



Somewhat Pairwise Fuzzy Precontinuous Mappings

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Abstract : The concept of somewhat pairwise fuzzy precontinuous mapping and somewhat pairwise fuzzy preopen mapping have been studied. Besides, some interesting properties of those mappings are given.

Keywords : somewhat pairwise fuzzy precontinuous mapping, somewhat pairwise fuzzy preopen mapping.

2010 Mathematics Subject Classification : 54A40; 03E72.

1 Introduction and Preliminaries

The fundamental concept of fuzzy sets was introduced by L.A. Zadeh [1] provided a natural foundation for building new branches. In 1968 C.L. Chang [2] introduced the concept of fuzzy topological spaces as a generalization of topological spaces.

The class of somewhat continuous mappings was first introduced by Karl. R. Gentry and others [3]. Later, the concept of “somewhat” in classical topology has been extended to fuzzy topological spaces. In fact, somewhat fuzzy continuous mappings and somewhat fuzzy semicontinuous mappings were introduced and studied by G. Thangaraj and G. Balasubramanian in [4] and [5] respectively. In

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1989, A. Kandil [6] introduced the concept of fuzzy bitopological spaces. The product related spaces and the graph of a function were found in Azad [7].

The concept of fuzzy precontinuous mappings on fuzzy topological space was introduced and studied by A. S. Bin Shahna in [8]. The concept of somewhat fuzzy precontinuous mappings was introduced and studied by Young Bin Im and others in [9]. The concept of somewhat pairwise fuzzy continuous mappings was introduced and developed by M. K. Uma and others in [10]. In this paper, we introduce the concept of somewhat pairwise fuzzy precontinuous mappings and somewhat pairwise fuzzy preopen mappings and study their properties.

Definition 1.1. A mapping $f : (X, \tau_1, \tau_2) \rightarrow (Y, \eta_1, \eta_2)$ is called *somewhat pairwise fuzzy continuous* [10] if there exists a τ_1 -fuzzy open or τ_2 -fuzzy open set $\mu \neq 0_X$ on (X, τ_1, τ_2) such that $\mu \leq f^{-1}(\nu) \neq 0_X$ for any η_1 -fuzzy open or η_2 -fuzzy open set on (Y, η_1, η_2) .

Definition 1.2. A mapping $f : (X, \tau_1, \tau_2) \rightarrow (Y, \eta_1, \eta_2)$ is called *somewhat pairwise fuzzy open* [10] if there exists an η_1 -fuzzy open or η_2 -fuzzy open set $\nu \neq 0_Y$ on (Y, η_1, η_2) such that $\nu \leq f(\mu) \neq 0_Y$ for any τ_1 -fuzzy open or τ_2 -fuzzy open set μ on (X, τ_1, τ_2) .

2 Somewhat Pairwise Fuzzy Precontinuous Mappings

In this section, we introduce a somewhat pairwise fuzzy precontinuous mapping which are weaker than a somewhat pairwise fuzzy continuous mapping. And we characterize those mappings.

Definition 2.1. A mapping $f : (X, \tau_1, \tau_2) \rightarrow (Y, \eta_1, \eta_2)$ is called *pairwise fuzzy precontinuous* if $f^{-1}(\nu)$ is a τ_1 -fuzzy preopen or τ_2 -fuzzy preopen set on (X, τ_1, τ_2) for any η_1 -fuzzy open or η_2 -fuzzy open set ν on (Y, η_1, η_2) .

Definition 2.2. A mapping $f : (X, \tau_1, \tau_2) \rightarrow (Y, \eta_1, \eta_2)$ is called *somewhat pairwise fuzzy precontinuous* if there exists a τ_1 -fuzzy preopen or τ_2 -fuzzy preopen set $\mu \neq 0_X$ on (X, τ_1, τ_2) such that $\mu \leq f^{-1}(\nu) \neq 0_X$ for any η_1 -fuzzy open or η_2 -fuzzy open set ν on (Y, η_1, η_2) .

From the definitions, it is clear that every somewhat pairwise fuzzy continuous mapping is a pairwise fuzzy precontinuous mapping and every pairwise fuzzy precontinuous mapping is a somewhat pairwise fuzzy precontinuous mapping. But the converses are not true in general as the following example shows.

Example 2.3. Let $X = Y = Z = \{a, b, c\}$. Then fuzzy sets $\lambda_1 = \frac{0.5}{a} + \frac{0.5}{b} + \frac{0.5}{c}$, $\lambda_2 = \frac{0.4}{a} + \frac{0.4}{b} + \frac{0.4}{c}$, $\sigma_1 = \frac{0.1}{a} + \frac{0.1}{b} + \frac{0.1}{c}$, $\sigma_2 = \frac{0.2}{a} + \frac{0.2}{b} + \frac{0.2}{c}$, $\mu_1 = \frac{0.3}{a} + \frac{0.3}{b} + \frac{0.3}{c}$, $\mu_2 = \frac{0.4}{a} + \frac{0.4}{b} + \frac{0.4}{c}$ are defined as follows: Consider $\tau_1 = \{0_X, 1_X, \lambda_1\}$, $\tau_2 = \{0_X, 1_X, \lambda_2\}$, $\eta_1 = \{0_X, 1_X, \sigma_1\}$, $\eta_2 = \{0_X, 1_X, \sigma_2\}$, $\gamma_1 = \{0_X, 1_X, \mu_1\}$, $\gamma_2 = \{0_X, 1_X, \mu_2\}$. Then (X, τ_1, τ_2) , (Y, η_1, η_2) and (Z, γ_1, γ_2) are fuzzy bitopologies. Define $f : (X, \tau_1, \tau_2) \rightarrow$

(Y, η_1, η_2) is an identity map. Then f is pairwise fuzzy precontinuous but not somewhat pairwise fuzzy continuous because there is no non-zero τ_1 -fuzzy open or τ_2 -fuzzy open set smaller than $f^{-1}(\sigma_2) = \sigma_2 \neq 0$. Define $g : (Y, \eta_1, \eta_2) \rightarrow (Z, \gamma_1, \gamma_2)$ is an identity map. Then g is somewhat pairwise fuzzy precontinuous but not pairwise fuzzy precontinuous because for any fuzzy set $g^{-1}(\mu_1) = \mu_1$ or $g^{-1}(\mu_2) = \mu_2$ which is not an η_1 -fuzzy preopen or η_2 -fuzzy preopen set on (Y, η_1, η_2) .

Definition 2.4. A fuzzy set μ on a fuzzy bitopological space (X, τ_1, τ_2) is called *pairwise predense fuzzy set* if there exists no τ_1 -fuzzy preclosed or τ_2 -fuzzy preclosed set ν in (X, τ_1, τ_2) such that $\mu < \nu < 1$.

Theorem 2.5. Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \eta_1, \eta_2)$ be a mapping. Then the following are equivalent:

- (1) f is somewhat pairwise fuzzy precontinuous.
- (2) If ν is an η_1 -fuzzy closed or η_2 -fuzzy closed set on (Y, η_1, η_2) such that $f^{-1}(\nu) \neq 1_X$, then there exists a proper τ_1 -fuzzy preclosed or τ_2 -fuzzy preclosed set $\mu \neq 1_X$ of (X, τ_1, τ_2) such that $f^{-1}(\nu) \leq \mu$.
- (3) If μ is a pairwise predense fuzzy set on (X, τ_1, τ_2) , then $f(\mu)$ is a pairwise predense fuzzy set on (Y, η_1, η_2) .

Proof. (1) \Rightarrow (2): Let ν be an η_1 -fuzzy closed or η_2 -fuzzy closed set on (Y, η_1, η_2) such that $f^{-1}(\nu) \neq 1_X$. Then ν^c is an η_1 -fuzzy open or η_2 -fuzzy open set in (Y, η_1, η_2) and $f^{-1}(\nu^c) = (f^{-1}(\nu))^c \neq 0_X$. Since f is somewhat pairwise fuzzy precontinuous, there exists a τ_1 -fuzzy preopen or τ_2 -fuzzy preopen set $\mu^c \neq 0_X$ on (X, τ_1, τ_2) such that $\mu^c \leq f^{-1}(\nu^c)$. Hence there exists a τ_1 -fuzzy preclosed or τ_2 -fuzzy preclosed set $\mu \neq 1_X$ on (X, τ_1, τ_2) such that $f^{-1}(\nu) = 1 - f^{-1}(\nu^c) \leq 1 - \mu^c = \mu$.

(2) \Rightarrow (3): Let μ be a pairwise predense fuzzy set on (X, τ_1, τ_2) and suppose that $f(\mu)$ is not pairwise predense fuzzy set on (Y, η_1, η_2) . Then there exists an η_1 -fuzzy preclosed or η_2 -fuzzy preclosed set ν on (Y, η_1, η_2) such that $f(\mu) < \nu < 1$. Since $\nu < 1$ and $f^{-1}(\nu) \neq 1_X$, there exists a τ_1 -fuzzy preclosed or τ_2 -fuzzy preclosed set $\delta \neq 1_X$ such that $\mu \leq f^{-1}(f(\mu)) < f^{-1}(\nu) \leq \delta$. This contradicts to the assumption that μ is a pairwise predense fuzzy set on (X, τ_1, τ_2) . Hence $f(\mu)$ is a pairwise predense fuzzy set on (Y, η_1, η_2) .

(3) \Rightarrow (1): Let ν be an η_1 -fuzzy open or η_2 -fuzzy open set on (Y, η_1, η_2) with $f^{-1}(\nu) \neq 0_X$. Suppose that there exists no τ_1 -fuzzy preopen or τ_2 -fuzzy preopen set $\mu \neq 0_X$ on (X, τ_1, τ_2) such that $\mu \leq f^{-1}(\nu)$. Then $(f^{-1}(\nu))^c$ is a τ_1 -fuzzy set or τ_2 -fuzzy set on (X, τ_1, τ_2) such that there is no τ_1 -fuzzy preclosed or τ_2 -fuzzy preclosed set δ on (X, τ_1, τ_2) with $(f^{-1}(\nu))^c < \delta < 1$. In fact, if there exists a τ_1 -fuzzy preopen or τ_2 -fuzzy preopen set δ^c such that $\delta^c \leq f^{-1}(\nu)$, then it is a contradiction. So $(f^{-1}(\nu))^c$ is a pairwise predense fuzzy set on (X, τ_1, τ_2) . Hence $f((f^{-1}(\nu))^c)$ is a pairwise predense fuzzy set on (Y, η_1, η_2) . But $f((f^{-1}(\nu))^c) = f(f^{-1}(\nu^c)) \neq \nu^c < 1$. This is a contradiction to the fact that $f((f^{-1}(\nu))^c)$ is

pairwise predense fuzzy set on (Y, η_1, η_2) . Hence there exists a τ_1 -preopen or τ_2 -preopen set $\mu \neq 0_X$ in (X, τ_1, τ_2) such that $\mu \leq f^{-1}(\nu)$. Consequently, f is somewhat pairwise fuzzy precontinuous. \square

Theorem 2.6. *Let $(X_1, \tau_1, \tau_2), (X_2, \omega_1, \omega_2), (Y_1, \eta_1, \eta_2), (Y_2, \sigma_1, \sigma_2)$ be fuzzy bitopological spaces. Let (X_1, τ_1, τ_2) be product related to $(X_2, \omega_1, \omega_2)$ and let (Y_1, η_1, η_2) be product related to $(Y_2, \sigma_1, \sigma_2)$. If $f_1 : (X_1, \tau_1, \tau_2) \rightarrow (Y_1, \eta_1, \eta_2)$ and $f_2 : (X_2, \omega_1, \omega_2) \rightarrow (Y_2, \sigma_1, \sigma_2)$ is a somewhat pairwise fuzzy precontinuous mappings, then the product $f_1 \times f_2 : (X_1, \tau_1, \tau_2) \times (X_2, \omega_1, \omega_2) \rightarrow (Y_1, \eta_1, \eta_2) \times (Y_2, \sigma_1, \sigma_2)$ is also somewhat pairwise fuzzy precontinuous.*

Proof. Let $\lambda = \bigvee_{i,j} (\mu_i \times \nu_j)$ be η_i -fuzzy open or σ_j -fuzzy open set on $(Y_1, \eta_1, \eta_2) \times (Y_2, \sigma_1, \sigma_2)$ where μ_i is η_i -fuzzy open set and ν_j is σ_j -fuzzy open set on (Y_1, η_1, η_2) and $(Y_2, \sigma_1, \sigma_2)$ respectively. Then $(f_1 \times f_2)^{-1}(\lambda) = \bigvee_{i,j} (f_1^{-1}(\mu_i) \times f_2^{-1}(\nu_j))$. Since f_1 is somewhat pairwise fuzzy precontinuous, there exists a τ_1 -fuzzy preopen or τ_2 -fuzzy preopen set $\delta_i \neq 0_{X_1}$ such that $\delta_i \leq f_1^{-1}(\mu_i) \neq 0_{X_1}$. And since f_2 is somewhat pairwise fuzzy precontinuous, there exists a ω_1 -fuzzy preopen or ω_2 -fuzzy preopen set $\gamma_j \neq 0_{X_2}$ such that $\gamma_j \leq f_2^{-1}(\nu_j) \neq 0_{X_2}$. Now $\delta_i \times \gamma_j \leq f_1^{-1}(\mu_i) \times f_2^{-1}(\nu_j) = (f_1 \times f_2)^{-1}(\mu_i \times \nu_j)$ and $\delta_i \times \gamma_j \neq 0_{X_1 \times X_2}$. Hence $\delta_i \times \gamma_j$ is a τ_i -fuzzy preopen or ω_j -fuzzy preopen set on $(X_1, \tau_1, \tau_2) \times (X_2, \omega_1, \omega_2)$ such that $\bigvee_{i,j} (\delta_i \times \gamma_j) \leq \bigvee_{i,j} (f_1^{-1}(\mu_i) \times f_2^{-1}(\nu_j)) = (f_1 \times f_2)^{-1}(\bigvee_{i,j} (\mu_i \times \nu_j)) = (f_1 \times f_2)^{-1}(\lambda) \neq 0_{X_1 \times X_2}$. Therefore, $f_1 \times f_2$ is somewhat pairwise fuzzy precontinuous. \square

Theorem 2.7. *Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \eta_1, \eta_2)$ be a mapping. If the graph $g : (X, \tau_1, \tau_2) \rightarrow (X, \tau_1, \tau_2) \times (Y, \eta_1, \eta_2)$ of f is somewhat pairwise fuzzy irresolute precontinuous, then f is also somewhat pairwise fuzzy irresolute precontinuous.*

Proof. Let ν be an η_1 -fuzzy open or η_2 -fuzzy open set on (Y, η_1, η_2) . Then $f^{-1}(\nu) = 1 \wedge f^{-1}(\nu) = g^{-1}(1 \times \nu)$. Since g is somewhat pairwise fuzzy precontinuous and $1 \times \nu$ is a τ_i -fuzzy open or η_j -fuzzy open set on $(X, \tau_1, \tau_2) \times (Y, \eta_1, \eta_2)$, there exists a τ_1 -fuzzy preopen or τ_2 -fuzzy preopen set $\mu \neq 0_X$ on (X, τ_1, τ_2) such that $\mu \leq g^{-1}(1 \times \nu) = f^{-1}(\nu) \neq 0_X$. Therefore, f is somewhat pairwise fuzzy precontinuous. \square

3 Somewhat Pairwise Fuzzy Preopen Mappings

In this section, we introduce a somewhat pairwise fuzzy preopen mapping which are weaker than a somewhat pairwise fuzzy open mapping. And we characterize those mappings.

Definition 3.1. A mapping $f : (X, \tau_1, \tau_2) \rightarrow (Y, \eta_1, \eta_2)$ is called *pairwise fuzzy preopen* if $f(\mu)$ is an η_1 -fuzzy preopen or η_2 -fuzzy preopen set on (Y, η_1, η_2) for any τ_1 -fuzzy open or τ_2 -fuzzy open set μ on (X, τ_1, τ_2) .

Definition 3.2. A mapping $f : (X, \tau_1, \tau_2) \rightarrow (Y, \eta_1, \eta_2)$ is called *somewhat pairwise fuzzy preopen* if there exists an η_1 -fuzzy preopen or η_2 -fuzzy preopen set $\nu \neq 0_Y$ on (Y, η_1, η_2) such that $\nu \leq f(\mu) \neq 0_Y$ for any τ_1 -fuzzy open or τ_2 -fuzzy open set μ on (X, τ_1, τ_2) .

From the definitions, it is clear that every somewhat pairwise fuzzy open mapping is a pairwise fuzzy preopen mapping and every pairwise fuzzy preopen mapping is a somewhat pairwise fuzzy preopen mapping. But the converses are not true in general as the following example shows.

Example 3.3. Let $X = Y = Z = \{a, b, c\}$. Then fuzzy sets $\lambda_1 = \frac{0.2}{a} + \frac{0.2}{b} + \frac{0.2}{c}$, $\lambda_2 = \frac{0.05}{a} + \frac{0.05}{b} + \frac{0.05}{c}$, $\sigma_1 = \frac{0.5}{a} + \frac{0.5}{b} + \frac{0.5}{c}$, $\sigma_2 = \frac{0.4}{a} + \frac{0.4}{b} + \frac{0.4}{c}$, $\mu_1 = \frac{0.01}{a} + \frac{0.01}{b} + \frac{0.01}{c}$, $\mu_2 = \frac{0.02}{a} + \frac{0.02}{b} + \frac{0.02}{c}$ are defined as follows: Consider $\tau_1 = \{0_X, 1_X, \lambda_1\}$, $\tau_2 = \{0_X, 1_X, \lambda_2\}$, $\eta_1 = \{0_X, 1_X, \sigma_1\}$, $\eta_2 = \{0_X, 1_X, \sigma_2\}$, $\gamma_1 = \{0_X, 1_X, \mu_1\}$, $\gamma_2 = \{0_X, 1_X, \mu_2\}$. Then (X, τ_1, τ_2) , (Y, η_1, η_2) and (Z, γ_1, γ_2) are fuzzy bitopologies. Define $f : (X, \tau_1, \tau_2) \rightarrow (Y, \eta_1, \eta_2)$ is an identity map. Then f is pairwise fuzzy preopen but not somewhat pairwise fuzzy open because there is no non-zero η_1 -fuzzy open or η_2 -fuzzy open set smaller than $f(\sigma_2) = \sigma_2 \neq 0$. Define $g : (X, \tau_1, \tau_2) \rightarrow (Z, \gamma_1, \gamma_2)$ be an identity map. Then g is somewhat pairwise fuzzy preopen but not pairwise fuzzy preopen because for the fuzzy set $f(\lambda_1) = \lambda_1$ is not pairwise fuzzy preopen set on (Z, γ_1, γ_2) .

Theorem 3.4. Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \eta_1, \eta_2)$ be a bijection. Then the following are equivalent:

- (1) f is pairwise somewhat fuzzy preopen.
- (2) If μ is a τ_1 -fuzzy closed or τ_2 -fuzzy closed set on (X, τ_1, τ_2) such that $f(\mu) \neq 1_Y$, then there exists an η_1 -fuzzy preclosed or η_2 -fuzzy preclosed set $\nu \neq 1_Y$ on (Y, η_1, η_2) such that $f(\mu) < \nu$.

Proof. (1) \Rightarrow (2): Let μ be a τ_1 -fuzzy closed or τ_2 -fuzzy closed set on (X, τ_1, τ_2) such that $f(\mu) \neq 1_Y$. Since f is bijective and μ^c is a τ_1 -fuzzy open or τ_2 -fuzzy open set on (X, τ_1, τ_2) , $f(\mu^c) = (f(\mu))^c \neq 0_Y$. From the definition, there exists an η_1 -preopen or η_2 -preopen set $\delta \neq 0_Y$ on (Y, η_1, η_2) such that $\delta < f(\mu^c) = (f(\mu))^c$. Consequently, $f(\mu) < \delta^c = \nu \neq 1_Y$ and ν is an η_1 -fuzzy preclosed or η_2 -fuzzy preclosed set on (Y, η_1, η_2) .

(2) \Rightarrow (1): Let μ be a τ_1 -fuzzy open or τ_2 -fuzzy open set on (X, τ_1, τ_2) such that $f(\mu) \neq 0_Y$. Then μ^c is a τ_1 -fuzzy closed or τ_2 -fuzzy closed set on (X, τ_1, τ_2) and $f(\mu^c) \neq 1_Y$. Hence there exists an η_1 -fuzzy preclosed or η_2 -fuzzy preclosed set $\nu \neq 1_Y$ on (Y, η_1, η_2) such that $f(\mu^c) < \nu$. Since f is bijective, $f(\mu^c) = (f(\mu))^c < \nu$. Thus $\nu^c < f(\mu)$ and $\nu^c \neq 0_X$ is an η_1 -fuzzy preopen or η_2 -fuzzy preopen set on (Y, η_1, η_2) . Therefore, f is somewhat pairwise fuzzy preopen. \square

Theorem 3.5. Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \eta_1, \eta_2)$ be a surjection. Then the following are equivalent:

- (1) f is somewhat pairwise fuzzy preopen.
- (2) If ν is a pairwise predense fuzzy set on (Y, η_1, η_2) , then $f^{-1}(\nu)$ is a pairwise predense fuzzy set on (X, τ_1, τ_2) .

Proof. (1) \Rightarrow (2): Let ν be a pairwise predense fuzzy set on (Y, η_1, η_2) . Suppose $f^{-1}(\nu)$ is not pairwise predense fuzzy set on (X, τ_1, τ_2) . Then there exists a τ_1 -fuzzy preclosed or τ_2 -fuzzy preclosed set μ on (X, τ_1, τ_2) such that $f^{-1}(\nu) < \mu < 1$. Since f is somewhat pairwise fuzzy preopen and μ^c is a τ_1 -fuzzy preopen or τ_2 -fuzzy preopen set on (X, τ_1, τ_2) , there exists an η_1 -fuzzy preopen or an η_2 -fuzzy preopen set $\delta \neq 0_Y$ on (Y, η_1, η_2) such that $\delta \leq f(Int\mu^c) \leq f(\mu^c)$. Since f is surjective, $\delta \leq f(\mu^c) \leq f(f^{-1}(\nu^c)) = \nu^c$. Thus there exists an η_1 -preclosed or η_2 -preclosed set δ^c on (Y, η_1, η_2) such that $\nu < \delta^c < 1$. This is a contradiction. Hence $f^{-1}(\nu)$ is pairwise predense fuzzy set on (X, τ_1, τ_2) .

(2) \Rightarrow (1): Let μ be a τ_1 -fuzzy open or τ_2 -fuzzy open set on (X, τ_1, τ_2) and $f(\mu) \neq 0_Y$. Suppose there exists no η_1 -fuzzy preopen or η_2 -fuzzy preopen set $\nu \neq 0_Y$ on (Y, η_1, η_2) such that $\nu \leq f(\mu)$. Then $(f(\mu))^c$ is an η_1 -fuzzy set or η_2 -fuzzy set δ on (Y, η_1, η_2) such that there exists no η_1 -fuzzy preclosed or η_2 -fuzzy preclosed set δ on (Y, η_1, η_2) with $(f(\mu))^c < \delta < 1$. This means that $(f(\mu))^c$ is pairwise predense fuzzy set on (Y, η_1, η_2) . Thus $f^{-1}((f(\mu))^c)$ is pairwise predense fuzzy set on (X, τ_1, τ_2) . But $f^{-1}((f(\mu))^c) = (f^{-1}(f(\mu)))^c \leq \mu^c < 1$. This is a contradiction to the fact that $f^{-1}(f(\nu))^c$ is pairwise predense fuzzy set on (X, τ_1, τ_2) . Hence there exists an η_1 -fuzzy preopen or η_1 -fuzzy preopen set $\nu \neq 0_Y$ on (Y, η_1, η_2) such that $\nu \leq f(\mu)$. Therefore, f is somewhat pairwise fuzzy preopen. \square

Conclusion: Young Bin [9] introduced somewhat fuzzy precontinuous mappings which we have studied and developed in this current paper some new results somewhat pairwise fuzzy precontinuous mappings. G. Thangaraj and G. Balasubramanian in [4] and [5] respectively introduced somewhat fuzzy continuous functions and somewhat fuzzy semi-continuous functions and subsequently in bitopology developed by M.K. Uma et al. in [10] somewhat pairwise fuzzy continuous functions which have been helpful us to develope this paper. Further research can be undertaken in this direction.

Acknowledgement : The Authors would like to thank anonymous referee and Dr.A.Vadivel, Department of Mathematics, Annamalai University, Chidambaram, for the valuable comments and suggestions for the improvement of the manuscript.

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(Received 12 April 2014)

(Accepted 15 September 2015)