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ON $k-NUC$ Property in Some Sequence Spaces Involving Lacunary Sequence

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Abstract : In this paper, we introduce a new sequence space involving Lacunary sequence and investigate $k-NUC$ property of this space which is equipped with the Luxemburg norm.

Keywords : *Cesaro* sequence space, Lacunary sequence, k-NUC property, Luxemburg norm
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1 Introduction

Let $X = (X, \|.\|)$ be a real Banach space and $B(X)$ and $S(X)$ be the closed unit ball and the unit sphere of X, respectively.

A Banach space X is called uniformly convex (UC) if for each $\varepsilon > 0$, there is $\delta > 0$ such that for $x, y \in S(X)$, the inequality $\|x - y\| > \varepsilon$ implies that

$$\left\| \frac{1}{2}(x+y) \right\| < 1 - \delta.$$

Clarkson introduced the concept of uniform convexity which implies reflexivity of Banach spaces.

For any $x \notin B(X)$, the drop determined by x is the set

$$D(x, B(X)) = conv(\{x\} \cup B(X))$$

A Banach space X has the drop property (D) if for every closed set C disjoint with $B(X)$, there exists an element $x \in$ C such that

$$D(x, B(X)) \cap C = \{x\}$$

A Banach space X is said to have the Kadec-Klee property (or $H-$ property) if every weakly convergent sequence on the unit sphere is convergent in norm.

In [5], Rolewicz proved that if the Banach space X has the drop property, then X is reflexive. Montesinos [4] extended this result by showing that X has the drop property if and only if X is reflexive and has the property (H).

Recall that a sequence is said to be $\varepsilon-$seperated sequence for some $\varepsilon > 0$ if

$$sep\,(x_n) = \inf \{\|x_n - x_m\|, n \neq m\} > \varepsilon$$

A Banach space is said to be nearly uniformly convex (NUC) if for every $\varepsilon > 0$ there exists $\delta \in (0,1)$ such that for every sequence $\{x_n\} \subseteq B(X)$ with $sep\,(x_n) > \varepsilon$, we have

$$conv\,(x_n) \cap ((1-\delta)\, B(X)) \neq \varnothing.$$

A Banach space is said to have the uniform Kadec-Klee property (UKK) if for every $\varepsilon > 0$, there exists $\delta > 0$ such that for every sequence (x_n) in $S(X)$ with $sep\,(x_n) > \varepsilon$ and $x_n \overset{w}{\rightarrow} x$, we have

$$\|x\| < 1 - \delta.$$

Every (UKK) Banach space has (H) property [2]. Huff [2] proved that every (NUC) Banach space is reflexive and it has property (H), he also proved that X is (NUC) if and only if X is reflexive and (UKK).

Kutzarova[3] has defined k-nearly uniformly convex Banach spaces. Let $k \geq 2$ be an integer. A Banach space X is said to be k-nearly uniformly convex $(k - NUC)$ if for every $\varepsilon > 0$, there exists $\delta > 0$ such that for any sequence $(x_n) \subset B(X)$ with $sep\,(x_n) > \epsilon$, there are $n_1, n_2, ..., n_k$ such that

$$\left\| \frac{x_{n_1} + x_{n_2} + ... + x_{n_k}}{k} \right\| < 1 - \delta.$$

Of course a Banach space is (NUC) whenever it is $(k - NUC)$ for some integer $k \geq 2$.

A Banach space X is said to have the Opial property if every sequence $\{x_n\}$ weakly convergent to x_0 satisfies

$$\liminf_{n\to\infty} \|x_n - x_0\| \leq \liminf_{n\to\infty} \|x_n - x\|$$

for every $x \in X$.

Opial has proved in [16] that the sequnce spaces $l_p\,(1 < p < \infty)$ have this property but $L_p\,[0, 2\pi]$ $(p \neq 2, 1 < p < \infty)$ do not have it.

A Banach space X is said to have the uniform Opial property if for every $\varepsilon > 0$ there exists $r > 0$ such that for each weakly null sequence $\{x_n\} \subset S(X)$ and $x \in X$ with $\|x\| \geq \varepsilon$, we have

$$1 + r \leq \liminf_{n\to\infty} \|x_n + x\|.$$

For a bounded subset $A \subset X$, the set-measure of noncompactness was defined in [17] by

$$\alpha\,(A) = \inf \{\varepsilon > 0 : A \text{ can be covered by finitely many sets of diameter } \leq \varepsilon\}.$$

The ball-measure of noncompactness is defined by

$$\beta(A) = \inf\{\varepsilon > 0 : A \text{ can be covered by finitely many balls of diameter } \le \varepsilon\}.$$

The functions α and β are called the Kuratowski measure of noncompactness and the Hausdorff measure of noncompactness in X, respectively.We can associate these functions with the notions of the set-contraction and the ball-contraction [15].

For each $\varepsilon > 0$ define

$$\Delta(\varepsilon) = \inf\{1 - \inf[\|x\| : x \in A] : A \text{ is a closed convex subset of } B(X) \text{ with } \beta(A) \ge \varepsilon\}.$$

The function Δ is called the modulus of noncompact convexity [18].

A Banach space X is said to have property (L) if $\lim_{\varepsilon\to 1^-} \Delta(\varepsilon) = 1$. It has been proved in [15] that property (L) is a useful tool in the fixed point theory and that a Banach space X has property (L) if and only if it is reflexive and has the uniform Opial property.

By a Lacunary sequence $(\theta) = (k_r)$ where $k_0 = 0$, we mean an increasing sequence of non-negative integers with $k_r - k_{r-1} \to \infty$ as $r \to \infty$. The intervals determined by θ will be denoted by $I_r = (k_{r-1}, k_r]$. We write $h_r = k_r - k_{r-1}$. The ratio $\frac{k_r}{k_{r-1}}$ will be denoted by q_r. The space of lacunary strongly convergent sequences N_θ was defined by Freedman and denoted by

$$N_\theta = \left\{ x = (x_k) : \lim_{r\to\infty} \frac{1}{h_r} \sum_{k\in I_r} |x_k - l| = 0, \text{for some l} \right\}.$$

It is well known that there exists very close connection between the space of lacunary strongly convergent sequences and the space of strongly Cesaro summable sequences. This connection can be found in [9], [10], [11].

2 BASIC FACTS AND DEFINITIONS

Let w be the space of all real sequences. Let $p = p_r$ be a bounded sequence of the positive real numbers. In this paper we define a new sequence space $l(p, s, \theta)$ involving lacunary sequence and denoted by:

$$l(p, s, \theta) = \left\{ x = (x_k) : \sum_{r=1}^{\infty} \left(\frac{1}{h_r} \sum_{k\in I_r} k^{-s} |x_k| \right)^{p_r} < \infty, \ s \ge 0 \right\}.$$

Paranorm on $l(p, s, \theta)$ is given by

$$\|x\|_{l(p,s,\theta)} = \left(\sum_{r=1}^{\infty} \left(\frac{1}{h_r} \sum_{k\in I_r} k^{-s} |x_k| \right)^{p_r} \right)^{\frac{1}{M}}$$

where $M = \max(1, H)$ and $H = \sup_r p_r$.

It is easy to see that the space $l(p, s, \theta)$ with $\|x\|_{l(p,s,\theta)}$ is a complete paranormed space. By using the properties of lacunary sequence in the space $l(p, s, \theta)$, we get the following sequence spaces: If $\theta = 2^r$, then $l(p, s, \theta) = C(s, p)$, $C(s, p)$ sequences space is introduced by T.Bilgin [7]. If we take $\theta = 2^r$ and $s = 0$ then we obtain $Ces(p)$ sequences space [12], [13].

For $x \in l(p, s, \theta)$, let

$$\rho(x) = \sum_{r=1}^{\infty} \left(\frac{1}{h_r} \sum_{k \in I_r} k^{-s} |x_k| \right)^{p_r}$$

and define the Luxemburg norm on $l(p, s, \theta)$ by

$$\|x\| = \inf \left\{ \varepsilon > 0 : \rho\left(\frac{x}{\varepsilon}\right) \le 1 \right\}.$$

The Luxemburg norm on $l(p, s, \theta)$ can be reduced to usual norm on $l(p, s, \theta)$, that is, $\|x\|_{l(p,s,\theta)} = \|x\|$. It is clear and we omit it [14].

The main purpose of this work is to show that the space $l(p, s, \theta)$ equipped with Luxemburg norm is a modular space and to investigate geometric property $k - NUC$ of this space.

3.*MAIN RESULTS*

We give a theorem which showing the connection between $l(p, s, \theta)$ and $C(s, p)$ [14].

Theorem 1 *If* $\liminf q_r > 1$, *then* $C(s, p) \subset l(p, s, \theta)$.

Proof. *: It is trivial and we omit it.* □

And now we give a property about ρ on $l(p, s, \theta)$ which is necessary for our consideration.

A modular ρ is said to satisfy the δ_2- condition if for any $\varepsilon > 0$, there exist constants $K \ge 2, a > 0$ such that

$$\rho(2u) \le K\rho(u) + \varepsilon$$

for all $u \in l(p, s, \theta)$ with $\rho(u) \le a$.

If satisfies ρ the δ_2- condition for all $a > 0$ with $K \ge 2$ dependent on a, we say that ρ satisfies the strong δ_2- condition $(\rho \in \delta_2^s)$.

For $x \in l(p, s, \theta)$, it is easy to see that; the modular ρ and Luxemburg norm $\|.\|$ on $l(p, s, \theta)$ satisfies the following properties and lemmas:

1. If $0 < \alpha < 1$; then $\alpha^H \rho\left(\frac{x}{\alpha}\right) \le \rho(x)$ and $\rho(\alpha x) \le \rho(x)$;

2. If $\alpha > 1$; then $\alpha^H \rho\left(\frac{x}{\alpha}\right) \geq \rho(x)$;
3. If $\alpha \geq 1$; then $\rho(\alpha x) \geq \alpha\rho(x) \geq \rho(x)$.

Lemma 2.1 *For any $x \in l(p,s,\theta)$, we have*

1. *If $\|x\| < 1$; then $\rho(x) \leq \|x\|$;*
2. *If $\|x\| > 1$; then $\rho(x) \geq \|x\|$;*
3. *$\|x\| = 1$ if and only if $\rho(x) = \|x\|$;*
4. *$\|x\| < 1$ if and only if $\rho(x) < \|x\|$;*
5. *$\|x\| > 1$ if and only if $\rho(x) > \|x\|$;*
6. *If $0 < \alpha < 1$ and $\|x\| > \alpha$; then $\rho(x) > \alpha^H$;*
7. *If $\alpha \geq 1$, $\|x\| < \alpha$, then $\rho(x) < \alpha^H$.*

Lemma 2.2 *If $\rho \in \delta_2^s$, then for any $L > 0$ and $\varepsilon > 0$, there exists $\delta > 0$ such that*

$$|\rho(u+v) - p(u)| < \varepsilon$$

whenever $u, v \in l(p,s,\theta)$ with $\rho(u) \leq L$ and $\rho(v) \leq \delta$.

Lemma 2.3 *1.If $\rho \in \delta_2^s$, then for any $x \in l(p,s,\theta)$, $\|x\| = 1$ if and only if $\rho(x) = 1$.*

2.If $\rho \in \delta_2^s$, then for any $\{x_n\} \subset l(p,s,\theta)$, $\|x_n\| \to 0$ if and only if $\rho(x_n) \to 0$.

Lemma 2.4 *If $\rho \in \delta_2^s$, then for any $\varepsilon \in (0,1)$, there exists $\delta \in (0,1)$ such that $\rho(x) \leq 1-\varepsilon$ implies $\|x\| \leq 1-\delta$.*

And now we give our basic theorem.

Theorem 2 *The space $l(p,s,\theta)$ is $k-NUC$ for any integer $k \geq 2$.*

Proof. *: Let $\varepsilon > 0$ and $(x_n) \subset B(l(p,s,\theta))$ with $sep(x_n) \geq \varepsilon$. For each $m \in \mathbb{N}$, let* □

$$(1.2.1) \qquad x_n^{(m)} = (0, 0, ..., x_n(m), x_n(m+1), ...)$$

Since for each $i \in \mathbb{N}$, $(x_n(i))_{i=1}^{\infty}$ is bounded, by using the diagonal method, we have for each $m \in \mathbb{N}$, we can find a subsequence $\left(x_{n_j}\right)$ of (x_n) such that $\left(x_{n_j}(i)\right)$ converges for each $i \in \mathbb{N}$, $1 \leq i \leq m$. Therefore, there exists an increasing sequence of positive integer t_m such that $sep\left(\left(x_{n_j}^m\right)_{j \geq t_m}\right) \geq \epsilon$. Hence, there is a sequence of positive integers $(r_m)_{m=1}^{\infty}$ with $r_1 < r_2 < ...$ such that $\left\|x_{r_m}^m\right\| \geq \frac{\varepsilon}{2}$ for all $m \in \mathbb{N}$. Then by Lemma 4 (2), we may assume that there exists $\eta > 0$ such that

$$\rho\left(x_{r_m}^m\right) \geq \eta, \ \forall m \in \mathbb{N} \tag{1.2.2}$$

Let $\alpha > 0$ be such that $1 < \alpha < \liminf_{n\to\infty} p_n$. For fixed integer $k \geq 2$, let $\varepsilon_1 = \frac{k^{\alpha-1}}{(k-1)k^{\alpha}}\left(\frac{\eta}{2}\right)$ .Then by Lemma 5, there is a $\delta > 0$ such that

$$\left|\rho\left(u+v\right)-\rho\left(u\right)\right| < \varepsilon_1 \tag{1.2.3}$$

whenever $\rho(u) \leq 1$ and $\rho(v) \leq \delta$. Since by Lemma 2 (1) for all $n \in \mathbb{N}$, there exist positive integers $m_i = (i = 1, 2, ..., k-1)$ with $m_1 < m_2 < ...m_{k-1}$ such that $\rho\left(x_i^{m_i}\right) \leq \delta$ and $\alpha \leq p_j$ for all $j \geq m_{k-1}$. Define $m_k = m_{k-1} + 1$. Since $\rho\left(x_{r_m}^m\right) \geq \eta, \forall m \in \mathbb{N}$, we have $\rho\left(x_{r_{m_k}}^{m_k}\right) \geq \eta$. Let $s_i = i$ for $1 \leq i \leq k-1$ and $s_k = r_{m_k}$. Then in virtue of (1.2.1), (1.2.2), (1.2.3) and the convexity of function

$$f_i\left(u\right) = \left|u\right|^{p_i} \ \left(i \in \mathbb{N}\right),$$

we have

$$\begin{aligned}
\rho\left(\frac{x_{s_1}+x_{s_2}+...+x_{s_k}}{k}\right) &= \sum_{r=1}^{\infty}\left(\frac{1}{h_r}\sum_{i\in I_r}\frac{1}{i^s}\left|\frac{x_{s_1}(i)+x_{s_2}(i)+...+x_{s_k}(i)}{k}\right|\right)^{p_r} \\
&= \sum_{r=1}^{m_1}\left(\frac{1}{h_r}\sum_{i\in I_r}\frac{1}{i^s}\left|\frac{x_{s_1}(i)+x_{s_2}(i)+...+x_{s_k}(i)}{k}\right|\right)^{p_r} \\
&+ \sum_{r=m_1+1}^{\infty}\left(\frac{1}{h_r}\sum_{i\in I_r}\frac{1}{i^s}\left|\frac{x_{s_1}(i)+x_{s_2}(i)+...+x_{s_k}(i)}{k}\right|\right)^{p_r} \\
&\leq \sum_{r=1}^{m_1}\left(\frac{1}{h_r}\sum_{i\in I_r}\frac{1}{i^s}\left|\frac{x_{s_1}(i)+x_{s_2}(i)+...+x_{s_k}(i)}{k}\right|\right)^{p_r} \\
&+ \sum_{r=m_1+1}^{\infty}\left(\frac{1}{h_r}\sum_{i\in I_r}\frac{1}{i^s}\left|\frac{x_{s_2}(i)+x_{s_3}(i)+...+x_{s_k}(i)}{k}\right|\right)^{p_r} + \varepsilon_1 \\
&\leq \sum_{r=1}^{m_1}\frac{1}{k}\sum_{j=1}^{k}\left(\frac{1}{h_r}\sum_{i\in I_r}\frac{1}{i^s}\left|x_{s_j}(i)\right|\right)^{p_r} \\
&+ \sum_{r=m_1+1}^{m_2}\left(\frac{1}{h_r}\sum_{i\in I_r}\frac{1}{i^s}\left|\frac{x_{s_2}(i)+x_{s_3}(i)+...+x_{s_k}(i)}{k}\right|\right)^{p_r} \\
&+ \sum_{r=m_2+1}^{\infty}\left(\frac{1}{h_r}\sum_{i\in I_r}\frac{1}{i^s}\left|\frac{x_{s_3}(i)+x_{s_4}(i)+...+x_{s_k}(i)}{k}\right|\right) + 2\varepsilon_1
\end{aligned}$$

$$\leq \sum_{r=1}^{m_1} \frac{1}{k} \sum_{j=1}^{k} \left(\frac{1}{h_r} \sum_{i \in I_r} \frac{1}{i^s} \left| x_{s_j}(i) \right| \right)^{p_r}$$

$$+ \sum_{r=m_1+1}^{m_2} \frac{1}{k} \sum_{j=2}^{k} \left(\frac{1}{h_r} \sum_{i \in I_r} \frac{1}{i^s} \left| x_{s_j}(i) \right| \right)^{p_r}$$

$$+ \sum_{r=m_2+1}^{m_3} \frac{1}{k} \sum_{j=3}^{k} \left(\frac{1}{h_r} \sum_{i \in I_r} \frac{1}{i^s} \left| x_{s_j}(i) \right| \right)^{p_r} + ...$$

$$+ ... + \sum_{r=m_{k-1}+1}^{m_k} \frac{1}{k} \sum_{j=k-1}^{k} \left(\frac{1}{h_r} \sum_{i \in I_r} \frac{1}{i^s} \left| x_{s_j}(i) \right| \right)^{p_r}$$

$$+ \sum_{r=m_k+1}^{\infty} \left(\frac{1}{h_r} \sum_{i \in I_r} \frac{1}{i^s} \left| \frac{x_{s_k}(i)}{k} \right| \right)^{p_r} + (k-1)\varepsilon_1$$

$$
\begin{aligned}
&\leq \left(\frac{\rho(x_{s_1})+\rho(x_{s_2})+...+\rho(x_{s_k})}{k}\right) \\
&\quad +\frac{1}{k}\sum_{r=1}^{m_k}\left(\frac{1}{h_r}\sum_{i\in I_r}\frac{1}{i^s}\left|x_{s_k}(i)\right|\right)^{p_r} \\
&\quad +\sum_{r=m_k+1}^{\infty}\left(\frac{1}{h_r}\sum_{i\in I_r}\frac{1}{i^s}\left|\frac{x_{s_k}(i)}{k}\right|\right)^{p_r}+(k-1)\varepsilon_1 \\
&\leq \frac{k-1}{k}+\frac{1}{k}\sum_{r=1}^{m_k}\left(\frac{1}{h_r}\sum_{i\in I_r}\frac{1}{i^s}\left|x_{s_k}(i)\right|\right)^{p_r} \\
&\quad +\frac{1}{k^{\alpha}}\sum_{r=m_k+1}^{\infty}\left(\frac{1}{h_r}\sum_{i\in I_r}\frac{1}{i^s}\left|x_{s_k}(i)\right|\right)^{p_r}+(k-1)\varepsilon_1 \\
&\leq 1-\frac{1}{k}+\frac{1}{k}\left(1-\sum_{r=m_k+1}^{\infty}\left(\frac{1}{h_r}\sum_{i\in I_r}\frac{1}{i^s}\left|x_{s_k}(i)\right|\right)^{p_r}\right) \\
&\quad +\frac{1}{k^{\alpha}}\sum_{r=m_k+1}^{\infty}\left(\frac{1}{h_r}\sum_{i\in I_r}\frac{1}{i^s}\left|x_{s_k}(i)\right|\right)^{p_r}+(k-1)\varepsilon_1 \\
&\leq 1+(k-1)\varepsilon_1-\left(\frac{k^{\alpha-1}-1}{k^{\alpha}}\right)\left(\sum_{r=m_k+1}^{\infty}\left(\frac{1}{h_r}\sum_{i\in I_r}\frac{1}{i^s}\left|x_{s_k}(i)\right|\right)^{p_r}\right) \\
&\leq 1+(k-1)\varepsilon_1-\left(\frac{k^{\alpha-1}-1}{k^{\alpha}}\right)\eta \\
&\leq 1+(k-1)\frac{k^{\alpha-1}}{(k-1)k^{\alpha}}\left(\frac{\eta}{2}\right)-\left(\frac{k^{\alpha-1}-1}{k^{\alpha}}\right)\eta
\end{aligned}
$$

By the Lemma 5 there exists $\gamma > 0$ such that $\left\|\frac{x_{s_1}+x_{s_2}+...+x_{s_k}}{k}\right\| \leq 1-\gamma$. Therefore, $l(p,s,\theta)$ is $k-NUC$. Since $k-NUC$ implies NUC and NUC implies property (H) and reflexivity holds, by the theorem 1.2, the following results are obtained.

Corollary 2.5 *The space $l(p,s,\theta)$ is NUC and UC and then is reflexive for $s \geq 0$.*

Corollary 2.6 *The space $l(p,s,\theta)$ is UKK and (H) property. Then it has drop property for $s \geq 0$.*

Proof. *Since $l(p,s,\theta)$ is NUC space, by [2] it is reflexive and UKK.Also, since every UKK space has (H) property, $l(p,s,\theta)$ has drop property for $s \geq 0$ by [4].* □

Corollary 2.7 *The space $l(p,s,\theta)$ has fixed point property for $s \geq 0$.*

Proof. *We have proved that the space $l(p,s,\theta)$ has uniform Opial property in [14] and since it is reflexive so it has property (L) by [15]. Therefore the space $l(p,s,\theta)$ has fixed point property for $s \geq 0$ [15].* □

References

[1] Clarkson J.A. , Uniformly Convex Spaces, Trans. Amer. Math. Soc. 40, (1936), 396-414

[2] Huff R. , Banach Spaces which are nearly uniformly convex, Rocky Mountain J.Math., 10, (1980), 743-749.

[3] Kutzarova D., k-β and k- nearly uniformly convex Banach spaces, J.Math. Anal. Appl. ,162, (1991), 322-338.

[4] Montesinos V., Drop property equals reflexivity, Studia Math. , 87, (1987), 93-100.

[5] Rolewicz S. ,On drop property, Studia Math. , 85, (1986), 27-35.

[6] Shiue J. , On the Cesaro sequence spaces, Tamkang J. Math. 1, (1970), 19-25.

[7] Bilgin T. , The sequence space C(s,p) and related matrix transformations, Punjab Univ.J.Math , (1997), 67-77.

[8] Sanhan W. , Suantai S. , On k-Nearly Uniform Convex Property In Generalized Cesaro Sequence Spaces, Hindawi Publishing Corp., (2003), 3599-3907.

[9] Das G. , Patel B. K. , Lacunary distribution of sequences, Indian J. Pure Appl. Math., 26, (1989), 64-74.

[10] Freedman A.R. , Sember J. J. , Raphael M. , Some Cesaro -Type Summability Spaces, Proc. London Math. Soc. , 37, (1978), 508-520.

[11] Mursaleen , Chishti T.A. , Some Spaces of Lacunary sequences defined by the modulus, J. Analysis, 41, (1996), 153-159.

[12] K.P.Lim, Matrix transformations in the Cesaro sequence spaces, Kyugpook Math.J.14 (1974), 221-227.

[13] K.P.Lim, Matrix transformation on certain sequence spaces, Tamkang J.Math.2 (1197), 213-220.

[14] Öztürk M., Başarır M.,On Opial Property in Some Sequence Space Involving Lacunary Sequence, submitted.

[15] Prus S., Banach Spaces with Uniform Opial Property, Nonlinear Anal. 8, (1992), 692-704.

[16] Opial Z., Weak Convergence Of the Sequence Of Successive Approximations For Nonexpansive Mappings, Bull. Amer. Math. Soc. 73, (1967), 591-597.

[17] Kuratowski K., Sur Les Espaces Completes, Fund. Math. 15, (1930), 301-309.

[18] Goebel R., Kirk W.A., Topics In Metric Fixed Point Theory, Cambridge University Press, (1990).

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