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Fixed Point Theorems for Generalized R-Contraction in b-Metric Spaces

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Abstract : In 2015, Roldán López de Hierro and Shahzad introduced the notion of R-function which contains simulation function and manageable function. They introduced R-contraction with respect to R-function. Following this line of research, we introduce a generalization of R-contraction in b-metric spaces and prove some fixed point theorems for such contraction in b-metric spaces. Our results extend and improve several well-known comparable results. Finally, we give examples to support our results.

Keywords : b-metric space; R-function; R-contraction; simulation function; fixed point.
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1 Introduction

The well-known Banach contraction principle assures the existence and uniqueness of fixed points of certain self-maps in metric spaces. This principle can be

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applied in various fields such as engineering, economics, computer science. Because of its wide applications, several researchers have extended, improved and generalized the result in many directions.

In 2015, Khojasteh et al. [4] introduced the notion of $\mathcal{Z}$-contraction defined by simulation function. Then, Khojasteh et al. proved a new fixed point theorem concerning $\mathcal{Z}$-contraction which generalizes Banach's contraction principle. Recently, Roldán López de Hierro and Shahzad [13] introduced the concept of R-contraction defined by R-function in order to generalize the previous results.

On the other hand, Bakhtin [1] and Czerwik [2] developed the notion of b-metric space and established some fixed point theorems in b-metric spaces. Subsequently, several results appeared in this direction [9, 14, 6, 5, 8, 10]. Recently, Mongkolkeha et al. [7] introduced the notion of a simulation function in the setting of b-metric spaces.

In this work, we define a generalization of R-contraction in b-metric spaces, called R'-contraction, via R'-function and prove the existence and uniqueness of a fixed point for such class of mappings in complete b-metric spaces. Furthermore, we provide examples to support our results.

2 Preliminaries

In this section, we recollect some basic definitions, notations and results which are needed in continuance.

In 1993, Czerwik [2] introduced a b-metric space shown below:

Definition 2.1. *[2] A b-metric on a set X is a mapping $d : X \times X \to [0, +\infty)$ satisfying the following conditions: for any $x, y, z \in X$,*

(b_1) *$d(x, y) = 0$ if and only if $x = y$;*

(b_2) *$d(x, y) = d(y, x)$;*

(b_3) *there exists $K \geq 1$ such that $d(x, y) \leq K(d(x, z) + d(z, y))$.*

Then (X, d) is known as a b-metric space with coefficient K.

Note that every metric space is a b-metric space with $K = 1$. Some examples of b-metric space are given below:

Example 2.2.

1. *Let $X = \mathbb{R}$. Define a mapping $d : X \times X \to [0, \infty)$ by*

$$d(x, y) = (x - y)^2 \quad \text{for all } x, y \in X.$$

   Then (X, d) is a b-metric space with coefficient $K = 2$.

2. *Let $X = \{1, 2, 3\}$. Define a mapping $d : X \times X \to [0, \infty)$ by $d(1, 1) = d(2, 2) = d(3, 3) = 0$, $d(1, 2) = d(2, 1) = 2$, $d(2, 3) = d(3, 2) = 1$ and $d(1, 3) = d(3, 1) = 6$. Then (X, d) is a b-metric space with coefficient $K = 2$.*

In 2015, Khojasteh et al. [4] introduced a simulation function shown below:

Definition 2.3. *[4] A simulation function is a mapping $\zeta : [0,\infty) \times [0,\infty) \to \mathbb{R}$ satisfying the following conditions:*

(ζ_1) $\zeta(0,0) = 0$;

(ζ_2) $\zeta(t,s) < s - t$, *for all* $t, s > 0$;

(ζ_3) *if* $\{t_n\}, \{s_n\}$ *are sequences in* $[0,\infty)$ *such that* $\lim_{n\to\infty} t_n = \lim_{n\to\infty} s_n > 0$, *then*

$$\limsup_{n\to\infty} \zeta(t_n, s_n) < 0.$$

The class of all simulation functions $\zeta : [0,\infty) \times [0,\infty) \to \mathbb{R}$ *is denoted by* $\mathcal{Z}$.

The following are examples of simulation functions given by Khojasteh [4].

Example 2.4.

1. *Let* $\lambda \in \mathbb{R}$ *be such that* $\lambda < 1$ *and define a mapping* $\zeta : [0,\infty) \times [0,\infty) \to \mathbb{R}$ *by*
$$\zeta(t,s) = \lambda s - t \quad \text{for all } s, t \in [0,\infty).$$
Then $\zeta \in \mathcal{Z}$.

2. *Define a mapping* $\zeta : [0,\infty) \times [0,\infty) \to \mathbb{R}$ *by* $\zeta(t,s) = \psi(s) - \phi(t)$ *for all* $t, s \in [0,\infty)$, *where* $\psi, \phi : [0,\infty) \to [0,\infty)$ *are two continuous functions such that* $\psi(t) = \phi(t) = 0$ *if and only if* $t = 0$ *and* $\psi(t) < t \leq \phi(t)$ *for all* $t > 0$, *then* $\zeta \in \mathcal{Z}$.

In 2015, Roldán López de Hierro and Shahzad [13] introduced R-function and R-contraction shown below:

Definition 2.5. *[13] Let* $A \subseteq \mathbb{R}$ *be a nonempty subset. A function* $\varrho : A \times A \to \mathbb{R}$ *is called R-function if it satisfies the following two conditions:*

(ϱ_1) *If* $\{a_n\} \subset (0,\infty) \cap A$ *is a sequence such that* $\varrho(a_{n+1}, a_n) > 0$ *for all* $n \in \mathbb{N}$, *then* $\{a_n\} \to 0$.

(ϱ_2) *If* $\{a_n\}, \{b_n\} \subset (0,\infty) \cap A$ *are two sequences converging to the same limit* $L \geq 0$ *and verifying that* $L < a_n$ *and* $\varrho(a_n, b_n) > 0$ *for all* $n \in \mathbb{N}$, *then* $L = 0$.

The class of all R-functions $\varrho : A \times A \to \mathbb{R}$ is denoted by R_A. They also consider the following property.

(ϱ_3) If $\{a_n\}, \{b_n\} \subset (0,\infty) \cap A$ are two sequences such that $\{b_n\} \to 0$ and $\varrho(a_n, b_n) > 0$ for all $n \in \mathbb{N}$, then $\{a_n\} \to 0$.

In [13], the authors showed that every simulation function is an $R-$function that satisfies (ϱ_3) but the converse is not true.

Definition 2.6. *[13] Let (X, d) be a metric space. A mapping $T : X \to X$ is called R-contraction if there exists an R-function $\varrho : A \times A \to \mathbb{R}$ such that ran(d)$\subseteq A$ and*

$$\varrho(d(Tx, Ty), d(x, y)) > 0 \text{ for all } x, y \in X \text{ such that } x \neq y.$$

Notice that if we take $\varrho(t, s) = \lambda s - t$ for all $s, t \geq 0$ and $\lambda \in [0, 1)$ in Definition 2.6, then R-contraction becomes the Banach contraction.

In 2017, Mongkolkeha et al. [7] introduced a simulation function in the framework of b-metric spaces shown below:

Definition 2.7. *[7] Let K be a given real number such that $K \geq 1$. A K-simulation function is a mapping $\zeta : [0, \infty) \times [0, \infty) \to \mathbb{R}$ satisfying the following conditions:*

(ζ_1') $\zeta(0, 0) = 0$;

(ζ_2') $\zeta(Kt, s) \leq s - Kt$, *for all* $t, s > 0$;

(ζ_3') *if* $\{t_n\}, \{s_n\}$ *are sequences in* $[0, \infty)$ *such that* $\limsup_{n\to\infty} Kt_n = \limsup_{n\to\infty} s_n > 0$ *and* $t_n < s_n$ *for all* $n \in \mathbb{N}$, *then*

$$\limsup_{n\to\infty} \zeta(Kt_n, s_n) < 0.$$

The class of all K-simulation functions $\zeta : [0, \infty) \times [0, \infty) \to \mathbb{R}$ is denoted by $\mathcal{Z}^*$.

Example 2.8. *[7] Let $\lambda, K \in \mathbb{R}$ be such that $\lambda < 1$ and $K \geq 1$. Define the mapping $\zeta : [0, \infty) \times [0, \infty) \to \mathbb{R}$ by*

$$\zeta(Kt, s) = \begin{cases} s - Kt & \text{if } s < t, \\ \dfrac{\lambda s - Kt}{Ks + 1} & \text{otherwise.} \end{cases}$$

Then $\zeta \in \mathcal{Z}^*$ but $\zeta \notin \mathcal{Z}$.

3 Main Results

In this section, we introduce the new family of functions in the setting of b-metric spaces.

Definition 3.1. *Let K be a given real number such that $K \geq 1$. A function $\varrho : [0, \infty) \times [0, \infty) \to \mathbb{R}$ is called $R'-$function if it satisfies the following two conditions:*

(ϱ_1') *If $\{a_n\} \subset (0, \infty)$ is a sequence such that $\varrho(Ka_{n+1}, a_n) > 0$ for all $n \in \mathbb{N}$, then $\{a_n\} \to 0$.*

(ϱ_2') *If* $\{a_n\}, \{b_n\} \subset (0, \infty)$ *are two sequences such that* $\limsup_{n\to\infty} Ka_n = \limsup_{n\to\infty} b_n = L \geq 0$ *and verifying that* $L < Ka_n$ *and* $\varrho(Ka_n, b_n) > 0$ *for all* $n \in \mathbb{N}$, *then* $L = 0$.

The class of all R'-functions $\varrho : [0, \infty) \times [0, \infty) \to \mathbb{R}$ is denoted by R^*. We also consider the following property.

(ϱ_3') If $\{a_n\}, \{b_n\} \subset (0, \infty)$ are two sequences such that $\{b_n\} \to 0$ and $\varrho(Ka_n, b_n) > 0$ for all $n \in \mathbb{N}$, then $\{a_n\} \to 0$.

Lemma 3.2. *Every* K*-simulation function is a* R'*-function that also verifies* (ϱ_3').

Proof. Let K be a given real number such that $K \geq 1$ and $\varrho : [0, \infty) \times [0, \infty) \to \mathbb{R}$ be a K-simulation function.

(ϱ_1') Let $\{a_n\} \subset (0, \infty)$ be a sequence such that $\varrho(Ka_{n+1}, a_n) > 0$ for all $n \in \mathbb{N}$. By condition (ζ_2'),

$$0 < \varrho(Ka_{n+1}, a_n) \leq a_n - Ka_{n+1} \leq a_n - a_{n+1},$$

for all $n \in \mathbb{N}$. So $\{a_n\}$ is a strictly decreasing sequence of positive real numbers. Then $\{a_n\}$ is convergent, given $L \geq 0$ such that $\{a_n\} \to L$. We will show that $L = 0$. By contradiction, assume $L > 0$. Let $t_n = \frac{a_{n+1}}{K}$ and $s_n = a_n$ for all $n \in \mathbb{N}$. By condition (ζ_3'),

$$0 \leq \limsup_{n\to\infty} \varrho(a_{n+1}, a_n) = \limsup_{n\to\infty} \varrho(Kt_n, s_n) < 0,$$

which is a contradiction. Therefore $\{a_n\} \to 0$.

(ϱ_2') Let $\{a_n\}, \{b_n\} \subset (0, \infty)$ be sequences such that $\limsup_{n\to\infty} Ka_n = \limsup_{n\to\infty} b_n = L \geq 0$ and satisfying that $L < Ka_n$ and $\varrho(Ka_n, b_n) > 0$ for all $n \in \mathbb{N}$. We will show that $L = 0$. By contradiction, assume $L > 0$. By condition (ζ_2'), $0 < \varrho(Ka_n, b_n) \leq b_n - Ka_n$. Then

$$a_n \leq Ka_n < b_n \text{ for all } n \in \mathbb{N}.$$

By condition (ζ_3'), $0 \leq \limsup_{n\to\infty} \varrho(Ka_n, b_n) < 0$, which is a contradiction. Therefore $L = 0$.

(ϱ_3') Let $\{a_n\}, \{b_n\} \subset (0, \infty)$ such that $\{b_n\} \to 0$ and $\varrho(Ka_n, b_n) > 0$ for all $n \in \mathbb{N}$. Since ϱ is a K-simulation function, $0 < \varrho(Ka_n, b_n) \leq b_n - Ka_n$ for all $n \in \mathbb{N}$. Hence $0 < Ka_n < b_n$ for all $n \in \mathbb{N}$, this implies that, $\{Ka_n\} \to 0$. Since $K \geq 1$, $\{a_n\} \to 0$.

□

Now, we use R'-function to define a new class of contractions in b-metric spaces.

Definition 3.3. *Let (X,d) be a b-metric space with coefficient $K \geq 1$ and let $T : X \to X$ be a mapping. We will say that T is a R'-contraction if there exists a R'-function $\varrho : [0,\infty) \times [0,\infty) \to \mathbb{R}$ such that*

$$\varrho(Kd(Tx,Ty), d(x,y)) > 0 \text{ for all } x,y \in X \text{ such that } x \neq y.$$

Now, we are ready to give the main theorem.

Theorem 3.4. *Let (X,d) be a complete b-metric space with coefficient $K \geq 1$. Let $T : X \to X$ be R'-contraction with respect $\varrho \in R^*$. If $\varrho(Kt,s) \leq s - Kt$ for all $s,t \in (0,\infty)$ then T has a unique fixed point.*

Proof. Let $x_0 \in X$ be a arbitrary point. Let $\{x_n\}$ be Picard sequence of T based on x_0, that is, $x_{n+1} = Tx_n$. If there exists $n_0 \in \mathbb{N}$ such that $x_{n_0+1} = x_{n_0}$, then $Tx_{n_0} = x_{n_0}$ which implies that x_{n_0} is a fixed point. Assume $x_n \neq x_{n+1}$ for all $n \in \mathbb{N}$. Let $\{a_n\} \subset (0,\infty)$ be a sequence defined by $a_n = d(x_n, x_{n+1}) > 0$ for all $n \in \mathbb{N}$. By R'-contraction,

$$\begin{aligned}\varrho(Ka_{n+1}, a_n) &= \varrho(Kd(x_{n+1}, x_{n+2}), d(x_n, x_{n+1}))\\ &= \varrho(Kd(Tx_n, Tx_{n+1}), d(x_n, x_{n+1}))\\ &> 0.\end{aligned}$$

From the condition (ϱ_1'),

$$\lim_{n\to\infty} d(x_n, x_{n+1}) = \lim_{n\to\infty} a_n = 0.$$

Next, we show that $\{x_n\}$ is a Cauchy sequence reasoning by contradiction. If $\{x_n\}$ is not a Cauchy sequence, then there exists $\varepsilon_0 > 0$ such that

$$d(x_{n_k}, x_{m_k}) > \varepsilon_0 \text{ and } d(x_{n_k}, x_{m_k-1}) \leq \varepsilon_0 \text{ for all } m_k > n_k \geq k. \tag{3.1}$$

Consider

$$\varepsilon_0 < d(x_{n_k}, x_{m_k}) \leq K(d(x_{n_k}, x_{m_k-1}) + d(x_{m_k-1}, x_{m_k})) \quad \text{for all } k \in \mathbb{N}.$$

Taking limit superior k to infinity,

$$\varepsilon_0 \leq \limsup_{k\to\infty} d(x_{n_k}, x_{m_k}) \leq K\varepsilon_0. \tag{3.2}$$

Since $d(x_{n_k-1}, x_{m_k-1}) \leq K(d(x_{n_k-1}, x_{n_k}) + d(x_{n_k}, x_{m_k-1}))$ for all $k \in \mathbb{N}$, taking limit superior k to infinity,

$$\limsup_{k\to\infty} d(x_{n_k-1}, x_{m_k-1}) \leq K\varepsilon_0. \tag{3.3}$$

If $d(x_{n_{k_0}-1}, x_{m_{k_0}-1}) = 0$, for some $k_0 \in \mathbb{N}$ then $x_{n_{k_0}} = x_{m_{k_0}}$, which contradict to (3.1). Therefore $x_{n_k-1} \neq x_{m_k-1}$ for all $k \in \mathbb{N}$. By R'-contraction,

$$0 < \varrho(Kd(x_{n_k}, x_{m_k}), d(x_{n_k-1}, x_{m_k-1})) \leq d(x_{n_k-1}, x_{m_k-1}) - Kd(x_{n_k}, x_{m_k}).$$

This implies that

$$Kd(x_{n_k}, x_{m_k}) < d(x_{n_k-1}, x_{m_k-1}) \quad \text{for all } k \in \mathbb{N}. \tag{3.4}$$

By (3.2), (3.3) and (3.4).

$$K\varepsilon_0 \leq \limsup_{k\to\infty} Kd(x_{n_k}, x_{m_k}) \leq \limsup_{k\to\infty} d(x_{n_k-1}, x_{m_k-1}) \leq K\varepsilon_0.$$

That is

$$\limsup_{k\to\infty} Kd(x_{n_k}, x_{m_k}) = \limsup_{k\to\infty} d(x_{n_k-1}, x_{m_k-1}) = K\varepsilon_0.$$

Since $K\varepsilon_0 < Kd(x_{n_k}, x_{m_k})$, for all $k \in \mathbb{N}$ and the condition (ϱ_2'), $K\varepsilon_0 = 0$. That is a contradiction. Thus $\{x_n\}$ is a Cauchy sequence. Since (X, d) is complete, there exists $z \in X$ such that $\{x_n\} \to z$.

Next, we will show that z is fixed point reasoning by contradiction. If z is not a fixed point, that is $z \neq Tz$. Let $\varepsilon = \frac{d(z,Tz)}{2} > 0$. Since $\{x_n\} \to z$,

$$\text{there exists } N \text{ such that } d(x_n, z) < \varepsilon \text{ for all } n > N. \tag{3.5}$$

Let $\Omega = \{n \in \mathbb{N} : d(x_n, z) = 0\}$. Assume that Ω is not finite, then we can find $n_0 > N$ such that $d(x_{n_0}, z) = 0$ i.e. $x_{n_0} = z$. By (3.5),

$$\varepsilon > d(x_{n_0+1}, z) = d(Tx_{n_0}, z) = d(Tz, z),$$

which is a contradiction. Therefore Ω is finite, there exists n_0 such that $d(x_n, z) > 0$ for all $n > n_0$. Since T is a R'-contraction,

$$0 < \varrho(Kd(Tx_n, Tz), d(x_n, z)) \leq d(x_n, z) - Kd(Tx_n, Tz), \text{ for all } n > n_0.$$

Hence,

$$Kd(Tx_n, Tz) < d(x_n, z), \text{ for all } n > n_0.$$

Taking limit n to infinity,

$$\lim_{n\to\infty} Kd(Tx_n, Tz) \leq \lim_{n\to\infty} d(x_n, z) = 0.$$

Thus $\lim_{n\to\infty} Kd(Tx_n, Tz) = 0$, that is, $\{x_{n+1} = Tx_n\} \to Tz$. By the uniqueness of the limit, $Tz = z$. Finally, let us show that z is unique fixed point of T. Assume $x = Tx$ and $y = Ty$ such that $x \neq y$. Let $a_n = d(x, y) > 0$ for all $n \in \mathbb{N}$. Consider

$$\varrho(Ka_{n+1}, a_n) = \varrho(Kd(x, y), d(x, y)) = \varrho(Kd(Tx, Ty), d(x, y)) > 0.$$

From (ϱ_1'), then $\{a_n\} \to 0$, which imply that $d(x, y) = 0$, which is a contradiction. So $x = y$. □

This following results are immediately true by our main result.

Corollary 3.5. *[4] Let (X, d) be a complete metric space and $T : X \to X$ be a $\mathcal{Z}$-contraction with respect to a certain simulation function ζ, that is,*

$$\zeta(d(Tx, Ty), d(x, y)) \geq 0, \quad \text{for all } x, y \in X.$$

Then T has a unique fixed point. Moreover, for every $x_0 \in X$, the Picard sequence $\{T^n x_0\}$ converges to this fixed point.

Corollary 3.6. *[3] Let (X, d) be a complete b-metric space and let $T : X \to X$ be a mapping. Suppose that there exists $\lambda \in (0, 1)$ such that*

$$d(Tx, Ty) \leq \lambda d(x, y) \text{ for all } x, y \in X.$$

Then T has a unique fixed point.

Corollary 3.7. *[12] Let (X, d) be a complete b-metric space and let $T : X \to X$ be a mapping. Suppose that there exists a lower semi-continuous function $\varphi : [0, \infty) \to [0, \infty)$ with $\varphi^{-1}(0) = 0$ such that*

$$d(Tx, Ty) \leq d(x, y) - \varphi(d(x, y)) \text{ for all } x, y \in X.$$

Then T has a unique fixed point.

Proof. The result follows from Theorem 3.4, by taking as simulation function

$$\zeta(t, s) = s - \varphi(s) - t \text{ for all } t, s \geq 0.$$

□

Corollary 3.8. *[11] Let (X, d) be a complete b-metric space and let $T : X \to X$ be a mapping. Suppose that there exists a function $\varphi : [0, \infty) \to [0, 1)$ with $\limsup_{t \to r^+} \varphi(t) < 1$ for all $r > 0$ such that*

$$d(Tx, Ty) \leq \varphi(d(x, y))d(x, y) \text{ for all } x, y \in X.$$

Then T has a unique fixed point.

Proof. The result follows from Theorem 3.4, by taking as simulation function

$$\zeta(t, s) = s\varphi(s) - t \text{ for all } t, s \geq 0.$$

□

The following examples support our main result.

Example 3.9. *Let $X=[0,1]$ and $d(x,y)=(x-y)^2$ for all $x,y\in X$, then (X,d) is a complete b-metric space with coefficient $K=2$. Let $T:X\to X$ be given by $T(x)=\dfrac{x}{\sqrt{5}}$ for all $x\in X$. Define $\varrho:[0,\infty)\times[0,\infty)\to\mathbb{R}$ by*

$$\varrho(2t,s)=\begin{cases}\frac{s}{2}-2t & \text{if } 2t\neq s,\\ 0 & \text{if } 2t=s.\end{cases}$$

Therefore, all the requirements of previous Theorem 3.4 are satisfied and $x=0$ is a unique fixed point in X.

Example 3.10. *Let $X=[0,1]$ and $d(x,y)=(x-y)^2$ for all $x,y\in X$, then (X,d) is a complete b-metric space with coefficient $K=2$. Let $T:X\to X$ be given by $T(x)=\dfrac{x}{\sqrt{10}(2+x)}$ for all $x\in X$. For all $x,y\in X$ such that $x\geq y$, we have*

$$\begin{aligned}d(Tx,Ty)&=\left(\frac{x}{\sqrt{10}(2+x)}-\frac{y}{\sqrt{10}(2+y)}\right)^2\\&=\frac{2}{5}\left(\frac{x-y}{(2+x)(2+y)}\right)^2\\&=\frac{2}{5}\left(\frac{(x-y)^2}{(4+2x+2y+xy)^2}\right)\\&\leq\frac{2}{5}\left(\frac{(x-y)^2}{(1+x-y)^2}\right)\\&\leq\frac{2}{5}\left(\frac{(x-y)^2}{1+(x-y)^2}\right).\end{aligned}$$

Define $\varrho:[0,\infty)\times[0,\infty)\to\mathbb{R}$ by $\varrho(2t,s)=\dfrac{s}{1+s}-2t$, then $\varrho\in R^*$. Therefore

$$\begin{aligned}\varrho(2d(Tx,Ty),d(x,y))&=\frac{(x-y)^2}{1+(x-y)^2}-2d(Tx,Ty)\\&\geq\frac{(x-y)^2}{1+(x-y)^2}-\frac{4}{5}\left(\frac{(x-y)^2}{1+(x-y)^2}\right)\\&=\frac{1}{5}\left(\frac{(x-y)^2}{1+(x-y)^2}\right)\\&>0.\end{aligned}$$

Therefore, T is a R'-contraction and $\varrho(2t,s)=\dfrac{s}{1+s}-2t\leq s-2t$ for all $s,t\in(0,\infty)$. By Theorem 3.4, T has a unique fixed point, that is, $x=0$.

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