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A Simple Thermodynamic Model of Radiation Fog Formation

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Abstract : A simple thermodynamic model for predicting fog at 0600 h local time at Don Muang Airport in the winter from surface observations at 0000 h (midnight) is presented. The basic parameters used are temperature and humidity of the surface air at 0000 h, and mean cloud amount for 0000 h to 0600 h. It is assumed that a layer of air with constant thickness at the surface loses heat by long-wave radiation, thereby falling in temperature and increasing its relative humidity. If the dew point is reached, water vapor then condenses, and the number of drops per unit volume with an assumed constant diameter (10 μm or 20 μm) is calculated. Estimates of the visibility are then calculated from the number of drops per unit volume. Different values of the basic parameters are used to investigate how they determine the number of drops formed. The results, found by numerical integration of the differential equations representing the thermal processes, are in reasonable order of magnitude agreement with actual meteorological observations. They give an indication of the likely depth of the surface layer under a temperature inversion formed by nocturnal radiation.

Keywords : Thermodynamic, visibility

1 Introduction

Fog at Don Muang Airport, Bangkok, sometimes reduces visibility, causing a hazard to aircraft, especially high speed fighter planes. The fog occurs mostly in the winter months December, January and February. It is usually caused by cooling of the surface air when there is a net loss of heat by long-wave radiation at night. The study reported in this paper was an attempt to calculate the visibility reduction by fog at dawn (0600 local time) using meteorological properties of the air at midnight (0000 local time) for predicting the formation of liquid fog droplets in a simple thermodynamic model. Previous work on the prediction of fog at Don Muang Airport includes studies by Thongphasuk [1], and Ruangjun and Exell [2].

Vinai Thongphasuk[1] studied the formation of fog over Don Muang Airport during the periods 9-11 February 1999 and 13-16 February 2000. His work found the conditions favourable for fog at sunrise were as follows:

- Pressure falling in a high pressure area over Thailand.
- An easterly wind above the surface with speed 5 knots.
- A clear sky during the night.
- Upper air soundings at 0700 local standard time (LST) at Bang Na about 30 km south of Don Muang showed a temperature inversion near the surface and stable air above.
- Dew point at the surface was almost equal to the air temperature giving a relative humidity almost 100 %.

The minimum visibility in thick fog was 50 meters at 0600 LST and 150 meters at 0630 LST.

Sathaporn Ruangjun and Exell [2] studied the method of predicting visibility over Don Muang Air Force Base at 0700 LST from meteorological observations at 0100 and 0500 LST using multiple linear regression. The observational data were received from the Royal Thai Air Force (RTAF) in January, February and December 2001-2003 and from the Thai Meteorological Department (TMD) in January, February and December 1999-2003. The multiple linear regression gave statistical models at 0100 and 0500 LST for predicting the visibility at 0700 LST. In each case, two models were found: one containing all the available independent variables, and one omitting the insignificant variables using the step-by-step method. The results from both the RTAF and TMD showed that the forecasting models using data at 0500 LST were better than those using data at 0100 LST and the TMD models were more accurate than the RTAF models.

Another paper on a radiation fog formation model has been published by Meyer and Rao[3]. The model forecast the diurnal variation of dry bulb temperature (T) and dew point temperature (T_d) to deduce the onset of radiation fog as the dew point depression fell to less than $1°C$. The model computed radiative cooling and turbulent diffusion of heat and vapor through the lower boundary layer to produce heat and vapor fluxes at the soil-atmosphere interface. The results of Meyer and Rao are of interest in connection with the models for fog at Don Muang, but the method is beyond the scope of the present paper. A further study of models for the formation of fog at Don Muang using turbulent transport in the boundary layer is planned.

2 Methods

2.1 Model Assumptions

The model used for the calculations reported in this paper is based on the following simplifying assumptions:

- The surface layer of the atmosphere is homogeneous and of constant height h, capped by a temperature inversion. The properties of the air are given per unit volume, or per unit horizontal area in the layer.
- The surface layer is a homogeneous mixture of dry air, water vapour, and liquid droplets. If the water vapour pressure is less than the saturation vapour pressure, there are no liquid droplets.
- If liquid droplets exist, they have a constant radius, which is assumed to be 5 μm or 10 μm.
- The surface layer is cooled by a net long-wave radiation heat loss during the period midnight to dawn. (The ground is cooled by radiation and this cooling is transferred to the air.)
- Liquid water on the ground is ignored: it is assumed that there is no evaporation from the surface, and no condensation on the surface as dew.
- Total pressure P is an external parameter determined by the weight of the air above, not by the properties of the air in the layer. This paper assumes that $P = 101,000\ Pa$. The partial pressure of the dry air P_d is the difference between the total pressure P and the vapour pressure e.
- The temperature T is uniform through the layer.

These assumptions are appropriate when there is enough movement of the air in the surface layer to maintain homogeneity, but not enough movement to break the surface layer up.

2.2 Model Equations

2.2.1 Thermodynamics

The surface air cools by radiation heat loss at night and its temperature T decreases gradually. If the air becomes saturated, the water vapor in the air condenses on small particles in the air to form liquid fog droplets. At this time, the model start calculating the number of water droplets (n_L) per unit volume of air by using the difference between the vapor pressure e and the saturation vapor pressure $e_s(T)$.

The vapor pressure e can be calculated from

$$e = \frac{RH \times e_s(T)}{100}, \tag{2.1}$$

where RH is the relative humidity (%) and the saturation vapour pressure $e_s(T)$ can be calculated from the equation

$$e_s(T) = 611.2exp[\frac{17.67(T)}{T + 243.5}], \tag{2.2}$$

where T is in degrees C.

The model calculates the change of temperature T at the surface every 10 minutes from the total energy transfer equation in the surface layer per unit area of the surface, which is the sum of the sensible heating and the latent heating rate :

$$\frac{dQ}{dt} = M_d C_{pd} \frac{dT}{dt} + M_v C_{pv} \frac{dT}{dt} + M_L C_L \frac{dT}{dt} + L(T) \frac{dM_v}{dt}. \tag{2.3}$$

where

T is the surface temperature in Kelvin,
M_d is the mass of dry air per unit area in the surface layer,

given by

$$M_d = \rho_d h = \frac{(P - e)h}{R_d h}, \tag{2.4}$$

ρ_d is the density of dry air,
R_d is the gas constant for dry air (287 J/kgK),
M_v is the mass of water vapor per unit area in the surface layer,

given by

$$M_v = \frac{eh}{R_v T}, \tag{2.5}$$

where

R_v is the gas constant of water vapour (461.5 J/kgK),
M_L is the mass of liquid water per unit area in the surface layer,
C_{pd} is the specific heat of dry air at constant pressure (1,005 J/kgK),
C_{pv} is the specific heat of water vapour at constant pressure (1,870 J/kgK),
C_L is the specific heat of liquid water (4,187 J/kgK),
$\frac{dT}{dt}$ is the rate of change of temperature in the surface layer,
$\frac{dQ}{dt}$ is the net rate of energy of the surface layer of air per unit area of surface,
$L(T)$ is the latent heat of vaporization of water,

given by

$$L(T) = [3,150 - 2.375(T)] \times 1,000 \quad J/kg, \tag{2.6}$$

where T is in degrees C.

When air is saturated, there is a phase change from water vapor to liquid water droplets as the air cools. The mass of liquid water per unit area M_L can be calculated from the mass transfer in the condensation process, namely

$$dM_L = \frac{-h de_s(T)}{R_v T}. \tag{2.7}$$

Let m_L be the mass of liquid water per unit volume of air, r_L be the liquid drop radius, and ρ_L be the density of water. For the volume of air with height h, the number n_L of water droplets per unit volume of air can be calculated from

$$M_L = m_L h = \rho_L V n_L h = (\frac{4}{3}\pi {r_L}^3)\rho_L n_L h. \quad (2.8)$$

while the air is not saturated, the terms $M_L C_L \frac{dT}{dt}$ and $L(T)\frac{dM_v}{dt}$ in (3) are ignored.

2.2.2 Radiation

The net rate of change of energy of the surface layer of air per unit area of surface $\frac{dQ}{dt}$, is derived from the net radiation at night. The calculation of downward long wave radiation in the presence of cloud is from Exell [4] and the calculation of upward long wave radiation is from the Stefan-Boltzmann law, so the net radiation can be expressed as

$$\frac{dQ}{dt} = (0.8n_c - 1)\sigma T^4 0.261 exp(-b(T - 273)^2) + 10 \quad (2.9)$$

where

σ	is the Stefan-Boltzmann constant ($5.67 \times 10^{-8}\ W/m^2K^4$),
T	is the absolute screen level temperature (in kelvin),
n_c	is cloud amount,
b	$= -0.000777(^\circ C)^{-2}$.

2.2.3 Visibility

The buildup of water droplets reduces visibility. Generally, fog is assumed to occur when visibility is less than 1,000 m. This paper studies the simple calculation of visibility and explains the relation between the visibility and the concentration of water droplets in the air. The calculation is developed following the formulation by Jacobson [5].

The following are the definitions of the important words in the equation:

Visibility is the furthest distance at which an observer can discern the outline of an object.
Contrast ratio, C_{ratio} is the lowest visually perceptible brightness contrast a person can see which is defined by the difference between the background intensity and the intensity in the viewer's line of sight, all relative to the background intensity. Normally, 0.02 has become an accepted minimal (lowest) contrast value for visibility calculations.
Extinction coefficient is a parameter that measures the loss of electromagnetic radiation due to some process per unit distance, and may be determined as the product of an effective cross section and a number concentration.

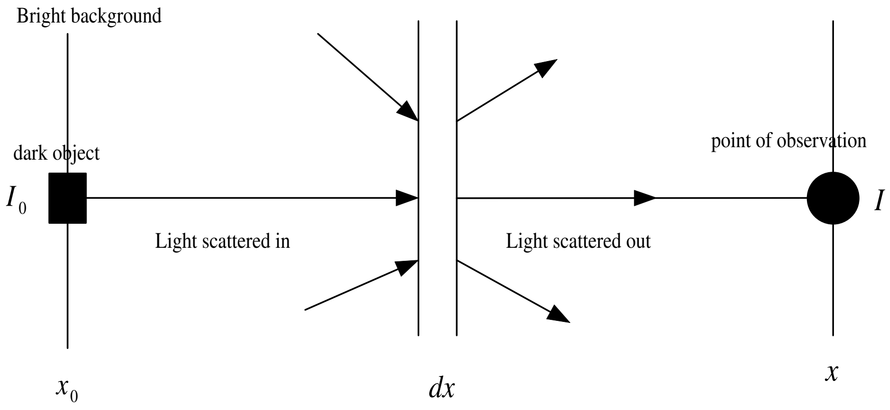

Figure 1: Light beam from a dark object against a bright back ground with scattering into and out of the beam.

Let

I_0	be the intensity of radiation originating from dark object at point x_0
I	be the intensity of radiation at the point of observation x,
r	be the radius of droplets,
n	be the concentration of droplets, and,
k	be the extinction coefficient due to absorbtion and scattering by droplets.

Suppose a perfectly absorbing dark object lies against a white background at a point x_0, and over a distance dx the layer is covered by drops as shown in Fig. 1. Because object is perfectly absorbing so $I_0 = 0$.

The intensity of radiation increases due to scattering of background light into the beam since water droplets scatter all wavelengths of visible light but absorb only a little light. In both cases, the added intensity is scattered out of, or absorbed in, the layer by water droplets. At point x , the net radiation of the beam has increased close to that of the background intensity. The visibility calculation can be derived from the equation for the change in object intensity along the path described in Fig. 1. The equation is

$$\frac{dI}{dx} = k(I_0 - I), \tag{2.10}$$

where k is the total extinction coefficient, kI_0 accounts for the scattering of background light radiation into the path, and $-kI$ accounts for the attenuation of radiation along the path due to scattering out of the path and absorption along the path.

Since k is the product of an effective across section of drops and a number concentration of drops, $k = \pi r^2 n$,
(10) becomes

$$\frac{dI}{I_0 - I} = \pi r^2 n dx; \tag{2.11}$$

integrating (11) from $I_0 = 0$ to I and $x_0 = 0$ to x gives

$$\frac{I_0 - I}{I_0} = e^{-\pi r^2 nx}. \tag{2.12}$$

Thus we see that is the visibility (as definition above) and by definition, $\frac{I_0 - I}{I_0}$ is the contrast ratio C_{ratio} . We choose $C_{ratio} = 0.02$ for calculating the visibility here, so (12) becomes

$$0.02 = e^{-\pi r^2 nx}. \tag{2.13}$$

3 The results of the thermodynamics radiation fog formation model.

The initial data for input in the model are surface temperature, relative humidity, cloud amount, and height of inversion in the layer. For the purpose of the model calculation, the height is assumed to have the values 100 m, 500m, 1,000m, and 2,000 m. The accuracy of this model was tested against actual meteorological observations and the heights giving the best results were found using, surface meteorological data from Don Muang Air Force Base [6] in the period from December 2000 - February 2001 at 0000 - 0600 LST. Fog was observed to occur on 11 January, 2 February, 13 February, 14 February, 15 February, and 20 February. The results after running the model are divided into two cases of water drops radius, 5 μm and 10 μm and this paper shows the results of some days that fog was formed and fog was not formed.

In the following tables:

T_0 is temperature at midnight (Celsius),
T_6 is temperature at 0600 LST (Celsius),
n_c is cloud amount,
RH_0 is relative humidity at midnight (%) ,
h is height of inversion base in the layer (m),
n_L is the number of droplets per unit volume of air ($million/m^3$),
VIS_6 is the visibility at 0600 LST (m),

3.1 Case fog forms when radius of droplets $= 10\mu m$

1. 11 January 2001

Day	Observation					Calculation			
11/1/01	T_0 ($^\circ C$)	RH_0 (%)	n_c	T_6 ($^\circ C$)	VIS_6 (m)	h (m)	T_6 ($^\circ C$)	n_L ($million/m^3$)	VIS_6 (m)
	26.4	94	0.34	24.2	8,000	100	25.35	14,890	0.83
						500	25.35	12,300	1.01
						1000	25.35	9,067	1.36
						2000	25.35	2,502	4.97

2. 2 February 2001

Day	Observation					Calculation			
02/2/01	T_0 ($^\circ C$)	RH_0 (%)	n_c	T_6 ($^\circ C$)	VIS_6 (m)	h (m)	T_6 ($^\circ C$)	n_L ($million/m^3$)	VIS_6 (m)
	26.2	92	0.12	22.00	2,000	100	24.79	14,230	0.87
						500	24.79	13,520	0.92
						1000	24.79	8,115	1.53
						2000	24.79	1,292	9.64

3. 13 February 2001

Day	Observation					Calculation			
13/2/01	T_0 ($^\circ C$)	RH_0 (%)	n_c	T_6 ($^\circ C$)	VIS_6 (m)	h (m)	T_6 ($^\circ C$)	n_L ($million/m^3$)	VIS_6 (m)
	28.5	67	0	24.70	1,000	100	21.78	9,457	1.32
						500	21.82	0	∞
						1000	24.83	0	∞
						2000	26.56	0	∞

4. 14 February 2001

Day	Observation					Calculation			
14/2/01	T_0 ($^\circ C$)	RH_0 (%)	n_c	T_6 ($^\circ C$)	VIS_6 (m)	h (m)	T_6 ($^\circ C$)	n_L ($million/m^3$)	VIS_6 (m)
	27.0	79	0	25.1	900	100	23.04	9,457	1.32
						500	23.04	0	∞
						1000	23.62	0	∞
						2000	25.23	0	∞

5. 15 February 2001

Day	Observation					Calculation			
15/2/01	T_0 $(°C)$	RH_0 $(\%)$	n_c	T_6 $(°C)$	VIS_6 (m)	h (m)	T_6 $(°C)$	n_L $(million/m^3)$	VIS_6 (m)
	25.7	86	0.25	22.6	4,000	100	23.18	11,891	1.04
						500	23.18	11,891	1.04
						1000	23.18	83	149.5
						2000	24.38	0	∞

6. 20 February 2001

Day	Observation					Calculation			
20/2/01	T_0 $(°C)$	RH_0 $(\%)$	n_c	T_6 $(°C)$	VIS_6 (m)	h (m)	T_6 $(°C)$	n_L $(million/m^3)$	VIS_6 (m)
	26.2	91	0.12	23.0	1,000	100	24.61	13,900	0.89
						500	24.61	10,900	1.14
						1000	24.61	7,080	1.76
						2000	24.67	0	∞

3.2 Case fog forms when radius of droplets $= 5\mu m$

1. 11 January 2001

Day	Observation					Calculation			
11/1/01	T_0 $(°C)$	RH_0 $(\%)$	n_c	T_6 $(°C)$	VIS_6 (m)	h (m)	T_6 $(°C)$	n_L $(million/m^3)$	VIS_6 (m)
	26.4	94	0.34	24.2	8,000	100	25.35	119,000	0.42
						500	25.35	98,600	0.50
						1000	25.35	72,500	0.68
						1000	25.35	20,020	2.49

2. 2 February 2001

Day	Observation					Calculation			
02/2/01	T_0 $(°C)$	RH_0 $(\%)$	n_c	T_6 $(°C)$	VIS_6 (m)	h (m)	T_6 $(°C)$	n_L $(million/m^3)$	VIS_6 (m)
	26.2	92	0.12	22.00	2,000	100	24.79	113,800	0.44
						500	24.79	921,700	0.05
						1000	24.79	64,920	0.77
						2000	24.79	10,340	4.82

3. 13 February 2001

Day	Observation					Calculation			
13/2/01	T_0 $(°C)$	RH_0 $(\%)$	n_c	T_6 $(°C)$	VIS_6 (m)	h (m)	T_6 $(°C)$	n_L $(million/m^3)$	VIS_6 (m)
	28.5	67	0	24.70	1,000	100	21.78	75,660	0.66
						500	21.82	0	∞
						1000	24.83	0	∞
						2000	26.56	0	∞

4. 14 February 2001

Day	Observation					Calculation			
14/2/01	T_0 ($^\circ C$)	RH_0 (%)	n_c	T_6 ($^\circ C$)	VIS_6 (m)	h (m)	T_6 ($^\circ C$)	n_L ($million/m^3$)	VIS_6 (m)
	27.0	79	0	25.1	900	100	23.04	92,105	0.54
						500	23.04	42,450	1.17
						1000	23.62	0	∞
						2000	25.23	0	∞

5. 15 February 2001

Day	Observation					Calculation			
15/2/01	T_0 ($^\circ C$)	RH_0 (%)	n_c	T_6 ($^\circ C$)	VIS_6 (m)	h (m)	T_6 ($^\circ C$)	n_L ($million/m^3$)	VIS_6 (m)
	25.7	86	0.25	22.6	4,000	100	23.18	95,133	0.52
						500	23.18	95,133	0.52
						1000	23.18	666	74.78
						2000	24.38	0	∞

6. 20 February 2001

Day	Observation					Calculation			
20/2/01	T_0 ($^\circ C$)	RH_0 (%)	n_c	T_6 ($^\circ C$)	VIS_6 (m)	h (m)	T_6 ($^\circ C$)	n_L ($million/m^3$)	VIS_6 (m)
	26.2	91	0.12	23.0	1,000	100	24.61	111,500	0.44
						500	24.61	87,280	0.57
						1000	24.61	56,670	0.87
						2000	24.67	0	∞

3.3 Case fog was not formed when radius of droplets = $10\mu m$

1. 3 December 2000

Day	Observation					Calculation			
03/12/00	T_0 ($^\circ C$)	RH_0 (%)	n_c	T_6 ($^\circ C$)	VIS_6 (m)	h (m)	T_6 ($^\circ C$)	n_L ($million/m^3$)	VIS_6 (m)
	27.6	86	0.53	24.4	10,000	100	25.04	13,200	0.94
						500	25.04	5,440	2.29
						1000	25.57	0	∞
						2000	26.55	0	∞

2. 27 December 2000

Day	Observation					Calculation			
27/12/00	T_0 ($^\circ C$)	RH_0 (%)	n_c	T_6 ($^\circ C$)	VIS_6 (m)	h (m)	T_6 ($^\circ C$)	n_L ($million/m^3$)	VIS_6 (m)
	22.4	85	0	19.8	5,000	100	29.75	9,041	1.37
						500	19.75	5,072	2.54
						1000	19.75	94	131.54
						2000	21.02	0	∞

3. 4 January 2001

Day	Observation					Calculation			
04/01/01	T_0 $(°C)$	RH_0 $(\%)$	n_c	T_6 $(°C)$	VIS_6 (m)	h (m)	T_6 $(°C)$	n_L $(million/m^3)$	VIS_6 (m)
	23.4	82	0.12	21.0	3,000	100	20.15	9,024	1.38
						500	20.15	3,604	3.45
						1000	20.86	0	∞
						2000	20.08	0	∞

4. 9 February 2001

Day	Observation					Calculation			
09/02/01	T_0 $(°C)$	RH_0 $(\%)$	n_c	T_6 $(°C)$	VIS_6 (m)	h (m)	T_6 $(°C)$	n_L $(million/m^3)$	VIS_6 (m)
	23.5	80	0	21.1	6,000	100	19.85	8,618	1.44
						500	19.85	2,658	4.68
						1000	20.95	0	∞
						2000	22.18	0	∞

3.4 Case fog was not formed when radius of droplets $= 5\mu m$

1. 3 December 2000

Day	Observation					Calculation			
03/12/00	T_0 $(°C)$	RH_0 $(\%)$	n_c	T_6 $(°C)$	VIS_6 (m)	h (m)	T_6 $(°C)$	n_L $(million/m^3)$	VIS_6 (m)
	27.6	86	0.53	24.4	10,000	100	25.04	105,922	0.47
						500	25.04	43,537	1.14
						1000	25.57	56,670	0.87
						2000	26.55	0	∞

2. 27 December 2000

Day	Observation					Calculation			
27/12/00	T_0 $(°C)$	RH_0 $(\%)$	n_c	T_6 $(°C)$	VIS_6 (m)	h (m)	T_6 $(°C)$	n_L $(million/m^3)$	VIS_6 (m)
	22.4	85	0	19.8	5,000	100	19.75	72,330	0.68
						500	19.75	40,570	1.22
						1000	19.75	750	65.77
						2000	21.02	0	∞

3. 4 January 2001

Day	Observation					Calculation			
04/01/01	T_0 $(°C)$	RH_0 $(\%)$	n_c	T_6 $(°C)$	VIS_6 (m)	h (m)	T_6 $(°C)$	n_L $(million/m^3)$	VIS_6 (m)
	23.4	82	0.12	21.0	3,000	100	20.15	72,192	0.69
						500	20.15	28,837	1.73
						1000	20.86	0	∞
						2000	20.08	0	∞

4. 9 February 2001

Day	Observation					Calculation			
09/02/01	T_0 ($°C$)	RH_0 (%)	n_c	T_6 ($°C$)	VIS_6 (m)	h (m)	T_6 ($°C$)	n_L ($million/m^3$)	VIS_6 (m)
	23.5	80	0	21.1	6,000	100	19.85	68,950	0.72
						500	19.85	21,270	2.34
						1000	20.95	0	∞
						2000	22.18	0	∞

4 Conclusion.

Fog is defined as a restriction of the surface visibility generally to less than 1,000 m. McIntosh and Thom [10] noticed that the typical cloud contains about 10^9 water droplets per cubic meter, the radii ranging from about 1 to 20 or 30 μm, the average being about 10 μm. For droplets of radii 10 and 20 μm the terminal velocities are about 1 and 5 cms^{-1}, respectively, in still air. Therefore, droplets of the size at the Earth's surface effectively remain suspended in the air as fog. From the formulation of the relation between the concentration of water droplets in the air and the visibility value which is given in equation (13), if we suppose fog occurs and choose drop radius is 10 μm and use the concentration of water droplets 10^9 drops per cubic meter, the visibility should be less than or equal to 12.45 meters. In the case of choosing droplets of radii 10 μm, the visibility should be less than or equal to 49.83 meters. The results after running the model by using initial data from the days that reported fog occurred, give the most of all cases of choosing drop radius 10 μm and drop radius 5 μm, produced water droplets exceed than 10^9 droplets per cubic meter. The temperature prediction in two cases are the same for all heights and they are not much higher than observation data. This shows that the size of water droplets does not effect the temperature prediction but it effects the concentration of water droplets. We see that smaller droplets gave the concentration of water droplets in the air greater than the bigger droplets and the concentration of small size droplets is eight times of the concentration of a big size. The concentration of droplets in the case of 10 μm and 5 μm are not in agreement with the observation data when it was reported that fog was not formed. Although the visibility of two cases are accordant to the definition of fog above that is less than the assumptions but the values are much lower than the observation data. The error in the model result occurs because this model does not consider the other parameters in the boundary layer such as wind speed, turbulent diffusion of heat and vapor, and evaporation from soil. These conditions have effect the occurrence of fog. Further development of the fog model should include these conditions.

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