



## On Generalized $\phi$ -Recurrent Generalized Sasakian-Space-Forms

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**Abstract :** The object of this present paper is to study generalized  $\phi$ -recurrent generalized Sasakian-space-forms and its various geometric properties. Among the results established here, it is shown that a generalized  $\phi$ -recurrent generalized Sasakian-space-form is an Einstein manifold. Further, we study generalized concircular  $\phi$ -recurrent generalized Sasakian-space-forms.

**Keywords :** Sasakian manifold; generalized Sasakian-space-form; generalized  $\phi$ -recurrent; concircular curvature tensor.

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### 1 Introduction

It is well known that in differential geometry the curvature of a Riemannian manifold plays a basic role and the sectional curvatures of a manifold determine the curvature tensor  $R$  completely. A Riemannian manifold with constant sectional

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curvature  $c$  is called a *real-space-form* and its curvature tensor  $R$  satisfies the condition

$$R(X, Y)Z = c\{g(Y, Z)X - g(X, Z)Y\}. \quad (1.1)$$

Models for these spaces are the Euclidean spaces ( $c = 0$ ), the spheres ( $c > 0$ ) and the hyperbolic spaces ( $c < 0$ ).

In contact metric geometry, a Sasakian manifold with constant  $\phi$ -sectional curvature is called Sasakian-space-form. As a generalization of Sasakian-space-form, Alegre et al [1] introduced and studied the notion of generalized Sasakian-space-form with the existence of such notions by several interesting examples. A generalized Sasakian-space-form is an almost contact metric manifold  $(M, \phi, \xi, \eta, g)$  whose curvature tensor is given by

$$\begin{aligned} R(X, Y)Z &= f_1\{g(Y, Z)X - g(X, Z)Y\} \\ &+ f_2\{g(X, \phi Z)\phi Y - g(Y, \phi Z)\phi X + 2g(X, \phi Y)\phi Z\} \\ &+ f_3\{\eta(X)\eta(Z)Y - \eta(Y)\eta(Z)X \\ &+ g(X, Z)\eta(Y)\xi - g(Y, Z)\eta(X)\xi\}, \end{aligned} \quad (1.2)$$

where  $f_1, f_2, f_3$  are differentiable functions on  $M$  and  $X, Y, Z$  are vector fields on  $M$ . In such case we will write the manifold as  $M(f_1, f_2, f_3)$ . This kind of manifolds appears as a natural generalization of the Sasakian-space-forms by taking:  $f_1 = \frac{c+3}{4}$  and  $f_2 = f_3 = \frac{c-1}{4}$ , where  $c$  denotes constant  $\phi$ -sectional curvature. The  $\phi$ -sectional curvature of generalized Sasakian-space-form  $M(f_1, f_2, f_3)$  is  $f_1 + 3f_2$ . Moreover, cosymplectic space-form and Kenmotsu space-form are also considered as particular types of generalized Sasakian-space-form. The generalized Sasakian-space-forms have also been studied in ([2–14]) and many others.

The notion of local symmetry of Riemannian manifolds has been weakened by many authors in several ways to a different extent. As a weaker version of local symmetry, Takahashi [15] introduced the notion of local  $\phi$ -symmetry on a Sasakian manifold. Generalizing the notion of local  $\phi$ -symmetry of Takahashi [15], De et al [16] introduced the notion of  $\phi$ -recurrent Sasakian manifold.

The notion of generalized recurrent manifolds introduced and studied by De and Guha [17]. A Riemannian manifold  $(M, g)$  is called generalized recurrent [17] if its curvature tensor  $R$  satisfies the condition

$$\nabla R = A \otimes R + B \otimes G, \quad (1.3)$$

where  $A$  and  $B$  are two non-vanishing 1-forms defined by  $A(X) = g(X, \rho_1)$ ,  $B(X) = g(X, \rho_2)$  and the tensor  $G$  is defined by

$$G(X, Y)Z = g(Y, Z)X - g(X, Z)Y \quad (1.4)$$

for all  $X, Y, Z \in \chi(M)$ ;  $\chi(M)$  being the Lie algebra of smooth vector fields and  $\nabla$  denotes the covariant differentiation with respect to the metric  $g$ . Here  $\rho_1$  and  $\rho_2$  are vector fields associated with 1-forms  $A$  and  $B$  respectively.

A Riemannian manifold  $(M, g)$  is called a *generalized Ricci-recurrent* [18] if its Ricci tensor  $S$  of type  $(0,2)$  satisfies the condition

$$\nabla R = A \otimes S + B \otimes g, \tag{1.5}$$

where  $A$  and  $B$  are defined as above.

Generalizing the notion of  $\phi$ -recurrent Sasakian manifold, Patil et al. [19] studied the notion of *generalized  $\phi$ -recurrent Sasakian manifolds*.

Motivated by the above studies, the object of the present paper is to study generalized  $\phi$ -recurrent generalized Sasakian-space-forms. The paper is organized as follows. Section 2 is concerned with preliminaries. Section 3 is devoted to the study of generalized  $\phi$ -recurrent generalized Sasakian-space-forms. Here it is shown that such a space form is generalized Ricci-recurrent. In section 4, we deal with generalized concircular  $\phi$ -recurrent generalized Sasakian-space-forms.

## 2 Preliminaries

In an almost contact metric manifold, we have [20]

$$\phi^2(X) = -X + \eta(X)\xi, \quad \phi\xi = 0, \tag{2.1}$$

$$\eta(\xi) = 1, \quad g(X, \xi) = \eta(X), \quad \eta(\phi X) = 0, \tag{2.2}$$

$$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y), \quad g(\phi X, Y) = -g(X, \phi Y). \tag{2.3}$$

In addition to the relation (1.2), for a  $(2n + 1)$ -dimensional  $(n > 1)$  generalized Sasakian-space-form  $M^{2n+1}(f_1, f_2, f_3)$  the following relations also hold [1]:

$$S(X, Y) = (2nf_1 + 3f_2 - f_3)g(X, Y) - \{3f_2 + (2n - 1)f_3\}\eta(X)\eta(Y), \tag{2.4}$$

$$r = 2n(2n + 1)f_1 + 6nf_2 - 4nf_3, \tag{2.5}$$

$$R(X, Y)\xi = (f_1 - f_3)\{\eta(Y)X - \eta(X)Y\}, \tag{2.6}$$

$$R(\xi, X)Y = (f_1 - f_3)\{g(X, Y)\xi - \eta(Y)X\}, \tag{2.7}$$

$$S(X, \xi) = 2n(f_1 - f_3)\eta(X), \tag{2.8}$$

$$\nabla_X \xi = -(f_1 - f_3)\phi X, \quad (\nabla_X \eta)(Y) = g(\nabla_X \xi, Y) = -(f_1 - f_3)g(\phi X, Y), \tag{2.9}$$

$$\begin{aligned} (\nabla_W R)(X, Y)\xi &= [df_1(W) - df_3(W)][\eta(Y)X - \eta(X)Y] \\ &+ (f_1 - f_3)^2[g(X, \phi W)Y - g(Y, \phi W)X] \\ &+ (f_1 - f_3)R(X, Y)\phi W. \end{aligned} \tag{2.10}$$

For a contact metric generalized Sasakian-space-form, we recall the following theorems due to P. Alegre et al [1]:

**Theorem 2.1** ([1]). *Let  $M(f_1, f_2, f_3)$  be a generalized Sasakian-space-form. If  $M$  is a contact metric manifold, then  $f_1 - f_3$  is constant on  $M$ .*

**Theorem 2.2** ([1]). *Let  $M(f_1, f_2, f_3)$  be a generalized Sasakian-space-form. If  $M$  is a contact metric manifold with  $f_3 = f_1 - 1$ , then it is a Sasakian manifold.*

The above results will be useful in later sections.

### 3 Generalized $\phi$ -Recurrent Generalized Sasakian-Space-Forms

**Definition 3.1.** A generalized Sasakian-space-form  $M^{2n+1}(f_1, f_2, f_3)$  is said to be *generalized  $\phi$ -recurrent* if the relation

$$\phi^2((\nabla_W R)(X, Y)Z) = A(W)R(X, Y)Z + B(W)G(X, Y)Z, \quad (3.1)$$

for any vector field  $X, Y, Z$  and  $W$ , where  $A$  and  $B$  are two non-vanishing 1-forms such that  $A(X) = g(X, \rho_1)$ ,  $B(X) = g(X, \rho_2)$ . Here,  $\rho_1$  and  $\rho_2$  are vector fields associated with 1-forms  $A$  and  $B$  respectively.

Now, if  $B = 0$  in (3.1), then  $M^{2n+1}(f_1, f_2, f_3)$  turns into the notion of  $\phi$ -recurrent generalized Sasakian-space-form.

If, in particular, the 1-forms  $A$  and  $B$  are vanishes in (3.1), then  $M^{2n+1}(f_1, f_2, f_3)$  reduces to be a  $\phi$ -symmetric generalized Sasakian-space-form.

Let us assume that a generalized Sasakian-space-form is generalized  $\phi$ -recurrent. Then by virtue of (2.1), we have from (3.1) that

$$\begin{aligned} & -(\nabla_W R)(X, Y)Z + \eta((\nabla_W R)(X, Y)Z)\xi \\ & = A(W)R(X, Y)Z + B(W)G(X, Y)Z, \end{aligned} \quad (3.2)$$

from which it follows that

$$\begin{aligned} & -g(\nabla_W R)(X, Y)Z, U) + \eta((\nabla_W R)(X, Y)Z)\eta(U) \\ & = A(W)g(R(X, Y)Z, U) + B(W)g(G(X, Y)Z, U). \end{aligned} \quad (3.3)$$

Taking an orthonormal frame field and then contracting (3.3) over  $X$  and  $U$  and then using (1.4), (2.2), (2.7), (2.9) and the relation  $g((\nabla_W R)(X, Y)Z), U) = -g((\nabla_W R)(X, Y)U), Z)$ , we have

$$\begin{aligned} (\nabla_W S)(Y, Z) & = -A(W)S(Y, Z) \\ & + [df_1(W) - df_3(W) - (2n - 1)B(W)]g(Y, Z) \\ & + [df_1(W) - df_3(W)]\eta(Y)\eta(Z), \end{aligned} \quad (3.4)$$

which can be written as

$$\nabla S = -A \otimes S + D \otimes g + E \otimes \eta \otimes \eta,$$

where  $D(W) = df_1(W) - df_3(W) - (2n - 1)B(W)$  and  $E(W) = df_1(W) - df_3(W)$ . Thus we can state the following:

**Theorem 3.2.** *A generalized  $\phi$ -recurrent generalized Sasakian-space-form is generalized Ricci-recurrent if and only if  $f_1 - f_3$  is constant.*

Let us notice that the assumption  $f_1 - f_3 = 1$  in Theorem 2.1 is coherent with Theorem 2.2. In that case, the constant is just 1. Hence by taking account of Theorem 2.1 and Theorem 2.2 in Theorem 3.2, we can state the following corollary:

**Corollary 3.3.** *A generalized  $\phi$ -recurrent Sasakian manifold is generalized Ricci-recurrent.*

Next, setting  $Z = \xi$  in (3.2), we obtain

$$-(\nabla_W R)(X, Y)\xi = A(W)R(X, Y)\xi + B(W)G(X, Y)\xi. \tag{3.5}$$

By virtue of (1.4) and (2.6) it follows from (3.5) that

$$-(\nabla_W R)(X, Y)\xi = [(f_1 - f_3)A(W) + B(W)]\{\eta(Y)X - \eta(X)Y\}. \tag{3.6}$$

From (2.10) and (3.6), we get

$$\begin{aligned} & -[df_1(W) - df_3(W)][\eta(Y)X - \eta(X)Y] \\ & - (f_1 - f_3)^2[g(X, \phi W)Y - g(Y, \phi W)X] - (f_1 - f_3)R(X, Y)\phi W \\ & = [(f_1 - f_3)A(W) + B(W)][\eta(Y)X - \eta(X)Y]. \end{aligned} \tag{3.7}$$

Replacing  $W$  by  $\phi W$  in (3.7) and using (2.1) and (2.6), we obtain

$$\begin{aligned} (f_1 - f_3)R(X, Y)W & = (f_1 - f_3)^2[g(Y, W)X - g(X, W)Y] \\ & + [df_1(\phi W) - df_3(\phi W) + (f_1 - f_3)A(\phi W) \\ & + B(\phi W)][\eta(Y)X - \eta(X)Y]. \end{aligned} \tag{3.8}$$

This leads to the following:

**Theorem 3.4.** *In a generalized  $\phi$ -recurrent generalized Sasakian-space-form, the curvature tensor  $R$  is given by (3.8).*

**Corollary 3.5.** *A generalized  $\phi$ -recurrent Sasakian manifold  $M^{2n+1}(\phi, \xi, \eta, g)$ ,  $n > 1$ , is a space of constant curvature, provided that  $X$  and  $Y$  are orthogonal to  $\xi$ .*

From (3.8) we have

$$\begin{aligned} (f_1 - f_3)S(Y, W) & = 2n(f_1 - f_3)^2g(Y, W) + 2n[df_1(\phi W) - df_3(\phi W) \\ & + (f_1 - f_3)A(\phi W) + B(\phi W)]\eta(Y). \end{aligned} \tag{3.9}$$

Setting  $Y = \xi$  in (3.9) and using (2.2) and (2.8), we obtain

$$df_1(\phi W) - df_3(\phi W) + (f_1 - f_3)A(\phi W) + B(\phi W) = 0. \tag{3.10}$$

By virtue of (3.10) it follows from (3.9) that

$$S(Y, W) = 2n(f_1 - f_3)g(Y, W), \tag{3.11}$$

provided  $f_1 \neq f_3$ . Hence, the manifold under consideration is Einstein.

Hence we can state the following:

**Theorem 3.6.** *A generalized  $\phi$ -recurrent generalized Sasakian-space-form is an Einstein manifold provided  $f_1 \neq f_3$ .*

Also, we can able to state:

**Corollary 3.7.** *A generalized  $\phi$ -reccurent Sasakian manifold is an Einstein manifold.*

The above Corollary was proved by Patil et al [11] in another way.

## 4 Generalized Concircular $\phi$ -Recurrent Generalized Sasakian-Space-Forms

**Definition 4.1.** A generalized Sasakian-space-form  $M^{2n+1}(f_1, f_2, f_3)$  is said to be *generalized concircular  $\phi$ -recurrent* if its concircular curvature tensor  $\tilde{C}$ , given by [21]

$$\tilde{C}(X, Y)Z = R(X, Y)Z - \frac{r}{2n(2n+1)}G(X, Y)Z \quad (4.1)$$

satisfies the relation

$$\phi^2((\nabla_W \tilde{C})(X, Y)Z) = A(W)\tilde{C}(X, Y)Z + B(W)G(X, Y)Z, \quad (4.2)$$

for any vector fields  $X, Y, Z$  and  $W$ , where  $A$  and  $B$  are defined as in (3.1) and  $r$  is the scalar curvature of the manifold.

In particular, if  $A = 0$  and  $B = 0$  then  $M^{2n+1}(f_1, f_2, f_3)$  is said to be concircular  $\phi$ -symmetric generalized Sasakian-space-form.

Let us consider a generalized concircular  $\phi$ -recurrent generalized Sasakian-space-form. Then by virtue of (2.1) it follows from (4.2) that

$$\begin{aligned} (\nabla_W \tilde{C})(X, Y)Z &= \eta((\nabla_W \tilde{C})(X, Y)Z)\xi \\ &- A(W)\tilde{C}(X, Y)Z - B(W)G(X, Y)Z, \end{aligned} \quad (4.3)$$

from which it follows that

$$\begin{aligned} g(\nabla_W \tilde{C})(X, Y)Z, U &= \eta((\nabla_W \tilde{C})(X, Y)Z)\eta(U) \\ &- A(W)g(\tilde{C}(X, Y)Z, U) - B(W)g(G(X, Y)Z, U). \end{aligned} \quad (4.4)$$

Contracting (4.4) over  $X$  and  $U$  and then using (4.1), we get

$$\begin{aligned} (\nabla_W S)(Y, Z) - \frac{dr(W)}{2n+1}g(Y, Z) & \quad (4.5) \\ &= g((\nabla_W \tilde{C})(\xi, Y)Z, \xi) - A(W)[S(Y, Z) \\ &- \frac{r}{2n+1}g(Y, Z)] - 2nB(W)g(Y, Z). \end{aligned}$$

By virtue of (1.4), (2.2), (2.7), (2.9), (4.1) and the relation  $g((\nabla_W \tilde{C})(X, Y)Z, U) = -g((\nabla_W \tilde{C})(X, Y)U, Z)$ , we have

$$\begin{aligned} g((\nabla_W \tilde{C})(\xi, Y)Z, \xi) &= [df_1(W) - df_3(W)] \\ &- \frac{dr(W)}{2n(2n+1)}[g(Y, Z) - \eta(Y)\eta(Z)]. \end{aligned} \quad (4.6)$$

Using (4.6) in (4.5) we get

$$\begin{aligned}
 (\nabla_W S)(Y, Z) &= -A(W)S(Y, Z) + \left[ \frac{2n-1}{2n(2n+1)}dr(W) \right. \\
 &+ \left. df_1(W) - df_3(W) + \frac{r}{2n+1}A(W) - 2nB(W) \right]g(Y, Z) \\
 &+ \left[ \frac{dr(W)}{2n(2n+1)} - \{df_1(W) - df_3(W)\} \right]\eta(Y)\eta(Z),
 \end{aligned}
 \tag{4.7}$$

which can be written as

$$\nabla S = -A \otimes S + \psi \otimes g + H \otimes \eta \otimes \eta,$$

where

$$\psi(W) = \frac{2n-1}{2n(2n+1)}dr(W) + df_1(W) - df_3(W) + \frac{r}{2n+1}A(W) - 2nB(W)$$

and

$$H(W) = \frac{dr(W)}{2n(2n+1)} - \{df_1(W) - df_3(W)\}.$$

Thus we can state the following:

**Theorem 4.2.** *A generalized concircular  $\phi$ -recurrent generalized Sasakian-space-form is generalized Ricci-recurrent if and only if  $H = 0$ .*

It is easy to see that,  $H = 0$  if and only if  $f_1 - f_3$  is constant and  $r$  is constant. Hence in view of Theorems 2.1 and 2.2, we are able to state the following corollaries:

**Corollary 4.3.** *A generalized concircular  $\phi$ -recurrent contact metric generalized Sasakian-space-form with constant scalar curvature is generalized Ricci-recurrent.*

**Corollary 4.4.** *A generalized concircular  $\phi$ -recurrent Sasakian manifold with constant scalar curvature is generalized Ricci-recurrent.*

Next, setting  $Z = \xi$  in (4.3), we obtain

$$(\nabla_W \tilde{C})(X, Y)\xi = -A(W)\tilde{C}(X, Y)\xi - B(W)G(X, Y)\xi. \tag{4.8}$$

Using (1.4), (2.6), (4.1) in (4.8), we get

$$\begin{aligned}
 (\nabla_W R)(X, Y)\xi &= \left[ \frac{dr(W)}{2n(2n+1)} + \frac{r}{2n(2n+1)}A(W) \right. \\
 &- \left. (f_1 - f_3)A(W) - B(W) \right] \{ \eta(Y)X - \eta(X)Y \}.
 \end{aligned}
 \tag{4.9}$$

By virtue of (2.10), (4.9) yields

$$\begin{aligned}
 -(f_1 - f_3)R(X, Y)\phi W &= \left[ df_1(W) - df_3(W) - \frac{dr(W)}{2n(2n+1)} \right. \\
 &- \frac{r}{2n(2n+1)}A(W) + (f_1 - f_3)A(W) \\
 &+ B(W) \left. \right] \{ \eta(Y)X - \eta(X)Y \} \\
 &+ (f_1 - f_3)^2 [g(X, \phi W)Y - g(Y, \phi W)X].
 \end{aligned} \tag{4.10}$$

Replacing  $W$  by  $\phi W$  in (4.10) and using (2.1), we get

$$\begin{aligned}
 (f_1 - f_3)R(X, Y)W &= \left[ df_1(\phi W) - df_3(\phi W) - \frac{dr(\phi W)}{2n(2n+1)} \right. \\
 &- \frac{r}{2n(2n+1)}A(\phi W) + (f_1 - f_3)A(\phi W) \\
 &+ B(\phi W) \left. \right] \{ \eta(Y)X - \eta(X)Y \} \\
 &+ (f_1 - f_3)^2 [g(Y, W)X - g(X, W)Y].
 \end{aligned} \tag{4.11}$$

This leads to the following:

**Theorem 4.5.** *In a generalized concircular  $\phi$ -recurrent generalized Sasakian-space-form, the curvature tensor  $R$  is given by (4.11).*

Contracting (4.11), we get

$$\begin{aligned}
 (f_1 - f_3)S(Y, W) &= 2n(f_1 - f_3)^2g(Y, W) + 2n[df_1(\phi W) - df_3(\phi W)] \\
 &- \frac{dr(\phi W)}{2n(2n+1)} - \frac{r}{2n(2n+1)}A(\phi W) \\
 &+ (f_1 - f_3)A(\phi W) + B(\phi W) \eta(Y).
 \end{aligned} \tag{4.12}$$

Setting  $Y = \xi$  in (4.12) and using (2.8), we obtain

$$\begin{aligned}
 df_1(\phi W) - df_3(\phi W) - \frac{dr(\phi W)}{2n(2n+1)} \\
 - \frac{r}{2n(2n+1)}A(\phi W) + (f_1 - f_3)A(\phi W) + B(\phi W) = 0.
 \end{aligned} \tag{4.13}$$

In view of (4.13), we have from (4.12) that

$$S(Y, W) = 2n(f_1 - f_3)g(Y, W), \quad \text{provided } f_1 \neq f_3. \tag{4.14}$$

This leads to the following:

**Theorem 4.6.** *A generalized concircular  $\phi$ -recurrent generalized Sasakian-space-form with  $f_1 \neq f_3$  is an Einstein manifold.*



**Corollary 4.7.** *A generalized concircular  $\phi$ -recurrent Sasakian manifold is an Einstein manifold.*

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