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MRA in the Space $H^1[0,\ 1]$

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Abstract: In this paper , we built Multiresolution Analysis (MRA for short) in the reproducing kernel space $H^1[0,\ 1]$. Moreover, we give a spline wavelet function in the space $H^1[0,1]$.

Keywords: Reproducing kernel spaces; Spline wavelet approximation ; Multiresolution Analysis (MRA for short).
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1 Introduction

Wavelet analysis is widely applied in many fields in engineering and technology. It is necessary to deeper study the theory of wavelet. All the one-dimensional wavelets constructions we have discussed so far lead to bases for $L_2\,(R)$. In many applications one is interested in only part of the real line:numerical analysis computations generally work on an interval, images are concentrated on rectangles, many systems to analyze sound divide it in chunks. All these involve decompositions of functions f supported on an interval. say $[0,1]$. One could, of course, decide to use standard wavelet bases to analyze f, setting the function equal to zero outside $[0,1]$, but this introduces an artificial"jump" at the edges. It is therefore useful to develop wavelets adapted to "life on an interval". The paper is first one to discuss multiresolution analysis in the reproducing kernel spaces $H^1[0,1]$. That will give a idea how to deal with MRA in the space $H^1[0,1]$ and find some applications with MRA in the space $H^1[0,1]$ in the future.

On the reproducing kernel space $H^1[0,1]$ comprising all complex valued satisfy the conditions that $f(x)$ is absolutely continuous functions, $f(0)=0$ and $f'(x)\in L_2[0,1]$, where the inner product is defined

$$(f,g)_{H^1}=\int_0^1 f'(x)\overline{g'(x)}dx.$$

And finite norm $\|f\|_{H^1}=(f,f)_{H^1}^{\frac{1}{2}}$ that generate by the inner product.

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It is well known that the space $H^1[0,1]$ is a reproducing kernel Hilbert space, and the reproducing kernel of $H^1[0,1]$ is

$$k(x,y) = \min\{x,y\}.$$

i.e., for each $f \in H^1[0,1]$ there holds the following

$$(f(x), k(x,y))_{H^1} = f(y).$$

2 MRA in $H^1[0,1]$

Definition 2.1 A closed subspaces $\{V_{-J}\}_{J\geq 0}$ of a Hilbert space X is called a MRA if the set satisfies the following conditions:

(1) $V_0 \subset V_{-1} \subset V_{-2} \subset \cdots \subset V_{-J} \cdots$;

(2) $\overline{\bigcup_{J\geq 0} V_{-J}} = X$;

(3) $f(x) \in V_{-J}$ if and only if $f(2^J x) \in V_0$;

(4) $f(x) \in V_{-J}$ if and only if $f(x-n) \in V_{-J}$ for $n = 0,1,2,\cdots,2^J-1$;

(5) $\{\varphi_{-J,k}(x) : k = 0,1,2,\cdots,2^J-1\}$ is a normal orthogonal bases of V_{-J}.

Next, we will built a MRA in the space $H^1[0,1]$. Put

$$h(x) = \begin{cases} 0 & x < 0 \\ x & 0 \leq x \leq 1 \\ 1 & x > 1 \end{cases}$$

and

$$\varphi_{-J,k}(x) = \begin{cases} 0, \quad z & 0 \leq x < 2^{-J}k \\ 2^{\frac{J}{2}}(x - 2^{-J}k), & 2^{-J}k \leq x < 2^{-J}(k+1) \\ 2^{-\frac{J}{2}}, & 2^{-J}(k+1) \leq x \leq 1 \end{cases}.$$

Then we have the following

$$2^{-\frac{J}{2}} h(2x-k) = \begin{cases} 0 & x < 0 \\ \varphi_{-J,k}(x) & 0 \leq x \leq 1 \\ 2^{-\frac{J}{2}} & x > 1 \end{cases}.$$

Let $V_{-J} = \overline{span}\{\varphi_{-J,k}(x); k = 0,1,2,\cdots,2^J-1\}$ for all $J = 0,1,2,\cdots$. Then $V_0 \subset V_{-1} \subset V_{-2} \subset \cdots \subset H^1[0,1]$ and $\{\varphi_{-J,k}(x); k = 0,1,2,\cdots,2^J-1\}$ is a normal orthogonal bases in the space V_{-J}.

Theorem 2.2 *The equality* $\overline{\bigcup_{J\geq 0} V_{-J}} = H^1[0,1]$ *holds.*

Proof. It is clear that $\overline{\bigcup_{J\geq 0} V_{-J}} \subset H^1[0,1]$ holds. We only need to prove that $\overline{\bigcup_{J\geq 0} V_{-J}} \supset H^1[0,1]$ holds. Let $f(x) \in \left(\bigcup_{J\geq 0} V_{-J}\right)^{\perp}$ be given. For any $\varepsilon > 0$ there exists a $f_0 \in C^\infty(0,1)$ such that

$$||f - f_0||_{H^1} = ||f' - f_0'||_{L^2} \leq \varepsilon.$$

Let P_{-J} be a projection from $H^1[0,1]$ to V_{-J}. Then

$$||P_{-J}f_0||_{H^1} = ||P_{-J}(f' - f_0')||_{H^1} \leq ||f - f_0||_{H^1} \leq \varepsilon \tag{1}$$

Since $\{\varphi_{-J,k}(x) : k = 0, 1, 2, \cdots, 2^J - 1\}$ is a normal orthogonal bases of V_{-J}, we have

$$||P_{-J}f_0||^2_{H^1} = \left\| \sum_{k=0}^{2^J-1} (f_0, \varphi_{-J,k})_{H^1} \varphi_{-J,k}(x) \right\|^2_{H^1}$$

$$= \sum_{k=0}^{2^J-1} |(f_0, \varphi_{-J,k})_{H^1}|^2 = \sum_{k=0}^{2^J-1} \left|(f_0', \varphi'_{-J,k})_{L^2}\right|^2.$$

Put

$$h_{-J,k}(x) = \begin{cases} 2^{\frac{J}{2}}, & 2^{-J}k < x < 2^{-J}(k+1) \\ 0, x \in (-\infty, 2^{-J}k] \cup [2^{-J}(k+1), +\infty) \end{cases}$$

and

$$g(x) = f_0'(x)\chi_{[0,1\}} + f_0'(2-x)\chi_{[1,2]}.$$

We denote by $\widehat{h}_{-J,k}(\omega)$ and $\widehat{g}(\omega)$ Fourier transform of $h_{-J,k}(x)$ and $g(x)$, respectively. Hence

$$\begin{aligned}\sum_{k=0}^{2^J-1} |(f_0, \varphi_{-J,k})_{H^1}|^2 &= \sum_{k=0}^{2^J-1} |(g, h_{-J,k})_{L^2}|^2 \\ &= \sum_{k=0}^{2^J-1} \left| \int_{-\infty}^{+\infty} g(x)\overline{h_{-J,k}(x)}dx \right|^2\end{aligned}$$

$$= \sum_{k=0}^{2^J-1} \left| \int_{-\infty}^{+\infty} \widehat{g}(\omega) 2^{-\frac{J}{2}} \overline{\widehat{h}_{0,0}(2^{-J}\omega)} e^{i2^{-J}k\omega} d\omega \right|^2$$

$$= \sum_{k=0}^{2^J-1} 2^{-J} \left| \sum_{l\in Z} \int_{2^{J+1}\pi l}^{2^{J+1}\pi(l+1)} \widehat{g}(\omega) 2^{-\frac{J}{2}} \overline{\widehat{h}_{0,0}(2^{-J}\omega)} e^{i2^{-J}k\omega} d\omega \right|^2$$

$$= \sum_{k=0}^{2^J-1} 2^{-J} \left| \int_{0}^{2^{J+1}\pi} e^{i2^{-J}k\omega} \sum_{l\in Z} \widehat{g}(\omega + 2^{J+1}\pi l) \overline{\widehat{h}_{0,0}(2^{-J}\omega + 2\pi l)} d\omega \right|^2$$

$$= 2\pi \int_{0}^{2^{J+1}\pi} \left| \sum_{l\in Z} \widehat{g}(\omega + 2^{J+1}\pi l) \overline{\widehat{h}_{0,0}(2^{-J}\omega + 2\pi l))} \right|^2 d\omega$$

$$= 2\pi \sum_{n\in Z} \int_{-\infty}^{+\infty} \left| \widehat{g}(\omega) \overline{\widehat{g}(\omega + 2^{J+1}\pi l)} \overline{\widehat{h}_{0,0}(2^{-J}\omega)} \widehat{h_{0,0}}(2^{-J}\omega + 2\pi n)) \right|^2 d\omega$$

$$= 2\pi \int_{-\infty}^{+\infty} |\widehat{g}(\omega)|^2 \left| \widehat{h}_{0,0}(2^{-J}\omega) \right|^2 d\omega + R, \tag{2}$$

where $R = 2\pi \sum\limits_{n\in Z, n\neq 0} \int\limits_{-\infty}^{+\infty} \left| \widehat{g}(\omega) \overline{\widehat{g}(\omega + 2^{J+1}\pi l)} \overline{\widehat{h}_{0,0}(2^{-J}\omega)} \widehat{h}_{0,0}(2^{-J}\omega + 2\pi n)) \right|^2 d\omega.$

Since $h_{0,0}(x) = \chi_{[0,1]}(x)$, we have $\widehat{h}_{0,0}(\omega) = \frac{\sin\frac{\omega}{2}}{\frac{\omega}{2}} e^{-i\frac{\omega}{2}}$, $\widehat{h}_{0,0}(0) = 1$ and $\left|\widehat{h}_{0,0}(\omega)\right| \leq 1$. Therefore

$$|R| \leq 2\pi \sum_{n\in Z, n\neq 0} \int_{-\infty}^{+\infty} \left|\widehat{g}(\omega)\right| \left|\overline{\widehat{g}(\omega + 2^{J+1}\pi l)}\right| \left|\overline{\widehat{h}_{0,0}(2^{-J}\omega)}\right| \left|\widehat{h}_{0,0}(2^{-J}\omega + 2\pi n))\right| d\omega$$

$$\leq 2\pi \sum_{n\in Z, n\neq 0} \int_{-\infty}^{+\infty} |\widehat{g}(\omega)| \left|\widehat{g}(\omega + 2^{J+1}\pi n)\right| d\omega.$$

Using $g(x) \in C^{\infty}(-\infty, +\infty)$, there exists a constant $C > 0$ such that $|\widehat{g}(\omega)| \leq C(1 + |\omega|^2)^{-\frac{3}{2}}$. Noting that

$$\sup_{x,y\in R} (1 + y^2)[1 + (x - y)^2]^{-1} < \infty,$$

$$
\begin{aligned}
|R| &\le 2\pi \sum_{n\in Z, n\neq 0} \int_{-\infty}^{+\infty} C^2(1+|\omega|^2)^{-\frac{3}{2}}(1+\left|\omega - 2^{-J}\pi n\right|^2)^{-\frac{3}{2}} d\omega \\
&\le C_1 \sum_{n\in Z, n\neq 0} \int_{-\infty}^{+\infty} (1+\left|\omega + 2^{J}\pi n\right|^2)^{-\frac{3}{2}}(1+\left|\omega - 2^{-J}\pi n\right|^2)^{-\frac{3}{2}} d\omega \\
&\le C_1 \sum_{n\in Z, n\neq 0} (1+2^{2J}\pi^2 n^2)^{-1} \left(\int_{-\infty}^{+\infty} (1+\left|\omega + 2^{J}\pi n\right|^2)^{-1} d\omega\right)^{\frac{1}{2}} \\
&\times \left(\int_{-\infty}^{+\infty} (1+\left|\omega - 2^{-J}\pi n\right|^2)^{-\frac{3}{2}}\right)^{\frac{1}{2}} d\omega \\
&= C_1 \sum_{n\in Z, n\neq 0} (1+2^{2J}\pi^2 n^2)^{-1} \int_{-\infty}^{+\infty} (1+|\omega|^2)^{-1} d\omega \le C_2 2^{-2J}. \qquad (3)
\end{aligned}
$$

By (1), (2) and (3), we get the follows.

$$
\begin{aligned}
2\pi \int_{-\infty}^{+\infty} |\widehat{g}(\omega)|^2 \left|\widehat{h}_{0,0}(2^{-J}\omega)\right|^2 d\omega &= \sum_{k=0}^{2^J-1} |(f_0, \varphi_{-J,k})_{H^1}|^2 - R \\
&= \sum_{k=0}^{2^J-1} |(f_0, \varphi_{-J,k})_{H^1}|^2 + |R| \le \varepsilon + C_2 2^{-2J}. \qquad (4)
\end{aligned}
$$

By dominated convergence theorem, we have

$$
2\pi \left\|\widehat{g}\right\|_{L^2(-\infty,+\infty)} = 2\pi \left|\widehat{h}_{0,0}(0)\right|^2 \left\|\widehat{g}\right\|^2_{L^2(-\infty,+\infty)} \le \varepsilon \text{ as } J \to \infty,
$$

that is

$$
\left\|\widehat{g}\right\|_{L^2(-\infty,+\infty)} \le \frac{\varepsilon}{2\pi}.
$$

Therefore

$$
\begin{aligned}
\|f\|_{H^1} = \|f'\|_{L^2[0,1]} &\le \|f' - f_0'\|_{L^2[0,1]} + \|f_0'\|_{L^2[0,1]} \\
&\le \varepsilon + \|g\|_{L^2(-\infty,+\infty)} \le \varepsilon + \left(\frac{\varepsilon}{2\pi}\right)^{\frac{1}{2}}.
\end{aligned}
$$

By the arbitrainess of ε, we have that $\|f\|_{H^1}$ holds, i.e., $f = 0$. Hence $\left(\bigcup\limits_{J\ge 0} V_{-J}\right)^{\perp} = \{0\}$, i.e., $\overline{\bigcup\limits_{J\ge 0} V_{-J}} = H^1[0,1]$. □

Summing up, we have $\{V_{-J}\}_{J\ge 0}$ is a MRA in the space $H^1[0,1]$.

Let W_{-J} orthogonal complemented subspace of V_{-J} in the space V_{J-1}, i.e., $W_{-J}\oplus V_{-J}=V_{-J-1}$ for all $J=0,1,2,\cdots$. We can easily to get

$$W_{-J}=\overline{span}\left\{\psi_{-J,k}(x);k=0,1,2,\cdots,2^J-1\right\},$$

where

$$\psi_{-J,k}(x)=\begin{cases} 2^{\frac{J}{2}}(x-\frac{2k}{2^{J+1}}), & \frac{2k}{2^{J+1}}\le x<\frac{2k+1}{2^{J+1}} \\ -2^{\frac{J}{2}}(x-\frac{2k+2}{2^{J+1}}), & \frac{2k+1}{2^{J+1}}\le x<\frac{2k+2}{2^{J+1}} \\ 0, & x\in[0,1]\backslash(\frac{2k}{2^{J+1}},\frac{2k+2}{2^{J+1}}) \end{cases}$$

for all $J=0,1,2,\cdots$. Then $\left\{\psi_{-J,k}(x);k=0,1,2,\cdots,2^J-1\right\}$ is also normal orthogonal bases in the space W_{-J} and the following properties hold.

1) $W_{-J}\perp W_{-i}$ for $i,J=0,1,2,\cdots$ with $i\neq J$;

2) $\bigcap\limits_{J\ge 0}W_{-J}=\{0\}$;

3) $H^1[0,1]=V_0\oplus(\bigoplus\limits_{J\ge 0}W_{-J})$.

We will often call $\varphi_{0,0}$ the "scaling function' of the multirsolution analysis. The function $\psi_{0,0}(x)$ is called to be a wavelet basis function generated by scaling function $\varphi_{0,0}$. Hence for any $f(x)\in H^1[0,1]$, we have

$$f(x)=f(1)x+\sum_{J=0}^{+\infty}\sum_{k=0}^{2^J-1}d_{-J,k}\psi_{-J,k}(x),$$

where

$$d_{-J,k}=(f,\psi_{-J,k})_{H^1}=2^{\frac{J}{2}}[2f(2^{-J-1}(2k+1))-f(2^{-J}k)-f(2^J(k+1))].$$

3 MRA in Two Dimension Space $H^1(\Omega)$

Put

$$H^1(Q)=\{u(x,y)=u_1(x)u_2(y),u_1(x),u_2(y)\in H^1[0,1]\},$$

where $Q=[0,1]\times[0,1]$. Then $H^1(Q)$ is two dimension tensor product space. we denote $\langle u,v\rangle_{H^1(Q)}$ by inner product in the space $H^1(\Omega)$, i..e,

$$\begin{aligned}\langle u,v\rangle_{H^1(Q)} &= \iint_Q \frac{\partial^2}{\partial x\partial y}u(x,y)\overline{\frac{\partial^2}{\partial x\partial y}v(x,y)}dxdy \\ &= \int_0^1\frac{d}{dx}u_1(x)\overline{\frac{d}{dx}v_1(x)}dx\int_0^1\frac{d}{dy}u_2(y)\overline{\frac{d}{dy}v_2(y)}dy \\ &= \langle u_1(x),v_1(x)\rangle_{H^1[0,1]}\langle u_2(y),v_2(y)\rangle_{H^1[0,1]},\end{aligned}$$

where $u=u(x,y)=u_1(x)u_2(y)$ and $u_1(x)$, $u_2(y)\in H^1[0,1]$, $v=v(x,y)=v_1(x)v_2(y)$ and $v_1(x)$, $v_2(y)\in H^1[0,1]$.

Theorem 3.1 *The function* $K(x,\xi;y,\eta) = K(x,\xi)K(y,\eta)$ *for* (x,ξ); $(y,\eta) \in Q$ *is a reproducing kernel function in the space* $H^1(\Omega)$. *Moreover,* $H^1(\Omega)$ *is a reproducing space.*

Proof. for any $u(x,y) \in H^1(Q)$, we have

$$\begin{aligned}
&\langle u(x,y), K(x,\xi;y,\eta)\rangle_{H^1(Q)} \\
&= \iint_Q \frac{\partial^2}{\partial x \partial y} u(x,y) \overline{\frac{\partial^2}{\partial x \partial y} K(x,\xi;y,\eta)} dxdy \\
&= \iint_Q \frac{\partial^2}{\partial x \partial y} u_1(x)u_2(y) \overline{\frac{\partial^2}{\partial x \partial y} K(x,\xi)K(y,\eta)} dxdy \\
&= \int_0^1 \frac{\partial}{\partial x} u_1(x) \overline{\frac{\partial}{\partial x} K(x,\xi)} dx \int_0^1 \frac{\partial}{\partial y} u_2(y) \overline{\frac{\partial}{\partial y} K(y,\eta)} dy \\
&= u_1(\xi)u_2(\eta) = u(\xi,\eta)
\end{aligned}$$

□

Theorem 3.2 *Let* $V_{-J}(x,y) = V_{-J} \otimes V_{-J}$, *where* $\{V_{-J}\}_{J\geq 0}$ *is a MRA in* $H^1[0,1]$. *Then* $\{V_{-J}(x,y)\}_{J\geq 0}$ *of* $H^1(\Omega)$ *have the following properties:*

1) $V_0(x,y) \subset V_{-1}(x,y) \subset V_{-2}(x,y) \subset \cdots$;

2) $\bigcap_{J=0}^{\infty} V_{-1}(x,y) = V_0(x,y)$;

3) $\overline{\bigcup_{J>0} V_{-1}(x,y)} = H^1(Q)$.

Proof. Since $V_0 \subset V_{-1} \subset V_{-2} \subset \cdots$ and $V_{-J}(x,y) = V_{-J} \otimes V_{-J}$, for any $J \geq 0$ we have

$$V_{-J}(x,y) = V_{-J} \otimes V_{-J} \subset V_{-J-1} \otimes V_{-J-1} = V_{-J-1}(x,y),$$

i.e., the 1) holds.

By $\bigcap_{J=0}^{\infty} V_{-J} = V_0$ and $\overline{\bigcup_{J\geq 0} V_{-J}} = H^1[0,1]$, we easy to get the 2) and 3) hold. □

Corollary 3.3 *Let* $W_{-J}(x,y)$ *is orthogonal complementary subspace* $V_{-J}(x,y)$ *in* $V_{-J-1}(x,y)$. *Then* $W_{-J}(x,y)$ *have the following properties :*

1) $W_{-J}(x,y) \oplus W_{-K}(x,y)$, *for* $J \neq K, J,\ K = 0,1,2,\cdots$;

2) $\bigcap_{J=0}^{\infty} W_{-J}(x,y) = \{(0,0)\}$;

3) $H^1(Q) = V_0(x,y) \oplus (\bigoplus_{J\geq 0} W_{-J}(x,y))$.

Proof. By Theorem 2.2 and the definition of $W_{-J}(x,y)$, we can obtain it. □

Let $\phi(x)$ be a scaling function in $H^1[0,1]$ and $\psi(x)$ be a wavelet function in $H^1[0,1]$. Define

$$\begin{cases} \psi^1(x,y) = \phi(x)\psi(y) \\ \psi^2(x,y) = \psi(x)\phi(y) \\ \psi^3(x,y) = \psi(x)\psi(y) \end{cases}.$$

Theorem 3.4 *The set of subspaces $\{V_{-J}(x,y)\}_{J>0}$ in $H^1(Q)$ is MRA of $H^1(Q)$ and for any $J \geq 0$ the set*

$$\begin{cases} \psi^1_{-J,k,m}(x,y) = \phi_{-J,k}(x)\psi_{-J,m}(y) \\ \psi^2_{-J,k.m}(x,y) = \psi_{-J,k}(x)\phi_{-J,m}(y) \\ \psi^3_{-J,k,m}(x,y) = \psi_{-J,k}(x)\psi_{-J,m}(y) \end{cases}$$

for $k,m = 0,1,\cdots 2^{-J}-1$ is normal orthogonal bases of $W_{-J}(x,y)$.

Moreover the set

$$\{\psi^{\varepsilon}_{-J,k,m}(x,y)\,|\varepsilon = 1,2,3,\ k,m = 0,1,2,\cdots,2^J-1; J \geq 0\} \cup \{\varphi_{0,k,m}(x,y)\}$$

is a normal orthogonal bases of $H^1(Q)$, where $\varphi_{0,k,m}(x,y) = \varphi_{0,k}(x)\varphi_{0,m}(y)$

Proof. By the definition, we have

$$\begin{aligned} V_{-J-1}(x,y) &= V_{-J-1} \otimes V_{-J-1} \\ &= (V_{-J} \oplus W_{-J}) \otimes (V_{-J} \oplus W_{-J}) \\ &= (V_{-J} \otimes V_{-J}) \oplus (V_{-J} \otimes W_{-J}) \oplus (W_{-J} \otimes V_{-J}) \oplus (W_{-J} \otimes W_{-J}) \\ &= V_{-J}(x,y) \oplus [(V_{-J} \otimes W_{-J}) \oplus (W_{-J} \otimes V_{-J}) \oplus (W_{-J} \otimes W_{-J}). \end{aligned}$$

Hence

$$W_{-J}(x,y) = (V_{-J} \otimes W_{-J}) \oplus (W_{-J} \otimes V_{-J}) \oplus (W_{-J} \otimes W_{-J}).$$

Since $\left\{\widetilde{\phi}_{-J,k}(x)\right\}$ for $k = 0,1,2,\cdots,2^J-1$ is a normal orthogonal bases of V_{-J} and $\left\{\widetilde{\psi}_{-J,k}(x)\right\}$ for $k = 0,1,2,\cdots,2^J-1$ is a normal orthogonal bases of $\widetilde{W}_{-J}$, we have that

$$\{\psi^1_{-J,k,m}(x,y), k,m = 0,1,2,\cdots,2^J-1\}$$
$$\{\psi^2_{-J,k,m}(x,y); k,m = 0,1,2,\cdots,2^J-1\},$$

and

$$(\psi^3_{-J,k,m}(x,y); k,m = 0,1,2,\cdots,2^J-1\}.$$

i.e., $\{\psi^{\varepsilon}_{-J,k,m}(x,y)\varepsilon = 1,2,3\ \ k,m = 0,1,2,\cdots,2^J-1\}$ is a normal orthogonal bases of $W_{-J}(x,y)$. Using again $\phi_{0,k,m}(x,y) = \widetilde{\phi}_{0,k}(x)\widetilde{\phi}_{0,m}(y),\ k,m = 0,1,2,\cdots,2^J-1\}$ is a normal orthogonal bases of $V_0(x,y)$, we have

$$\{\psi^{\varepsilon}_{-J,k,m}(x,y) : \varepsilon = 1,2,3,\ k,m = 0,1,2,\cdots,2^J-1; J \geq 0\} \cup \{\phi_{0,k,m}(x,y)\}$$

is a normal orthogonal bases of $H^1(Q)$ thanks to the corollary 1. □

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