



On Extended Hypergeometric Function

Dharmendra Kumar Singh

Department of Mathematics, University Institute of Engineering
and Technology, Chhatrapati Shahu Ji Maharaj University
Kanpur (U.P.) India.
e-mail : drdksignhabp@gmail.com

Abstract : The principal aim of the paper is to establish some theorems for the extended hypergeometric function due to Emine, Mehmet and Abdullah [1] in Wright function form due to E.M. Wright [2]. The result provide connection to the extended Gauss hypergeometric function due to M.A. Chaudhry et al. [3] and Gauss hypergeometric function.

Keywords : extended hypergeometric functions; Wright function; Gauss hypergeometric function.

2010 Mathematics Subject Classification : 33C20.

1 Introduction

The subject of hypergeometric function and its extension form has gained considerable importance and popularity in different area of the science. Extension of the some well known functions have been considerable by several authors ([3, 4, 5, 6, 7]). In 2011, ÖZergin et al. ([1], p.4606) introduced the generalized Gauss hypergeometric function $F_p^{(\alpha, \beta)}$ which is defined for $p \geq 0$ and $\Re(c) > \Re(b) > 0$ as:

$$F_p^{(\alpha, \beta)} = \sum_{n=0}^{\infty} (a)_n \frac{B_p^{(\alpha, \beta)}(b+n, c-b)}{B(b, c-b)} \frac{z^n}{n!}, \quad (1.1)$$

where

$$B_p^{(\alpha, \beta)}(x, y) = \int_0^1 t^{x-1} (1-t)^{y-1} {}_1F_1 \left(\alpha; \beta; \frac{-pt}{t(1-t)} \right) dt, \quad (1.2)$$

$$(\Re(p) > 0, \Re(x) > 0, \Re(y) > 0, \Re(\alpha) > 0, \Re(\beta) > 0)$$

represent generalized beta function ([1], p.4602, equ.4). An integral representation for the generalized Gauss hypergeometric function (1.1) with the generalized beta function (1.2) defined as ([1], p.4606, equ.10)

$$F_p^{(\alpha,\beta)}(a, b; c; z) = \frac{1}{B(b,c-b)} \int_0^1 t^{b-1}(1-t)^{c-b-1} {}_1F_1\left(\alpha; \beta; \frac{-p}{t(1-t)}\right) (1-zt)^{-a} dt, \tag{1.3}$$

where

$$\Re(p) > 0; p = 0, |arg(1-z)| < \pi; \Re(c) > \Re(b) > 0.$$

Observe that

$$F_p^{(\alpha,\alpha)}(a, b; c; z) = F_p(a, b; c; z)$$

and

$$F_0^{(\alpha,\alpha)}(a, b; c; z) = {}_2F_1(a, b; c; z)$$

$$F_0^{(\alpha,\beta)}(a, b; c; z) = {}_2F_1(a, b; c; z),$$

where $F_p^{(\alpha,\alpha)}$ represent extended hypergeometric function due to Chaudhry et al. [3] and ${}_2F_1$ represent Gauss hypergeometric function.

The generalized hypergeometric Wright function ${}_r\Psi_s(z)$ defined for $z \in C$, $a_i, b_j \in C$ and real $\alpha_i, \beta_j \in \Re = (-\infty, \infty)$ ($\alpha_i, \beta_j \neq 0; i = 1, 2, \dots, r; j = 1, 2, \dots, s$) by the series

$${}_r\Psi_s(z) \equiv {}_r\Psi_s \left[\begin{matrix} (a_i, \alpha_i)_{1,r} \\ (b_j, \beta_j)_{1,s} \end{matrix} \middle| z \right] = \sum_{m=0}^{\infty} \frac{\prod_{i=1}^r \Gamma(a_i + \alpha_i m)}{\prod_{j=1}^s \Gamma(b_j + \beta_j m)} \frac{z^m}{m!} \tag{1.4}$$

where C is the set of the complex numbers and $\Gamma(z)$ is the Euler gamma function ([8], section 1.1) and the function (1.4) was introduced by Wright [2] and is known as generalized hypergeometric Wright function ([2], section 4.1). Condition for the existence of (1.4) together with its representation in terms of Mellin-Barnes integral and of the H-function were established in [9]. In particular, ${}_r\Psi_s(z)$ is an entire function if there hold the condition

$$\sum_{j=1}^s \beta_j - \sum_{i=1}^r \alpha_i > -1. \tag{1.5}$$

2 Theorems

Theorem 2.1. *If $\Re(p) > 0; p = 0$ and $|arg(1-z)| < \pi; \Re(c) > \Re(b) > 0$ and satisfied the condition (1.5) then*

$$F_p^{(\alpha,\beta)}(a, b, ; c; 1) = \frac{\Gamma(c)\Gamma(\beta)}{\Gamma(b)\Gamma(c-b)\Gamma(\alpha)} {}_3\Psi_2 \left[\begin{matrix} (\alpha, 1), (b, -1), (c-b-a, -1) \\ (\beta, 1), (c-a, -2) \end{matrix} \middle| -p \right]. \tag{2.1}$$

Proof. Using equation(1.3) is reshaped to the form

$$F_p^{(\alpha,\beta)}(a, b; c; 1) = \frac{\Gamma(c)}{\Gamma(b)\Gamma(c-b)} \int_0^1 t^{b-1}(1-t)^{c-b-1} {}_1F_1\left(\alpha; \beta; \frac{-p}{t(1-t)}\right) (1-t)^{-a} dt.$$

Using series representation for confluent hypergeometric function ${}_1F_1(\cdot)$ in the inner integral and Interchanging the order of integration and summation, which is permissible under the condition, stated with the theorem due to convergence of the integrals involved in this process, we obtain

$$= \frac{\Gamma(c)}{\Gamma(b)\Gamma(c-b)} \sum_{m=0}^{\infty} \frac{(\alpha)_m (-p)^m}{(\beta)_m m!} \int_0^1 t^{b-m-1} (1-t)^{c-b-a-m-1} dt. \tag{2.2}$$

On applying the known Beta integral formula

$$B(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)} = \int_0^1 t^{m-1}(1-t)^{n-1} dt \tag{2.3}$$

left side of equation (2.2), we have

$$= \frac{\Gamma(c)\Gamma(\beta)}{\Gamma(b)\Gamma(c-b)\Gamma(\alpha)} \sum_{m=0}^{\infty} \frac{\Gamma(\alpha+m)\Gamma(b-m)\Gamma(c-b-a-m)}{\Gamma(\beta+m)\Gamma(c-a-2m)} \frac{(-p)^m}{m!}. \tag{2.4}$$

Finally, employing the generalized hypergeometric Wright function (1.4), the desired result is obtained. \square

Remark 2.2. *If we put $\alpha = \beta$ in result (2.1), we obtain*

$$F_p^{(\alpha,\alpha)}(a, b; c; 1) = \frac{\Gamma(c)}{\Gamma(b)\Gamma(c-b)} {}_2\Psi_1 \left[\begin{matrix} (b, -1), (c-b-a, -1) \\ (c-a, -2) \end{matrix} \middle| -p \right]. \tag{2.5}$$

Remark 2.3. *If we take $m=0$ and $\beta = \alpha$ in equation (2.4), we arrive at*

$$F_p^{(\alpha,\alpha)}(a, b; c; 1) = \frac{\Gamma(c)\Gamma(c-b-a)}{\Gamma(c-b)\Gamma(c-a)}. \tag{2.6}$$

Remark 2.4. *If we set $p=0$ in our result (2.6), it reduces in it to well known result [7, p. 280]*

$${}_2F_1(a, b; c; 1) = \frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)}. \tag{2.7}$$

Theorem 2.5. *If $\Re(p) > 0; p = 0$ and $|\arg(1-z)| < \pi; \Re(c) > \Re(b) > 0$ and satisfied the condition (1.5) then*

$$F_p^{(\alpha,\beta)}(-n, a+n; c; 1) = \frac{\Gamma(c)\Gamma(\beta)}{\Gamma(a+n)\Gamma(c-a-n)\Gamma(\alpha)} \times {}_3\Psi_2 \left[\begin{matrix} (\alpha, 1), (a+n, -1), (c-a, -1) \\ (\beta, 1), (c+n, -2) \end{matrix} \middle| -p \right]. \tag{2.8}$$

Proof. By using equation (1.3), we have

$$F_p^{(\alpha,\beta)}(-n, a + n; c; 1) = \frac{\Gamma(c)}{\Gamma(a + n)\Gamma(c - a - n)} \times \int_0^1 t^{a+n-1}(1-t)^{c-a-n-1} \sum_{m=0}^{\infty} \frac{(\alpha)_m(-p)^m}{(\beta)_m t^m (1-t)^m m!} (1-t)^n dt.$$

Changing the order of integration and summation which is permissible under the condition stated with the theorem, we have

$$= \frac{\Gamma(c)}{\Gamma(a + n)\Gamma(c - a - n)} \sum_{m=0}^{\infty} \frac{(\alpha)_m(-p)^m}{(\beta)_m m!} \int_0^1 t^{a+n-m-1}(1-t)^{c-a-m-1} dt.$$

Employing the equation (2.3) and (1.4) on the right side of the the above expression, we arrive at

$$= \frac{\Gamma(c)\Gamma(\beta)}{\Gamma(a + n)\Gamma(c - a - n)\Gamma(\alpha)} \sum_{m=0}^{\infty} \frac{\Gamma(\alpha + m)\Gamma(a + n - m)\Gamma(c - a - m)}{\Gamma(\beta + m)\Gamma(c + n - 2m)} \frac{(-p)^m}{m!}. \tag{2.9}$$

Which is required result. □

Remark 2.6. If we take $\alpha = \beta$ in (2.8), we obtain result for extended hypergeometric function

$$F_p^{(\alpha,\alpha)}(-n, a+n; c; 1) = \frac{\Gamma(c)}{\Gamma(a + n)\Gamma(c - a - n)} {}_2\Psi_1 \left[\begin{matrix} (a + n, -1), (c - a, -1) \\ (c + n, -2) \end{matrix} \middle| -p \right]. \tag{2.10}$$

Remark 2.7. If we set $\alpha = \beta, m=0$ and using the formula $(a)_{-k} = \frac{(-1)^k}{(1-a)_k}, k = 1, 2, \dots$ in equation (2.9). Then we get

$$F_p^{(\alpha,\alpha)}(-n, a + n; c; 1) = (-1)^n \frac{(1 - c + a)_n}{(c)_n}. \tag{2.11}$$

Remark 2.8. If we set $p=0$ in equation (2.11), we get the following result ([10], p. 283])

$${}_2F_1(-n, a + n; c; 1) = (-1)^n \frac{(1 - c + a)_n}{(c)_n}. \tag{2.12}$$

Theorem 2.9. If $\Re(p) > 0; p = 0$ and $|\arg(1 - z)| < \pi; \Re(c) > \Re(b) > 0$ and satisfied the condition (1.5) then

$$F_p^{(\alpha,\beta)} \left[-\frac{1}{2}n, -\frac{1}{2}n + \frac{1}{2}; b + \frac{1}{2}; 1 \right] = \frac{2^n \Gamma(2b)\Gamma(b + \frac{1}{2}n + \frac{1}{2})\Gamma(\beta)}{\Gamma(b)\Gamma(2b + n)\Gamma(\frac{1}{2} - \frac{1}{2}n)\Gamma(\alpha)} {}_3\Psi_2 \left[\begin{matrix} (\frac{1}{2} - \frac{1}{2}n, -1), (b + n, -1), (\alpha, 1) \\ (\beta, 1), (\frac{1}{2} + \frac{1}{2}n + b, -2) \end{matrix} \middle| -p \right]. \tag{2.13}$$

Proof. With the equation (1.3), we get

$$\begin{aligned}
 & F_p^{(\alpha, \beta)} \left[-\frac{1}{2}n, -\frac{1}{2}n + \frac{1}{2}; b + \frac{1}{2}; 1 \right] \\
 &= \frac{\Gamma(b + \frac{1}{2})}{\Gamma(-\frac{1}{2}n + \frac{1}{2})\Gamma(b + \frac{1}{2} + \frac{1}{2}n - \frac{1}{2})} \\
 &\times \int_0^1 t^{-\frac{1}{2}n + \frac{1}{2} - 1} (1-t)^{b + \frac{1}{2} + \frac{1}{2}n - \frac{1}{2} - 1} {}_1F_1 \left(\alpha; \beta; \frac{-p}{t(1-t)} \right) (1-t)^{\frac{1}{2}n} dt \\
 &= \frac{\Gamma(b + \frac{1}{2})}{\Gamma(-\frac{1}{2}n + \frac{1}{2})\Gamma(b + \frac{1}{2} + \frac{1}{2}n - \frac{1}{2})} \\
 &\times \int_0^1 t^{-\frac{1}{2}n + \frac{1}{2} - 1} (1-t)^{b + \frac{1}{2} + \frac{1}{2}n - \frac{1}{2} - 1} \sum_{k=0}^{\infty} \frac{(\alpha)_k}{(\beta)_k} \frac{(-p)^k}{(t(1-t))^k k!} (1-t)^{\frac{1}{2}n} dt \\
 &= \frac{\Gamma(b + \frac{1}{2})}{\Gamma(-\frac{1}{2}n + \frac{1}{2})\Gamma(b + \frac{1}{2}n)} \sum_{k=0}^{\infty} \frac{(\alpha)_k}{(\beta)_k} \frac{(-p)^k}{k!} \\
 &\times \int_0^1 t^{\frac{1}{2} - \frac{1}{2}n - k - 1} (1-t)^{b + \frac{1}{2}n - k + \frac{1}{2}n - 1} dt.
 \end{aligned}$$

By using (2.3), above equation becomes

$$= \frac{\Gamma(b + \frac{1}{2})}{\Gamma(-\frac{1}{2}n + \frac{1}{2})\Gamma(b + \frac{1}{2}n)} \sum_{k=0}^{\infty} \frac{(\alpha)_k}{(\beta)_k} \frac{\Gamma(\frac{1}{2} - \frac{1}{2}n - k)\Gamma(b + n - k)}{\Gamma(\frac{1}{2} + \frac{1}{2}n + b - 2k)} \frac{(-p)^k}{k!}. \tag{2.14}$$

Now using Legendre’s duplication formula

$$\Gamma(b)\Gamma(b + \frac{1}{2}) = 2^{1-2b} \sqrt{\pi} \Gamma(2b)$$

in (2.14), we arrive at

$$\begin{aligned}
 &= \frac{2^{1-2b} \sqrt{\pi} \Gamma(2b)}{\Gamma(b)\Gamma(\frac{1}{2} - \frac{1}{2}n)} \frac{\Gamma(b + \frac{1}{2}n + \frac{1}{2})}{2^{1-2b-n} \sqrt{\pi} \Gamma(b + 2n)} \\
 &\times \sum_{k=0}^{\infty} \frac{(\alpha)_k}{(\beta)_k} \frac{\Gamma(\frac{1}{2} - \frac{1}{2}n - k)\Gamma(b + n - k)}{\Gamma(\frac{1}{2} + \frac{1}{2}n + b - 2k)} \frac{(-p)^k}{k!} \\
 &= \frac{2^n \Gamma(2b)\Gamma(b + \frac{1}{2}n + \frac{1}{2})\Gamma(\beta)}{\Gamma(b)\Gamma(2b + n)\Gamma(\frac{1}{2} - \frac{1}{2}n)\Gamma(\alpha)} \\
 &\times \sum_{k=0}^{\infty} \frac{\Gamma(\alpha + k)}{\Gamma(\beta + k)} \frac{\Gamma(\frac{1}{2} - \frac{1}{2}n - k)\Gamma(b + n - k)}{\Gamma(\frac{1}{2} + \frac{1}{2}n + b - 2k)} \frac{(-p)^k}{k!}. \tag{2.15}
 \end{aligned}$$

Finally, using (1.4) we arrive at required result. □

Remark 2.10. If we take $\alpha = \beta$, in (2.13), these result reduce to

$$F_p^{(\alpha, \alpha)} \left[-\frac{1}{2}n, -\frac{1}{2}n + \frac{1}{2}; b + \frac{1}{2}; 1 \right] = \frac{2^n \Gamma(2b) \Gamma(b + \frac{1}{2}n + \frac{1}{2})}{\Gamma(b) \Gamma(2b + n) \Gamma(\frac{1}{2} - \frac{1}{2}n)} {}_2\Psi_1 \left[\begin{matrix} (\frac{1}{2} - \frac{1}{2}n, -1), (b + n, -1) \\ (\frac{1}{2} + \frac{1}{2}n + b, -2) \end{matrix} \mid -p \right]. \quad (2.16)$$

Remark 2.11. On the other hand, when $\alpha = \beta$ and $k=0$ in (2.15), we get the result

$$F_p^{(\alpha, \alpha)} \left[-\frac{1}{2}n, -\frac{1}{2}n + \frac{1}{2}; b + \frac{1}{2}; 1 \right] = \frac{2^n (b)_n}{(2b)_n}. \quad (2.17)$$

Remark 2.12. If we set $p=0$ in result (2.17) reduces immediately to Gauss hypergeometric function ([10], p. 283)

$${}_2F_1 \left[-\frac{1}{2}n, -\frac{1}{2}n + \frac{1}{2}; b + \frac{1}{2}; 1 \right] = \frac{2^n (b)_n}{(2b)_n}. \quad (2.18)$$

Theorem 2.13. If $\Re(p) > 0; p = 0$ and $|\arg(1 - z)| < \pi; \Re(c) > \Re(b) > 0$ and satisfied the condition (1.5) then

$$F_p^{(\alpha, \beta)}(-n, 1 - b - n; c; 1) = \frac{\Gamma(c)\Gamma(\beta)}{\Gamma(1 - b - n)\Gamma(c - 1 + b + n)\Gamma(\alpha)} \times {}_3\Psi_2 \left[\begin{matrix} (\alpha, 1), (1 - b - n, -1), (c - 1 + b + 2n, -1) \\ (\beta, 1), (c + n, -2) \end{matrix} \mid -p \right]. \quad (2.19)$$

Proof. From (1.3), we get

$$F_p^{(\alpha, \beta)}(-n, 1 - b - n; c; 1) = \frac{\Gamma(c)}{\Gamma(1 - b - n)\Gamma(c - 1 + b + n)} \int_0^1 t^{1 - b - n - 1} (1 - t)^{c - 1 + b + n - 1} \times {}_1F_1 \left(\alpha; \beta; \frac{-p}{t(1 - t)} \right) (1 - t)^n dt. = \frac{\Gamma(c)}{\Gamma(1 - b - n)\Gamma(c - 1 + b + n)} \sum_{m=0}^{\infty} \frac{(\alpha)_m (-p)^m}{(\beta)_m m!} \times \int_0^1 t^{1 - b - n - m - 1} (1 - t)^{c - 1 + b + 2n - m - 1} dt.$$

The simplification of above equation gives

$$= \frac{\Gamma(c)\Gamma(\beta)}{\Gamma(1 - b - n)\Gamma(c - 1 + b + n)\Gamma(\alpha)} \times \sum_{m=0}^{\infty} \frac{\Gamma(\alpha)\Gamma(1 - b - n - m)\Gamma(c - 1 + b + 2n - m)}{\Gamma(\beta)\Gamma(c + n - 2m)} \frac{(-p)^m}{m!}. \quad (2.20)$$

Which is required result. □

Remark 2.14. By putting $\alpha = \beta$ in (2.19) we shall have the result

$$F_p^{(\alpha,\alpha)}(-n, 1 - b - n; c; 1) = \frac{\Gamma(c)}{\Gamma(1 - b - n)\Gamma(c - 1 + b + n)} \times {}_2\Psi_1 \left[\begin{matrix} (1 - b - n, -1), (c - 1 + b + 2n, -1) \\ (c + n, -2) \end{matrix} \middle| -p \right]. \tag{2.21}$$

Remark 2.15. On setting $\beta = \alpha$, $m=0$ and using the calculation

$$\frac{\Gamma(c - 1 + b + 2n)}{\Gamma(c - 1 + b + n)} = \frac{(c - 1 + b)_{2n}\Gamma(c - 1 + b)}{(c - 1 + b)_n\Gamma(c - 1 + b)} = \frac{(c + b - 1)_{2n}}{(c + b - 1)_n}$$

in result (2.20) we can produce the result

$$F_p^{(\alpha,\alpha)}(-n, 1 - b - n; c; 1) = \frac{(c - 1 + b)_{2n}}{(c)_n(c - 1 + b)_n}. \tag{2.22}$$

Remark 2.16. On taking $p=0$ in result (2.22), we obtain the known result [7, p. 283]

$${}_2F_1(-n, 1 - b - n; c; 1) = \frac{(c - 1 + b)_{2n}}{(c)_n(c - 1 + b)_n}. \tag{2.23}$$

Theorem 2.17. If $\Re(p) > 0; p = 0$ and $|\arg(1 - z)| < \pi; \Re(c) > \Re(b) > 0$ and satisfied the condition (1.5) then

$$F_p^{(\alpha,\beta)}[-n, b; c; 1] = \frac{\Gamma(c)\Gamma(\beta)}{\Gamma(b)\Gamma(c - b)\Gamma(\alpha)} {}_3\Psi_2 \left[\begin{matrix} (\alpha, 1), (b, -1), (c - b + n, -1) \\ (\beta, 1), (c + n, -2) \end{matrix} \middle| -p \right]. \tag{2.24}$$

Proof. By using the equation (1.3), equation (1.4) and proceeding similarly to the proof of above theorems, we obtain

$$\begin{aligned} F_p^{(\alpha,\beta)}[-n, b; c; 1] &= \frac{1}{B(b, c - b)} \int_0^1 t^{b-1}(1 - t)^{c-b+n-1} {}_1F_1 \left(\alpha; \beta; \frac{-p}{t(1 - t)} \right) dt \\ &= \frac{1}{B(b, c - b)} \int_0^1 t^{b-1}(1 - t)^{c-b+n-1} \sum_{m=0}^{\infty} \frac{(\alpha)_m}{(\beta)_m} \frac{(-p)^m}{(t(1 - t))^m m!} dt \\ &= \sum_{m=0}^{\infty} \frac{(\alpha)_m}{(\beta)_m} \frac{(-p)^m}{m!} \frac{1}{B(b, c - b)} \int_0^1 t^{b-m-1}(1 - t)^{c-b+n-m-1} dt \\ &= \frac{\Gamma(c)\Gamma(\beta)}{\Gamma(b)\Gamma(c - b)\Gamma(\alpha)} \sum_{m=0}^{\infty} \frac{\Gamma(b - m)\Gamma(c - b + n - m)\Gamma(\alpha + m)}{\Gamma(c + n - 2m)\Gamma(\beta + m)}. \end{aligned}$$

This complete the proof. □

Remark 2.18. When $\alpha = \beta$ in (2.24) reduces to an elegant result

$$F_p^{(\alpha,\alpha)}[-n, b; c; 1] = \frac{\Gamma(c)}{\Gamma(b)\Gamma(c - b)} {}_2\Psi_1 \left[\begin{matrix} (b, -1), (c - b + n, -1) \\ (c + n, -2) \end{matrix} \middle| -p \right]. \tag{2.25}$$

Remark 2.19. *If we take $m=0$ and $\alpha = \beta$ in result (2.25), we obtain*

$$F_p^{(\alpha, \alpha)}[-n, b; c; 1] = \frac{(c-b)_n}{(c)_n}. \quad (2.26)$$

Remark 2.20. *On setting $p=0$, in result (2.27) we arrive at given result ([10], p.283)*

$${}_2F_1(-n, b; c; 1) = \frac{(c-b)_n}{(c)_n}. \quad (2.27)$$

References

- [1] E. ÖZergin, M.A. Özarslan, A. Altin, Extension of gamma, beta and hypergeometric functions, *J. Comp. and Appl. Math.* 235 (2011) 4601-4610.
- [2] E.M. Wright, The asymptotic expansion of the generalized hypergeometric functions, *J. London Math. Soc.* 10 (1935) 286-293.
- [3] M.A. Chaudhry, A. Qadir, H.M. Srivastava, R.B. Paris, Extended hypergeometric and confluent hypergeometric functions, *Appl. Math. Comput.* 159 (2004) 589-602.
- [4] A.R. Miller, Reduction of a generalized incomplete gamma function, related Kampe de Fariet functions, and in complete Weber integrals, *Rocky Mountain J. Math.* 30 (2002) 703-714.
- [5] F. AL-Musallam, S.L. Kalla, Further results on a generalized gamma function occurring in diffraction theory, *Integral transform and special function* 7 (3-4) (1998) 175-190.
- [6] M.A. Chaudhry, S.M. Zubair, Extended incomplete gamma functions with applications, *J. Math. Anal. Appl.* 274 (2002) 725-745.
- [7] M.A. Chaudhry, S.M. Zubair, Generalized incomplete gamma function with applications, *J. Comp. Appl. Math.* 55 (1994) 99-124.
- [8] A. Erdlyi, W. Magnus, F. Oberhettinger, F.G. Tricomi, *Higher Transcendental Functions*, Vol. I. McGraw-Hill, New York-Toronto-London (1953).
- [9] A.A. Kilbas, M. Saigo, J.J. Trujillo, On the generalized Wright function, *Frac. Calc. Appl. Anal.* Vol. 5, No. 4 (2002) 437-460.
- [10] E. George Andrews, C. Larry, *Special function for engineering and applied mathematicians*, Macmillan, New York (1985).

(Received 21 August 2012)

(Accepted 20 October 2013)