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A random generalized $\mathcal{S}$-contraction mapping for finding a random p-common best proximity point

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Abstract : In this paper, we constract the two existence theorems for finding a p-common best proximity point and a random p-common best proximity point, respectively. A part of our process is to define two contraction mappings which are so-called a generalized $\mathcal{S}$-contraction and a random generalized $\mathcal{S}$-contraction by using a generalized distance on complete metric-type spaces.

Keywords : random p-common best proximity point; random generalized $\mathcal{S}$-contraction; proximally commuting mappings; generalized $\mathcal{S}$-contraction

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1 Introduction

Fixed point theory is essential tool for finding a solution of nonlinear equations of the form $Tx = x$ for self mapping T defined on non-empty subset of metric spaces or other spacses. When T is a non-self mapping the equation $Tx = x$ dose not necessarily have a solution. However, many researchers try to find an element which is closed a privious solution. Best proximity point therorems provide sufficient conditions, these theorems can confirm the existence of a complete approximate solution to the equation.

Moreover, best proximity point theorems for several types using different contraction mappings have been taken in [3], [4], [5], [6], [7], [2]. The theorems develop the generalization of fixed points by considering self mappings. For sets $A, B \neq \phi$ of (X, d), with the mappings $S : A \to B$ and $T : A \to B$, the equations $Tx = x$ and $Sx = x$ have no common fixed point of the mappings S and T. In such situation when there does not exist any type of a common solution then it is essential to find an element that is closely distant to Sx and Tx, such optimal approximate solution is known as common best proximity point of the given non-self mappings. If x is such element which gives global minimum value for these two mappings S and T, then we write as: $d(x, Sx) = d(x, Tx) = d(A, B)$. Similarly, for random common fixed points that are given in like [2], we notice that $x = F(\omega, x(\omega))$ which is the equation for random fixed point where $F : \Omega \times X \to X$ be a random mapping on metric-type spaces.

In this article, we find the p-common and random p-common best proximity results with the help of generalized $\mathcal{S}$-contraction and random generalized $\mathcal{S}$-contraction. Motivated from [1] and [2] we constract the existence theorems by using a generalized distance on metric-type spaces.

2 Preliminaries

In this section, we gather and give some definitions for an our article on a metric-type space X.

Definition 2.1. *[10] Let X be a nonempty set, $K \geq 1$ be a real number, and let the function $D : X \times X \to \mathbb{R}$ satisfy the following properties:*

(i) $D(x, y) = 0$ if and only if $x = y$;

(ii) $D(x, y) = D(y, x)$ for all $x, y \in X$;

(iii) $D(x, z) \leq K(D(x, y) + D(y, z))$ for all $x, y, z \in X$

Then (X, D) is called a metric-type space.

Definition 2.2. *[10] Let X be a metric-type space, A and B two non-empty subsets of X. Define*

$$D(A, B) = \inf\{D(a, b) : a \in A, b \in B\},$$

$$A_{D,0} = \{a \in A : \text{there exists some } b \in B \text{ such that } D(a, b) = D(A, B)\},$$

$$B_{D,0} = \{b \in B : \text{there exists some } a \in A \text{ such that } D(a, b) = D(A, B)\}.$$

Definition 2.3. *[12] Let (X, D) be a metric type space with constant $K \leq 1$. Then the function $p : X \times X \to [0, \infty)$ is called wt-distance on X if the following are satisfied:*

(i) $p(x, z) \leq K(p(x, y) + p(y, z))$ *for all* $x, y, z \in X$*;*
(ii) for any $x \in X$, $p(x, \cdot) : X \to [0, \infty)$ *is* K*-lower semi-continuous;*
(iii) for any $\epsilon > 0$*, there exists* $\delta > 0$ *such that* $p(z, x) \leq \delta$ *and* $p(z, y) \leq \delta$ *imply* $D(x, y)\epsilon$*.*

Lemma 2.1. *[12] Let (X, D) be a metric type space with constant $K \geq 1$ and p be a wt-distance on X. Let $\{x_n\}$ and $\{y_n\}$ be sequences in X, let $\{\alpha_n\}$ and $\{\beta_n\}$ be sequences in $[0, \infty)$ converging to zero, and let $x, y, z \in X$. Then the following hold:*

(1) If $p(x_n, y) \leq \alpha_n$ *and* $p(x_n, z) \leq \beta_n$ *for any* $n \in \mathbb{N}$*, then* $y = z$*.*
Inparticular, if $p(x, y) = 0$ *and* $p(x, z) = 0$*, then* $y = z$;
(2) if $p(x_n, y_n) \leq \alpha_n$ *and* $p(x_n, z) \leq \beta_n$ *for any* $n \in \mathbb{N}$*, then* $D(y_n, z) \to 0$;
(3) if $p(x_n, x_m) \leq \alpha_n$ *for any* $n, m \in \mathbb{N}$ *with* $m > n$*, then* $\{x_n\}$ *is a Cauchy sequence;*
(4) if $p(y, x_n) \leq \alpha_n$ *for any* $n \in \mathbb{N}$*, then* $\{x_n\}$ *is a Cauchy sequence.*

Definition 2.4. *[9] Let (Ω, Σ) be a measurable space with Σ be a σ-algebra of subsets of Ω, and let K be a non-empty subset of a metric-type space (X, D).*

i) A mapping $\xi : \Omega \to X$ *is measurable if* $\xi^{-1}(U) \in \Sigma$ *for any open subset* U *of* X*;*

ii) the operator $T : \Omega \times K \to K$ *is a random mapping iff for any fixed* $x \in K$, $T(\cdot, x) : \Omega \to K$ *is measurable and continuous if* $\forall \omega \in \Omega$, $T(\omega, x) : K \to X$ *is continuous;*

iii) a measurable mapping $\xi : \Omega \to X$ *is a random fixed point of the random operator* $T : \Omega \times X \to X$ *iff* $T(\omega, \xi(\omega)) = \xi(\omega)$, $\forall \omega \in \Omega$*.*

3 On generalized $\mathcal{S}$-contraction mappings

Definition 3.1. *Let X be a metric type space, p a wt-distance, A and B two non-empty subsets of X. Define*

$$p(A, B) = \inf\{p(a, b) : a \in A, b \in B\},$$

$$A_0 = \{a \in A : \text{there exists some } b \in B \text{ such that } p(a, b) = p(A, B)\},$$

$$B_0 = \{b \in B : \text{there exists some } a \in A \text{ such that } p(a, b) = p(A, B)\}.$$

Definition 3.2. *Given non-self mappings $S : A \to B$ and $T : A \to B$, an element x^* is called p-common best proximity point of the mappings if this condition satisfied:*

$$p(x^*, Sx^*) = p(x^*, Tx^*) = p(A, B).$$

We noticed here that p-common best proximity point is that element at which both functions S and T attain their global minimum, since $p(x, Sx) \geq p(A, B)$ and $p(x, Tx) \geq p(A, B)$ for all x.

Definition 3.3. *Given mappings $S : A \to B$ and $T : A \to B$, are said to commute proximally if this condition satisfied:*

$$p(u, Sx) = p(v, Tx) = p(A, B) \quad \Rightarrow \quad Sv = Tu,$$

for all x, u and v in A.

Definition 3.4. *Let $A_0 \neq \emptyset$.Then the pair (A,B) is said to have the P_p-property if and only if for any $x_1, x_2 \in A_0$ and $y_1, y_2 \in B_0$*

$$\left.\begin{aligned} p(x_1, y_1) &= p(A, B) \\ p(x_2, y_2) &= p(A, B) \end{aligned}\right\} \Rightarrow p(x_1, x_2) = p(y_1, y_2).$$

Definition 3.5. *Let (X, D) be a complete metric type space and p be a wt-distance, then the function $\mathcal{S} : A \to B$, where A and B are subsets of (X, D), is called a generalized $\mathcal{S}$-function in X if it obeys certain hypothesis as:*

1. *if there exists another mapping $F : A \to B$ in (X, D) then $p(Fx, Fy) < p(\mathcal{S}x, \mathcal{S}y)$ with $F(A_0) \subseteq \mathcal{S}(A_0)$;*
2. *for any $A, B \subseteq (X, D)$, if A_0 and B_0 are non-empty, then $\mathcal{S}(A_0) \subseteq B_0$;*
3. *for any sequence $\{x_m\}$, in A if $\lim_{m\to\infty} x_m = x \in A$, then $\lim_{m\to\infty} \mathcal{S}x_m = \mathcal{S}x \in B$ where $A \subseteq X$ and $m \in \mathbb{N}$.*

Definition 3.6. *Let (X, D) be a complete metric type space and p be a wt-distance, a mapping $T : A \to B$ with $T(A_0) \subseteq B_0$ is called a generalized $\mathcal{S}$-contraction in (X, D) if there is some generalized $\mathcal{S}$-function in (X, D) such that:*

$$p(Tx, Ty) \leq \beta(p(x, y))p(\mathcal{S}x, \mathcal{S}y),$$

where $x, y \in A$ and $\beta \in \mathcal{F}$, where we denote $\mathcal{F}$ the collection of all mappings $\beta : [0, \infty) \to [0, 1)$ satisfying $\beta(t_n) \to 1$, implies $t_n \to 0$ as $n \to \infty$.

The following theorem can assert the existence of a unique p-common best proximity point by setting T is a continuous generalized $\mathcal{S}$-contraction with a wt-distance on a complete metric-type space.

Theorem 3.7. *Let (X, D) be a complete metric type space and p be a wt-distance, A and B nonempty closed subset of X, and T a continuous $\mathcal{S}$-contraction for any $x_0 \in A_0$. If T and S are commute proximally and P_p-Property, then there exists a unique p-common best proximity point in A such that $p(x, Tx) = p(A, B)$ and $p(x, \mathcal{S}x) = p(A, B)$*

Proof. Let $x_0 \in A_0$. We note that T is a generalized $\mathcal{S}$-contraction, it obtains $T(A_0) \subseteq \mathcal{S}(A_0)$. So, there exists $x_1 \in A_0$ such that $Tx_0 = Sx_1$. From the same reason, we also get $x_2 \in A_0$. Thus $Tx_1 = \mathcal{S}x_2$. By induction, we have a sequence $\{x_n\}$ which

$$Tx_{n-1} = \mathcal{S}x_n \tag{3.1}$$

for every $n \in \mathbb{R}^+ \cup \{0\}$. By (3.1), we consider for every $m > n$

$$p(Tx_m, Tx_n) \leq \beta p(\mathcal{S}x_m, \mathcal{S}x_n) = \beta p(Tx_{m-1}, Tx_{n-1})$$

Therefore, $\{Tx_n\}$ is a Cauchy sequence. Since (X, D) is complete, there exists $y \in B$ which $\{Tx_n\} \to y$. Similarly, the sequence $\{\mathcal{S}x_n\}$ also converges to $y \in B$. We known that $T(A_0) \subseteq B_0$ and A, B are closed, it can imply that there is $u_n \in A_0$ such that

$$p(u_n, Tx_n) = p(A, B) \tag{3.2}$$

for every $n \in \mathbb{R}^+ \cup \{0\}$. For any $x_n \in A_0$, we have

$$p(u_{n-1}, \mathcal{S}x_n) = p(u_{n-1}, Tx_{n-1}) = p(A, B)$$

for every $n \in \mathbb{R}^+ \cup \{0\}$. By hypothesis, we have $\mathcal{S}u_n = Tu_{n-1}$ for any n. Therefore, S and T are continuous mappings. This implies that $Tu = \lim_{n\to\infty} Tu_{n-1}$ and $Su_n = \lim_{n\to\infty} Su_n$. From $T(A_0) \subseteq B_0$, it obtains that

$$p(x, Tu) = p(A, B) \quad \text{and} \quad p(x, \mathcal{S}u) = p(A, B).$$

Since $\mathcal{S}$ and T are commute proximally, we receive $\mathcal{S}x = Tx$. Obviously,

$$p(Tu, Tx) \leq \beta p(\mathcal{S}u, \mathcal{S}x) = \beta p(Tu, Tx). \tag{3.3}$$

That is $Tu = Tx$ and $\mathcal{S}u = \mathcal{S}x$. It means

$$p(x, Tx) = p(x, Tu) = p(A, B) \quad \text{and} \quad p(x, \mathcal{S}x) = p(x, \mathcal{S}u) = p(A, B). \tag{3.4}$$

We can conclude that x is a p-common best proximity point of the mappings T and $\mathcal{S}$. Hereafter, we are going to show the uniqueness of p-common optimal approximate solution. Suppose that x^* is an another p-common best proximity point of mappings T and $\mathcal{S}$. We have

$$p(x^*, Tx^*) = p(A, B) \quad \text{and} \quad p(x^*, \mathcal{S}x^*) = p(A, B). \tag{3.5}$$

Since $\mathcal{S}$ and T commute proximally, we obtain $\mathcal{S}x = Tx$ and $\mathcal{S}x^* = Tx^*$. It can imply that $Tx = Tx^*$. By (3.4), (3.5), and p is a P_p-property, we have $p(x, x^*) = p(Tx, Tx^*)$ and $p(x, x^*) = p(\mathcal{S}x, \mathcal{S}x^*)$. Consider

$$p(x, x^*) = p(Tx, Tx^*) \leq \beta p(Sx, Sx^*) = \beta p(x, x^*).$$

This is a contradiction, thus $x = x^*$. We can conclude that there exists a unique p-common best proximity point for mappings $T, \mathcal{S}$. □

By setting $\beta(t) = k \in [0, 1)$ and $A = B = X$ in Theorem 3.7, we recevie the following corollary.

Corollary 3.8. *Let (X, D) be a complete metric type space with p be a wt-distance. Assume that $\mathcal{S} : X \to X$ is a generalized $\mathcal{S}$-contraction and $T : X \to X$ satifies the following conditions:*

1. *There exists a nonnegative real number $k < 1$ such that*

$$p(Tx_n, Tx_{n+1}) \leq kp(\mathcal{S}x_n, \mathcal{S}x_{n+1})$$

for every x_n and x_{n+1} in A.

2. *$\mathcal{S}$ and T commute and are continuous.*
3. *$T(X) \subseteq \mathcal{S}(X)$.*

Then the mappings $\mathcal{S}$ and T have a unique common fixed point.

Furthermore, we set $K = 1$ in Theorem 3.7, it can imply Theorem 3.1 and Corollary 3.1 in [1].

4 On random generalized $\mathcal{S}$-contraction mappings

In this section, we define a random generalized $\mathcal{S}$-contraction and find out a random p-common best proximity point for given non-self mappings. Let (ϕ, Σ) be a measurable space with Σ being a sigma-algebra of subsets of ϕ.

Definition 4.1. *An equation of the type $F(\rho, x(\rho)) = p(A, B) = G(\rho, x(\rho))$ for all $\rho \in \phi$, the mappings $F, G : \Omega \times A \to B$ is called random p-common best proximity point equation where A, B are non-empty subsets of given metric-type space (X, D).*

Definition 4.2. *Let (X, D) be a complete metric type space, and p be a wt$-$distance. A mapping $T : \phi \times A \to B$ with $T(A_0) \subseteq B_0$ is called a random generalized $\mathcal{S}$-contraction if there exists $0 < k < 1$ such that*

$$p(T(\rho, x(\rho)), T(\rho, y(\rho))) \leq kp(S(\rho, x(\rho)), S(\rho, y(\rho))),$$

for every $\rho \in \phi$ where S is $\mathcal{S}$-function.

Theorem 4.3. *Let $(X, \leq)$ be a partially ordered set and let there exists a metric D on X such that (X, D) is a complete separable metric type space with A, B are closed subsets of (X, d) and (ϕ, Σ, σ) is complete probability measure space. Let $T : \phi \times A \to B$ be a continuous random generalized $\mathcal{S}$-contraction with the condition that $\mathcal{S}$ and T commute proximally, where $\mathcal{S} : A \to B$. Then there exists a random p-common best proximity point of mappings $\mathcal{S}, T$.*

Proof. Consider an arbitrary measurable mapping $\eta_0 : \phi \to A$. Since A and B are non-empty subsets of X, and $T(A_0) \subseteq \mathcal{S}(A_0)$. Let us choose an element $\eta(\rho)$ from A_0, We know $T(A_0) \subseteq \mathcal{S}(A_0)$. There is an $\eta_1(\rho)$ from A_0 with the condition that $T(\rho,\eta_0(\rho)) = \mathcal{S}(\rho,\eta_1(\rho))$ for all $\rho \in \phi$. Based on $\eta_0(\rho)$ we define a sequence $\{\eta_n(\rho)\}$ from ϕ to A with

$$T(\rho,\eta_{2n}(\rho)) = \mathcal{S}(\rho,\eta_{2n+1}(\rho)), \quad \forall \rho \in \phi, \quad n = 0,1,2,\dots \ .$$

Since $T(A_0) \subseteq B_0$ then there exists a point $\xi_n \in A_0$ such that

$$p(\xi_n, T(\rho,\eta_n(\rho))) = d(A,B)$$

for any non-negative integer $n \in \mathbb{N}$. We will show that for $\{\eta_n(\rho)\}$ is a Cauchy sequence in X, since $T(\rho,\eta_{n-1}(\rho)) = \mathcal{S}(\rho,\eta_n(\rho))$. Since $k < 1$. It follows from any $\eta_m(\rho)$ and $\eta_n(\rho)$ that

$$\begin{aligned} p(T(\rho,\eta_m(\rho)),T(\rho,\eta_n(\rho))) &\leq kp(S(\rho,\eta_m(\rho)),S(\rho,\eta_n(\rho))) \\ &= kp(T(\rho,\eta_{m-1}(\rho)),T(\rho,\eta_{n-1}(\rho))) \\ &< p(T(\rho,\eta_{m-1}(\rho)),T(\rho,\eta_{n-1}(\rho))). \end{aligned}$$

Thus $T(\rho,\eta_n(\rho))$ is Cauchy sequence and converges to some η in B. Similarly, $\mathcal{S}(\rho,\eta_n(\rho))$ is a Cauchy sequence and converges to some η in B. Since A, B are both closed sets, it means that if we take any sequence from these then it will obviously converges in same sets. Since $T(A_0) \subseteq B_0$, there exists a point $\xi_n \in A_0$ such that the pair $(T,\mathcal{S})$ is weakly increasing. Thus

$$\begin{aligned} \eta_1(\rho) = T(\rho,\eta_0(\rho)) &\leq \mathcal{S}(\rho,T(\rho,\eta_0(\rho))) \\ &= \mathcal{S}(\rho,\eta_1(\rho)) = \eta_2(\rho), \end{aligned}$$

$$\begin{aligned} \eta_2(\rho) = \mathcal{S}(\rho,\eta_1(\rho)) &\leq \mathcal{S}(\rho,T(\rho,\eta_1(\rho))) \\ &= \mathcal{S}(\rho,\eta_1(\rho)) = \eta_3(\rho). \end{aligned}$$

Continuing in this same manner, we obtain

$$\eta_{2n+1}(\rho) = T(\rho,\eta_{2n}(\rho)) \leq \mathcal{S}(\rho,T(\rho,\eta_0(\rho))) = \mathcal{S}(\rho,\eta_1(\rho)) = \eta_2(\rho)$$

$$\eta_{2n+2}(\rho) = \mathcal{S}(\rho,\eta_{2n+1}(\rho)) \leq \mathcal{S}(\rho,T(\rho,\eta_1(\rho))) = \mathcal{S}(\rho,\eta_1(\rho)) = \eta_3(\rho).$$

Hence for each $n \geq 1$. We have $T(\rho,\eta_{n-1}(\rho)) \subseteq \mathcal{S}(\rho,\eta_n(\rho))$, that is $\eta_{n-1}(\rho) \leq \eta_n(\rho)$. We will prove that for $\{\eta_n(\rho)\}$ is a Cauchy sequence in X. It is sufficient to prove that $\{\eta_{2n}(\rho)\}$ is a Cauchy sequence. We proceed by negation, suppose that $\{\eta_{2n}(\rho)\}$ is not a Cauchy sequence, then there exists $\epsilon > 0$ for which we can find two sequences of positive integers $\{m_k\},\{n_k\}$ such that for positive integer k, we have $m(k) > n(k) > k,\ d(\eta_{2n(k)}(\rho),\eta_{2m(k)}(\rho)) \geq \epsilon,\ k \geq 1$. We also assume m_k to be smallest integer with $m_k > n_k$. Therefore

$$p(\xi_n, T(\rho,\eta_n(\rho))) = p(A,B)$$

for any non-negative integer $n \in \mathbb{N}$. For any $\eta_n \in A_0$,

$$p(\xi_{n-1}, S(\rho,\eta_n(\rho))) = p(\xi_{n-1}, T(\rho,\eta_{n-1}(\rho))) = p(A,B)$$

for any non-negative integer $n \in \mathbb{N}$. Since S and T commute proximally, we obtain $\mathcal{S}(\rho_n, \xi(\rho)) = T(\rho, \xi_{n-1}(\rho))$. Since $\mathcal{S}$ and T are continuous mappings, we have

$$\lim_{n\to\infty} T(\rho, \eta_n(\rho)) = T(\rho, \eta(\rho)), \quad \rho \in \phi,$$

$$\lim_{n\to\infty} T(\rho, \eta_n(\rho)) = \mathcal{S}(\rho, \eta(\rho)), \quad \rho \in \phi.$$

From $T(A_0) \subseteq B_0$, we get an element $\xi \in A$ such that $p(\xi, T(\rho, \eta(\rho))) = p(A, B)$ and $p(\xi, \mathcal{S}(\rho, \eta(\rho))) = p(A, B)$. By assumption, $\mathcal{S}$ and T are commute proximally, it can imply that $T(\rho, \xi) = \mathcal{S}(\rho, \xi)$. Thus

$$\begin{aligned} p(T(\rho, \eta(\rho)), T(\rho, \xi(\rho))) &\leq kp(\mathcal{S}(\rho, \eta(\rho)), S(\rho, \xi(\rho))) \\ &= kp(T(\rho, \xi(\rho)), T(\rho, \xi(\rho))) \end{aligned} \tag{4.1}$$

which contradicts our supposition. Thus, $T(\rho, \eta(\rho)) = T(\rho, \xi(\rho))$ and $\mathcal{S}(\rho, \eta(\rho)) = \mathcal{S}(\rho, \xi(\rho))$. We have

$$p(\eta, T(\rho, \eta(\rho))) = p(\eta, T(\rho, \xi(\rho))) = p(A, B), \quad \text{and}$$

$$p(\eta, \mathcal{S}(\rho, \eta(\rho))) = p(\eta, \mathcal{S}(\rho, \xi(\rho))) = p(A, B).$$

Thus η is random p-common best proximity point of the mappings $\mathcal{S}$ and T. □

Corollary 4.4. *Let $(X, \leq)$ be a partially ordered set and let there exists a metric d on X such that (X, d) is a complete separable metric type space with (ϕ, Σ, σ) is complete probability measure space. Let $T : \phi \times X \to X$ be a continuous random generalized $\mathcal{S}$-contraction w.r.t self mapping with the condition that S and T commute proximally, where $\mathcal{S} : A \to A$. Then there exists a common random fixed point of the mappings $\mathcal{S}, T$.*

Proof. By putting $A = B = X$ and $p = D$ in Theorem 4.3, we receive a common random fixed point of the mappings $\mathcal{S}, T$. □

5 Conclusions

In this article the authors introduced the notions of generalized $\mathcal{S}$-contraction and generalized random $\mathcal{S}$-contraction. These contractions and results in this article developed the techniques for finding out the optimal approximate and global optimal approximate solutions in metric-type spaces and ordered metric-type spaces.

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