &\quad +2t\langle u - z_t, z_t - x_n\rangle. \end{aligned} \tag{3.3}$$

Since $\{x_n\}$, $\{Tx_n\}$ and z_t are bounded, there exists constant $M > 0$ such that $\|z_t - x_n\| \leq M$ and $2\|z_t - x_n\| + \|Tx_n - x_n\| \leq M$ for all positive integer n and $t \in (0,1)$, respectively. It follows from (3.3) that

$$\langle u - z_t, x_n - z_t\rangle \leq \frac{t}{2}M^2 + \frac{\|Tx_n - x_n\|}{2t}M. \tag{3.4}$$

Taking lim sup as $n \to \infty$ in the inequality (3.4), then it follows from (3.2) that

$$\limsup_{n\to\infty}\langle u - z_t, x_n - z_t\rangle \leq \frac{t}{2}M^2. \tag{3.5}$$

Letting $t \to 0$ in (3.5), we have

$$\limsup_{n\to\infty}\langle u - q, x_n - q\rangle \leq 0. \tag{3.6}$$

We now show that $\{x_n\}$ converges strongly to the fixed point q of T.
By Lemma 2.3, we have

$$\begin{aligned} \|x_{n+1} - q\|^2 &= \|\alpha_n(u - q) + \beta_n(x_n - q) + \gamma_n(T^{\lambda_{n+1}}x_n - q)\|^2 \\ &\leq \|\beta_n(x_n - q) + \gamma_n(T^{\lambda_{n+1}}x_n - q)\|^2 + 2\alpha_n\langle u - q, x_{n+1} - q\rangle \\ &\leq (\beta_n\|x_n - q\| + \gamma_n\|T^{\lambda_{n+1}}x_n - T^{\lambda_{n+1}}q\| \\ &\quad +\gamma_n\|T^{\lambda_{n+1}}q - q\|)^2 + 2\alpha_n\langle u - q, x_{n+1} - q\rangle \\ &\leq [(\beta_n + \gamma_n)\|x_n - q\| + \gamma_n\lambda_{n+1}\mu\|F(q)\|]^2 + 2\alpha_n\langle u - q, x_{n+1} - q\rangle \\ &\leq (1-\alpha_n)^2\|x_n - q\|^2 + \lambda_{n+1}[2(\beta_n + \gamma_n)\gamma_n\mu\|x_n - q\| \cdot \|F(q)\| \\ &\quad +\gamma_n^2\lambda_{n+1}\mu^2\|F(q)\|^2] + 2\alpha_n\langle u - q, x_{n+1} - q\rangle. \end{aligned}$$

Since $\{x_n\}$ is bounded, there exists constant M_1 such that $2(\beta_n + \gamma_n)\gamma_n\mu\|x_n - q\| \cdot \|F(q)\| + \gamma_n^2\lambda_{n+1}\mu^2\|F(q)\|^2 \leq M_1$. Thus, we have

$$\|x_{n+1} - q\|^2 \leq (1-\alpha_n)\|x_n - q\|^2 + \lambda_{n+1}M_1 + 2\alpha_n\langle u - q, x_{n+1} - q\rangle.$$

It follows from Lemma 2.4 that $\lim_{n\to\infty}\|x_n - q\| = 0$. This implies that $\{x_n\}$ converges strongly to the fixed point q of T.

Finally, we show that q solves the following variational inequality

$$\langle u - q, p - q\rangle \leq 0, \quad \forall p \in F(T).$$

For any $p \in F(T)$, we have

$$\begin{aligned} \|z_t - p\|^2 &= \|t(u-p) + (1-t)(Tz_t - p)\|^2 \\ &\leq (1-t)^2\|Tz_t - p\|^2 + 2t\langle u - p, z_t - p\rangle \\ &\leq (1-t)^2\|z_t - p\|^2 + 2t\langle u - q, z_t - p\rangle + 2t\langle q - p, z_t - p\rangle. \end{aligned}$$

Further, we have

$$\langle u - q, p - z_t\rangle \leq -\|z_t - p\|^2 + \frac{t}{2}\|z_t - p\|^2 + \langle q - p, z_t - p\rangle. \tag{3.7}$$

Letting $t \to 0$, it follows from (3.7) that $\langle u - q, p - q\rangle \leq 0$ for all $p \in F(T)$. This completes the proof.

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(Received 20 July 2007)

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