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A Novel Pairs Trading Model with Mean Reversion and Coefficient of Variance

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Abstract : One of the problems of stock trading is that the stock prices are not consistent, depending on market condition. The Mean Reversion process of Pairs Trading is a market neutral strategy, which is independent of market movements and carried an assumption that any trading index will reverse to its mean value. This paper proposes a novel algorithm, called multiclass Pairs Trading, which is an advance of co-integration method in Pairs Trading technique. The proposed model uses Mean Reversion and coefficient of variance (CV) to analyze and classify a series of stocks to have different distribution. It provides a buffer-trading zone when the paired stocks are about to change their directions from high to low and vice versa. Moreover, this model extends an opportunity for any highly correlated and paired stocks to cross-trade with any lowly correlated and paired stocks. It serves to improve performance in portfolio trading. The 10-year data were collected from 127 stocks listed in the Global Dow. The results show that using the proposed model for the co-integrated Pairs Trading outperforms those of the conventional

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co-integrated Pairs Trading outstandingly. Thus, benefits of this model are not only mitigating stock trading risk but also maximise returns of them.

Keywords : Pairs Trading; Mean Reversion; coefficient of variance; risk mitigation.
2010 Mathematics Subject Classification : 91G99.

1 Introduction

An early attempt at Pairs Trading is credited to Nunzio Tartaglia, a quantitative analyst at Morgan Stanley in the 1980s. Tartaglia gathered a group of professionals with the aim of forming a quantitative arbitrage strategy using statistical techniques. One technique that they implemented was trading pairs of securities. The procedure distinguishes between pairs of security prices that move together. The abnormality in the relationship indicates that the pair will be traded with anticipation that the abnormality will be neutralised in the future. Different schools of thought offer an alternative that is Mean Reversion. In normal circumstance, positive and negative returns on financial assets is temporary. It is because return reverses to the mean from time to time and the speed of the reversing process can vary from one day to one year (Hillebrand, 2004[1]). Lo and Mackinlay (1998)[2], Fama and French (1988)[3], and Poterba and Summers (1988) demonstrated using empirical evidence that positive market return persists over the short term. However, in the long term, profit opportunity is reverted. Wachter (2002)[4]; and Campbell Chan and Viceira (2003)[5] confirmed that Mean Reversion possesses the characteristics of equity index return over the long term.

Additionally, Bessembinder, Coughenour, Seguin, and Smoller (1995)[6] determined that Mean Reversion that exists in the financial markets uses empirical evidence from the term structure of future prices. The data sample of the authors study was based on 11 different future markets including financial, metals, and agriculture markets. The daily settlement price from January 1982 to December 1991 was used. The disadvantage of the study methodology is that it can only spot Mean Reversion in the equilibrium condition of the market, and it cannot be applied when the market is in disequilibrium. Gatev, Goetzmann, and Rouwenhorst (2006)[7] conducted an investigation into the risk and return characteristics of Pairs Trading using data from 1962 to 2002. The authors showed that simple Mean Reversion for a single stock index could not produce clear values. However, the values can be generated when trading suitably formulate pairs of stocks. Perlin (2007)[8] proposed a multivariate version of Pairs Trading, which developed an artificial pair for a stock based on the information of assets. This method assessed the performance of three versions of the multivariate approach for the Brazilian stock market using data for 57 assets from 2000 to 2006. The examination of performance was conducted using the calculation of raw returns, excessive returns, beta, and alpha. Mudchanatongsuk, Primbs and Wong (2008)[9] investigated a uniform and analytical framework to implement Pairs Trading on arbitrary pairs and suggested an asset pricing-based model to parameterise Pairs Trading that included theoretical considerations rather than statistical history. Huck (2010)[10] proposed a general and flexible framework for the selection of random pairs. Multiple return forecasts based on bivariate information sets and multi-criteria decision techniques were implemented. Currently, there are four main approaches to Pairs Trading: i) the co-integration method (Vidyamurthy, 2004[11]), ii) the distance method (Gatev et al, 2006[7]), iii) the stochastic spread method (Elliott, Van Der Hoek, and Malcolm, 2004[12]), and iv) Combined Forecasts and Multi-Criteria Decision Methods (MCDM) (Huck, 2010[10]).

The objective of this study is to introduce an advanced model of the current co-integration using in the Pairs Trading technique, called, multiclass Pairs Trading. It analyses and classifies a series of stocks to have different distribution. This newly invented technique improves risk mitigation by providing a buffer-trading zone when the paired stocks are about to change their directions from high to low and vice versa. Moreover, the proposed model extends an opportunity for a highly correlated and paired stock to cross-trade with any lowly correlated and paired stock. It serves to improve performance in portfolio trading.

2 Theoretical Considerations

2.1 The Proposed Multiclass Pairs Trading

The methodology of this research based on Pairs Trading using Mean Reversion and CV. The Mean Reversion technique analyses any dataset whose distributions move from upward to downward directions and vice versa. Following, we introduce classification technique using coefficient of variance to group the stock indexes (variable datasets, and now called datasets), followed by the Mean Reversion technique.

In theory, the conventional co-integrated Pairs Trading method identifies two stocks that move in time series together and calculate a correlation between them. The model begins by normalising the datasets using the mean and standard deviation followed to co-integration them with Pearsons correlation coefficient(ρ), and it represents by

$$\frac{cov(x_i, y_i)}{\rho_{x_i}, \rho_{y_i}} = \frac{E[(x_i - \mu_{x_i})(y_i - \mu_{y_i})]}{\rho_{x_i}\rho_{y_i}} \tag{2.1}$$

where $cov(x_i, y_i)$ represents the covariance of x_i, and y_i, when $i = 1, 2, ..., n$. Following, we select the paired stocks in order from high to low.

Next, we introduce the Mean Reversion and CV to analyse and group the datasets. The Mean Reversion algorithm is expressed(Premanode et al., 2013[13]) as follows:

i) Compute the mean $\mu_i(t)$ of $x_i(t)$, where $i = 1, 2, ..., n$.

ii) Compute the variance $V_i(t)$ of $x_i(t)$.

iii) By normalising each $V_i(t)$ using $\mu_i(t)$, we obtain $\frac{V_i(t)}{\mu_i(t)}$.

iv) Using the datasets $x_i(t)$ from the upward scenario,we calculate and plot $V_1(t) > V_2(t) > \ldots > V_{i-1}(t) > V_i(t)$.

v) The same process is applied to the downward scenario where $V_1(t) < V_2(t) < \ldots < V_{i-1}(t) < V_i(t)$.

vi) If $\frac{V_i(t)}{\mu_i(t)} = \frac{V_{i-1}(t)}{\mu_{i-1}(t)}$, ignore the calculation, but move the plot one step forward.

vii) Repeat the steps in items iv) to vi) and stop when $i = n$.

viii) We obtain a curve of $x_i(t)$ that marks points of local maxima and minima.

In the next process, we introduce the coefficient of variance (CV) to compute the datasets, at which is represented by

$$CV_i = \frac{\rho_i}{\mu_i} \tag{2.2}$$

where ρ_i represents standard deviation and μ_i represents mean. Consequent to applying the Mean Reversion and CV, we derived a number of groups of datasets and termed them to CV. Each CV may then have different normal distribution, reflecting different values for the paired stock indices. Following to plotting standard deviation, we divide the datasets into six classes in time series; namely, $CV_1, CV_2, CV_3, CV_4, CV_5$ and CV_6. we then plot the mean of CV_1 to CV_6 between the mean of CV_3 and CV_4. Hence, in the normal distribution, standard deviation of the CV_1 should be significantly deviated greater than the CV_2. Applying the same rationale, standard deviation of the CV_6 is significantly deviated greater than CV_5. In each CV, we calculate the return Pairs Trading (Perlin, 2007[8]) using Eq.(2.3). The co-integrated Pairs Trading formula is expresses as follows:

$$R_{CO} = \sum_{t=1}^{T}\sum_{i=1}^{n} R_i(t) \cdot I_i^{L\&S}(t) \cdot W_i + \left(\sum_{t=1}^{T}\sum_{i=1}^{n} Tc_i(t) \cdot \left[ln\left(\frac{1-C}{1+C} \right) \right] \right) \tag{2.3}$$

where $R_i(t)$ represents the real return of asset i at time t, calculated by $ln\left(\frac{P_i(t)}{P_i(t-1)}\right)$; $I_i^{L\&S}(t)$ represents the dummy variable with a value of 1 if a Long position is created for the asset i, a value of -1 if a short position is created, and 0 otherwise; $Tc_i(t)$ represents the dummy variable that takes a value of 1 if a transaction is made for the asset i at time t and 0 otherwise; C represents the transaction cost per operation (by percentage); T represents the number of observations on the whole trading period, and

$$W_i(t) = \frac{1}{\sum_{i=1}^{n} |I_i^{L\&S}(t)|} \qquad \text{for} \quad |I_i^{L\&S}(t)| = \begin{cases} 1 & \text{if trade exist;} \\ 0 & \text{if no trade.} \end{cases} \tag{2.4}$$

where $W_i(t)$ is the weighting variable that controls for portfolio construction at time t, assuming that the same weight is applied to each transaction.

2.2 Benefits of the Multiclass Pairs Trading

Since the co-integrated Pairs Trading is used to buying a stock, commodity or currency under the expectation that the asset will rise or fall in value from time to time. As a result, the long position is exercised when the curve of a paired stock is at high peak (maxima). Whereas, the short position is exercised when the paired stock is moving at the low peak (minima). With the proposed multiclass Pairs Trading, there are two extra benefits, which are follows:

i) By applying the proposed model to the historical trading datasets, we then found that a number of paired stocks could distribute to any CV, depending on there values of Mean Reversion and CV. An example is given that the highest correlated paired stock may locate in CV_1. Once the trade begins within any CV, we can exercise either long or short positions in time series until the existing CV starts to change the new CV. In the situation where the stock starts to diverge, we then analyse the new CV and compile it with the historical CV datasets. Hence, the trading can resume. Since the stocks are traded within the same CV from time to time, the returns are maximised. Without using the proposed model, we will never know when the correlation of any paired indices is about to diverge.

ii) With respect to portfolio trading, there is a possibility that any stock indices in the different correlation can be cross-paired and cross-traded among them, provide that they have shared the same CV. Thus, it creates additional trading opportunities inasmuch as risk is minimised.

3 Data

The datasets were composed of 127 daily stocks recorded in the Global Dow. Table 1 presents the blue chip stocks of companies with a national reputation for reliability, quality, and the capability to operate profitably under extreme market conditions. The stocks are among the most widely and actively traded ones. The datasets contain daily stock prices over a 10-year period from 1 August 2002 as shown in Table 1. Saturday and Sunday price observations were removed prior to the analysis to avoid any bias in the results from weekend market closures.

4 Simulation and Results

4.1 Generating the Mean Regression and CV

Referring to Bloomberg terminal, Table 1 summarises the 127 datasets of Global Dow indices in the year 2013. Following, Fig. 1 presents simulation procedure of the proposed multi-class Pairs Trading model using Mean Reversion and CV, and it is expressed in order as follows:

Table 1: The 127 listed companies in Global Dow indices in the year 2013

3M Co.	China Mobile Ltd.	Infosys Technologies	Royal Bank of Canada
ABB Ltd.	China Unicom Ltd.	Intel Corp.	Samsung Electronics
Abbott Laboratories	Cisco Systems Inc.	IBM Corp.	SAP AG
Alcoa Inc.	CLP Holdings Ltd.	Johnson & Johnson	Schlumberger Ltd.
Allianz SE	Coca-Cola Co.	JPMorgan Chase & Co.	Siemens AG
Amazon.com Inc.	Colgate-Palmolive Co.	Komatsu Ltd.	Societe Generale S.A.
America Movil S.A.B. de C.V. Series L	Compagnie de Saint-Gobain S.A.	L.M. Ericsson Telephone Co. Series B	Sony Corp. Sony Corp.
American Express Co.	Companhia Energetica de Minas Gerais-CEMIG Pr	LVMH Moet Hennessy Louis Vuitton	Southwest Airlines Co.
Amgen Inc.	ConocoPhillips	McDonald's Corp.	Taiwan Semiconductor Manufacturing Co. Ltd.
Anglo American PLC	Credit Suisse Group	Medtronic Inc.	Takeda Pharmaceutical Co. Ltd.
Anheuser-Busch InBev	Daimler AG	Merck & Co. Inc.	Tata Steel Ltd.
N.V.			
Apple Inc.	Deere & Co.	Microsoft Corp.	Telefonica S.A.
Assicurazioni Generali	Deutsche Bank AG	Mitsubishi Corp.	Tesco PLC
S.p.A.			
AstraZeneca PLC	E.I. DuPont de Nemours & Co.	Mitsui & Co. Ltd.	Time Warner Inc.
AT&T Inc.	E.ON AG	Monsanto Co.	Toshiba Corp.
BAE Systems PLC	eBay Inc.	National Australia Bank Ltd.	Total S.A.
Banco Bilbao Vizcaya Argentaria S.A.	EDP-Energias de Portugal S.A.	National Grid PLC	Toyota Motor Corp.
Banco Santander S.A.	Esprit Holdings Ltd.	Nestle S.A.	Travellers Cos. Inc.
Bank of America Corp.	Express Scripts. Inc.	News Corp. ClA	UBS AG
Bank of New York Mellon Corp.	Exxon Mobil Corp.	Nike Inc. ClB	UniCredit S.p.A.
BASF S.E.	FedEx Corp.	Nintendo Co. Ltd.	United Parcel Service Inc.
Baxter International Inc.	Freeport-McMoRan Copper & Gold Inc.	Nippon Steel Corp.	United Tech. Corp.
BHP Billiton Ltd.	General Electric Co.	Nokia Corp.	Vale S.A. Pref A
BNP Paribas S.A.	Gilead Sciences Inc.	Panasonic Corporation	Veolia Environment S.A.
Boeing	GlaxoSmithKline PLC	Petroleo Brasileiro S/A Pref	Verizon Com. Inc.
BP PLC	Goldman Sachs Group Inc.	Pfizer Inc.	Vestas Wind Systems A/S
Bridgestone Corp.	Hewlett-Packard Co.	Potash Corp. of Saskatchewan Inc.	Vinci S.A.
Canon Inc.	Home Depot Inc.	Procter & Gamble Co.	Vodafone Group PLC
Carnival Corp.	Honda Motor Co. Ltd.	Reliance Industries Ltd.	Wall-Mart Stores Inc.
Carrefour S.A.	Honeywell International Inc.	Research in Motion Ltd.	Walt Disney Co.
Caterpillar Inc.	HSBC Holdings PLC UK Re	Rio Tinto PLC	Wells Fargo
Chevron Corp.	Hutchison Whampoa Ltd.	Roche Holding AG Part.	N.A.

i) Assign a matrix $x_{ki}(t)$ where k represents the number of columns, $k = 127$ and i represents the number of rows, $i = 3961$

ii) By normalising the matrix of $x_{ki}(t)$, we obtain $A_{ki}(t)$

iii) Calculate $A_{ki}(t)$ for $k = 127$ and $i = 3961$

iv) By selecting the highest return of $A_{ki}(t)$ using the Person's correlation coefficient, we obtain $x_{p1}(t)$ and $x_{p2}(t)$ in time series, see results in Table 3 and 4

v) Use the Mean Reversion algorithm in section 2.1 to compute each point of reverse of $x_{p1}(t)$ and $x_{p2}(t)$ in time series. Then mark the reversed local maxima and minima of $x_{p1}(t)$ and $x_{p2}(t)$ in time series

vi) Compute each local $x_{p1}(t)$ and $x_{p2}(t)$ in time series with the coefficient of variance (CV)

vii) Thus, the local $x_{p1}(t)$ and $x_{p2}(t)$ in time series are grouped into different $CV_1, CV_2, \ldots, CV_n$, and termed to $x_{p1}(t_{CV})$ and $x_{p2}(t_{CV})$

viii) Calculate expected returns of the local $x_{p1}(t)$, $x_{p2}(t)$, $x_{p1}(t_{CV})$, and $x_{p2}(t_{CV})$

ix Next, we compare the expected returns of $x_{p1}(t)$ and $x_{p2}(t)$ (the original datasets) with the returns of $x_{p1}(t_{CV})$ and $x_{p2}(t_{CV})$ (the datasets, which are applied the Mean Reversion and CV). The probabilities for calculating the expected returns of $x_{p1}(t)$, $x_{p2}(t)$, $x_{p1}(t_{CV})$ and $x_{p2}(t_{CV})$ using Markov chain are listed in Table 5 and 6. Moreover, the expected returns of $x_{p1}(t)$, $x_{p2}(t)$, $x_{p1}(t_{CV})$ and $x_{p2}(t_{CV})$ are shown in Table 7 and 8

x) For robustness test, use the same procedures listed in item v) and item vi) calculating the expected returns of another ten cross-pairing that listed in table 2. Then compare the expected returns of ten cross-pairing stocks of $x_{p1}(t)$ and $x_{p2}(t)$ (the original datasets) with the $x_{p1}(t_{CV})$ and $x_{p2}(t_{CV})$, the datasets which have applied the Mean Reversion and CV, are also shown in Table 9 and 10.

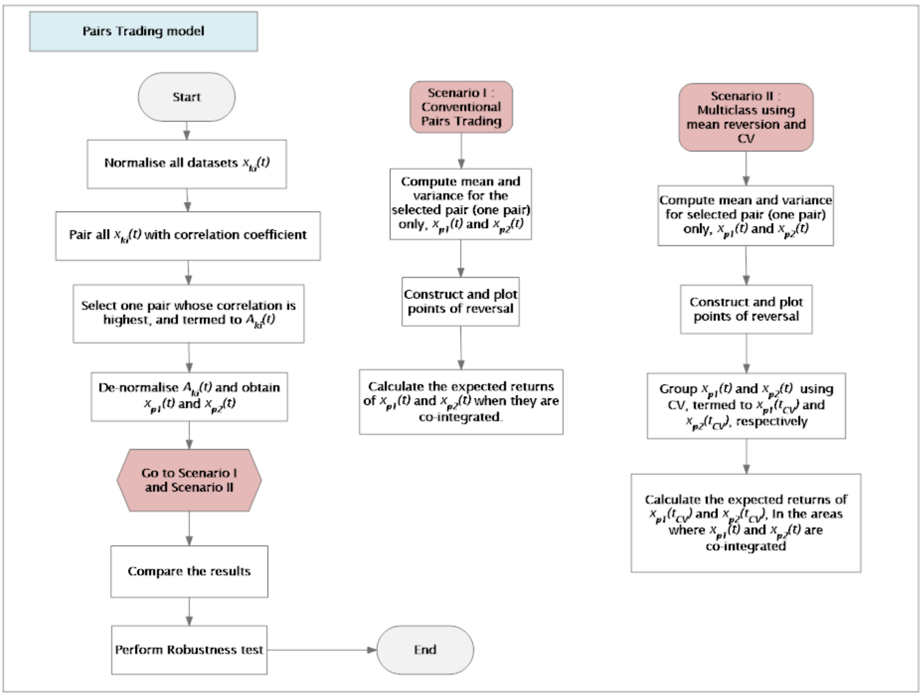

Figure 1: Procedure of the multiclass Pairs Trading model

The workflow of the multi-class Pairs Trading demonstrated in Figure 1 is started by normalising all the datasets $x_{ki}(t)$, pairing $x_{ki}(t)$ with Pearson's coefficient. Then, we select the pair that has the highest value of CV and term to $A_{ki}(t)$, and de-normalising the paired of $A_{ki}(t)$. Finally, we obtain $x_{p1}(t)$ and $x_{p2}(t)$. The next step is to calculate the multi-class Pairs Trading using Scenario II. The results of Scenario II are then subject to compare with Scenario I which is the conventional co-integration of the paired trading.

In Scenario I, we calculate the expected returns of co-integrated $x_{p1}(t)$ and $x_{p2}(t)$, see Table 7 and 8, using probability in Table 5 and 6 whereas we process Scenario II with the following:

- compute mean and variance of $x_{p1}(t)$ and $x_{p2}(t)$
- construct point of reversal using items i) to viii) under Section 2.1
- group $x_{p1}(t)$ and $x_{p2}(t)$ and use Equation 2.2 to compute Mean Reversion and CV, then termed to $x_{p1}(t_{CV})$ and $x_{p2}(t_{CV})$. Next, we calculate probabilities and the expected returns of $x_{p1}(t_{CV})$ and $x_{p2}(t_{CV})$, resulted in Table 5, 6, 7 and 8, respectively.

4.2 Results in Pairing the Normalised Datasets

Consequent to the procedural workflow presented in Figure 1, all of the datasets are normalised. We introduce the Pearson's correlation coefficient to measure the degree of correlation among the paired stock indices. Because there are 127 datasets, we cross-map each stock price and neglect redundant pairings.

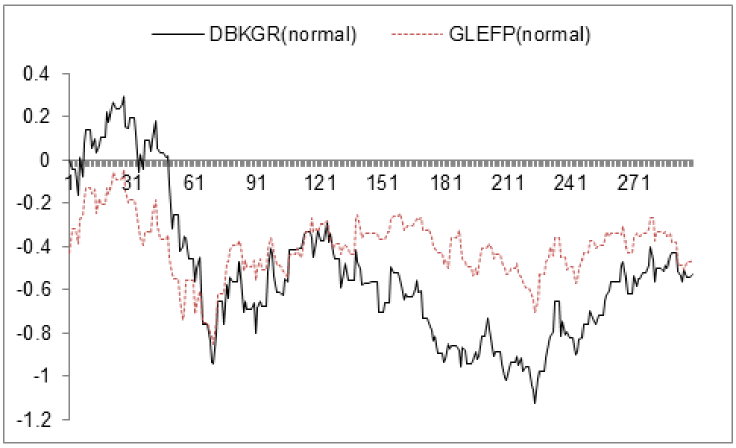

Figure 2: Performance of the highest correlation coefficient, DBKGR and GLEFP

Because of pairing, there are 8001 pairs. We have found that Deutsche Bank AG (DBKGR) and Societe Generale S.A. (GLEFP) stock share the highest correlation coefficient of 0.973049. Figure 2 presents two graphs, DBKGR and GLEFP.

To ease a presentation, the x-axis represents a sample of 300 datasets, whereas the y-axis represents the normalised values ranging from 0.04 to -1.00. This implies that the pairs of DBKGR and GLEFP performed close to the mean comparing to the standard deviation at the scale of ± 3. We present the ranking of top ten pairs out of 8001 pairs and their correlation coefficients in Table 2.

Table 2: Top ten pairs from the Global Dow Index that share a high correlation coefficient value

Rank	Stock #1	Stock #2	Correlation Coefficient
1	DBKGR	GLEFP	0.9730
2	AMZNUS	IBMUS	0.9718
3	NKEUS	MCDUS	0.9717
4	VALE5BZ	BHPAU	0.9683
5	ABBSS	BHPAU	0.9654
6	BBVASM	GLEFP	0.9650
7	X8058JP	X8031JP	0.9629
8	AAPLUS	IBMUS	0.9627
9	UBSNVX	GLEFP	0.9609
10	FCXUS	RIOLN	0.9596

4.3 Results in Using Mean Reversion and CV

Referring to Table 2, we select the highest correlation coefficient pair, the DBKGR and GLEFP and simulate those datasets separately with Mean Reversion and CV. They are outlined in the items i) to viii) in section 2.1. At this stage, the datasets have been partitioned into different CV values in time series.

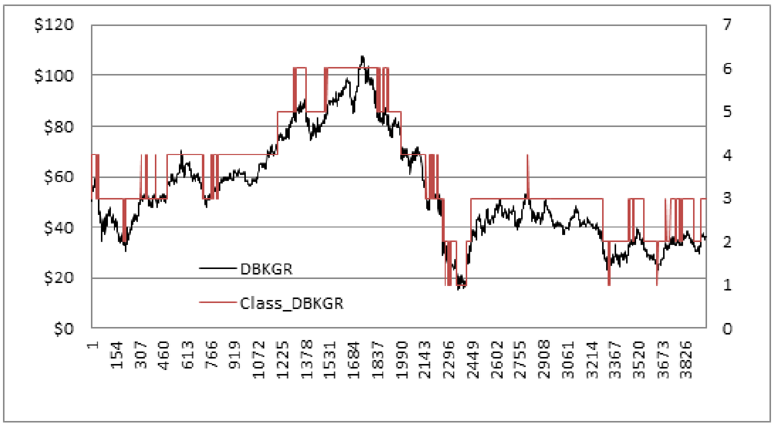

Figure 3: DBKGR showing the different CVs comparing the original datasets

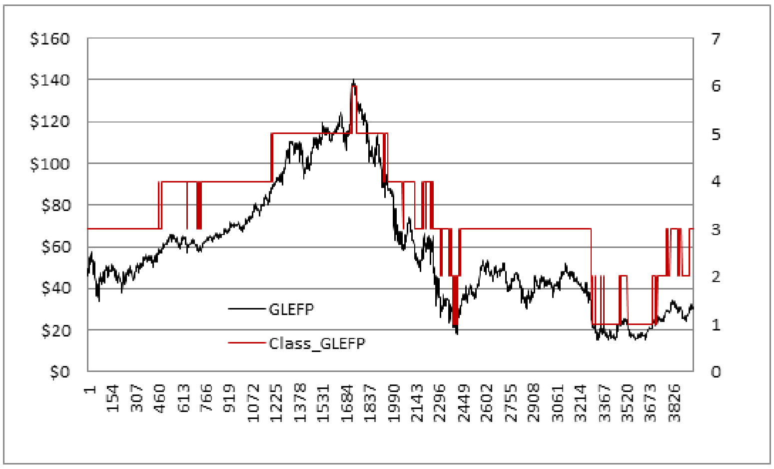

Figure 4: GLEFP showing the different CVs comparing the original datasets

Figure 3 and 4 show the performance of Mean Reversion and CV by plotting six different CV classes, and two original datasets, DBKGR and GLEFP. Of those six CV classes, the x-axis represents the entire datasets in time series; whereas, the y_1-axis represents the stock values of DBKGR and GLEFP, and the CV values use the scale of the y_2-axis.

4.4 Risk Mitigation Using Mean Reversion and CV

There are six CV classes showing the minimum to maximum values of datasets in each class. Apparently, it is illustrated in Table 3 and 4. With the remark, the current DBKGR and GLEFP datasets have no longer formatted in time series.

For risk mitigation of any stock trading, we utilise contents in Table 1 starting from the following:

i) Collect historical minimum and maximum records/units of Pairs Trading for a particular period, e.g., 500 daily records/units of DBKGR and GLEFP.

ii) Match the present observed prices of DBKGR and GLEFP with one of the CV classes.

   a) In case of non-volatility, the future price will behave and situate in the same CV class, use Long and Short positions for trading. It is because we assume that the future stock prices of DBKGR and GLEFP will probability fit into the existing CV class.

   b) If the new observed prices are highly volatile and run out of the situated CV class, stop trading.

   c) If the new observed prices are equal to the previous prices, continue to trade by using the last position.

iii) Update Table 3 and 4 and going step i).

iv) Check the new volatility with variance changes.

v) To continue trading, loop the procedures in step ii) to step iv).

Table 3: Detailed classification of DBKGR, prices in US dollars

DBKGR					
Class	CV	Range	Units	Mean	Variance
1	0.14	15-30	435	25.50	13.22
2	0.12	31-48	1436	39.65	21.87
3	0.03	49-53	418	50.98	2.97
4	0.03	54-59	357	57.04	3.12
5	0.12	60-87	986	72.41	78.75
6	0.05	88-107	329	94.75	28.94

Table 4: Detailed classification of GLEFP, prices in US dollars

GLEFP					
Class	CV	Range	Units	Mean	Variance
1	0.16	15-26	522	20.52	10.18
2	0.17	27-50	1395	40.38	45.14
3	0.05	51-57	351	53.61	6.54
4	0.02	58-64	341	61.62	2.23
5	0.19	65-109	963	82.54	227.19
6	0.07	110-141	389	117.8	59.62

4.5 Proof Concept of the Mean Reversion and CV

This section is to proof that in the co-integrated Pairs Trading using the proposed Mean Reversion and CV model can outperform the conventional co-integrated Pairs Trading (without using the Mean Reversion and CV).

Initially, we calculate probabilities of the DBKGR and GLEFP assuming that the chance of the future stock prices moving either upward or downward is equal, at which both probabilities are 0.5. On contrary, the probabilities of the DBKGR and GLEFP using the Mean Reversion and CV are better than those of the conventional co-integrated Pairs Trading as displayed in Table 5 and 6.

In terms of comparison, the expected returns of the model using Mean Reversion and CV shown in Table 8 are better than the conventional Pairs Trading, at which listed in Table 7.

Additionally, we conduct robustness test by using other pairs of prices from the Global Dow indices which have shared a high correlation coefficient values listed in Table 1. We found that the expected returns using the conventional Pairs Trading, shown in Table 9 are less than those of Mean Reversion and CV. Thus, we conclude that the proposed model is robust.

4.5.1 Calculation of Probabilities of DBKGR and GLEFP

Using Equation 2.2 and Equation 2.4 to calculate of the expected returns of the co-integrated conventional Pairs Trading, and the co-integrated Pairs Trading using Mean Reversion and CV, we subtitute the value of some elements as follows

- $I_i^{L\&S}(t)$ is 1 if a long position is created for individual return, a value of -1 if a short position is created, and 0 otherwise;
- t represents the dummy variable that takes the value of 1 if a transaction is made for individuals at time t and 0 otherwise;
- C represents the transaction cost per operation and set to 0.25%;
- T represents the number of observations with 3961 data points;
- $W_i(t)$ is weight at position 1.

Each expected returns of the co-integrated $x_{p1}(t)$ and $x_{p2}(t)$ are calculated by using the value of the present observed variables multiplies with the probability of the lag and repeats infinitely in time series. The expected returns of any co-integrated Pairs Trading can be expressed by

$$ER_{CO} = \sum_{i=1}^{n} R_{CO}^i(t) p_{CO}^i(t) \tag{4.1}$$

where $R_{CO}^i(t)$ is the return of co-integrated $x_{p1}(t)$ and $x_{p2}(t)$ in scenario i, $p_{CO}^i(t)$ is the probability for the return $R_{CO}^i(t)$ in scenario i, and i counts the number of scenarios. However, we omit to calculate the first two observations after the stocks reverted. It is because we have taken into consideration that some stock can be highly volatile and immediately reverted. Additionally, the returns of co-integrated $x_{p1}(t_{CV})$ and $x_{p2}(t_{CV})$ can be termed to $R_{CO}^i(t_{CV})$; and the results are listed in Table 7. The expected returns of $R_{CO}^i(t_{CV})$ are inevitably similar to those of the expected returns of $R_{CO}^i(t)$. We calculate probability for expected returns of the conventional co-integrated by assuming that each stock in the same pair can revert to the co-integrated line and vice versa with a probability of 0.5. The total probability reversion of co-integrated pair is calculated to 0.5 multiplies with 0.5, equalling to 0.25. Hence, the total probability of non-reverted pairs moving along time series is 1.00 minus 0.25, equalling 0.75 as illustrated in Table 5.

The difference is that the calculation of the expected returns of $R_{CO}^i(t)$ used the probability listed in Table 6 rather than the fixed of probability employed in the calculation of $R_{CO}^i(t)$, in which is given to 0.75. It is because we assume that any stock prices during the trade can equally move up and down. We introduce Markov chain to calculate probabilities of the conventional co-integrated Pairs Trading the used Mean Reversion and CV. In the Markov chain's process, the value of the present observation is multiplied with the probability of the lag, and it repeats an infinite number of times. Table 6 indicates, DBKGR and GLEFP are ranging from 0.887955 to 0.982759. Whereas the probability of the conventional co-integrated Pairs Trading (without Mean Reversion and CV) remains to 0.75 as illustrated in Table 5.

Table 5: Calculations of the probabilities of conventional co-integrated Pairs Trading (without Mean Reversion and CV)

Index	Probabilities of conventional co-integrated Pairs Trading (without Mean Reversion and CV)
DBKGR	0.75
GLEFP	0.75

Table 6: Calculations of the probabilities of co-integrated Pairs Trading using Mean Reversion and CV

Index	Probabilities of co-integrated Pairs Trading Mean Reversion and CV					
Class	CV_1	CV_2	CV_3	CV_4	CV_5	CV_6
DBKGR	0.9770	0.9805	0.8995	0.8880	0.9757	0.9787
GLEFP	0.9828	0.9770	0.9003	0.8915	0.9626	0.9743

4.5.2 Calculation of Expected Returns

This section consists of two parts, at which the first part represents a calculation for expected returns of co-integrated $x_{p1}(t)$ and $x_{p2}(t)$, $R^i_{CO}(t)$, and the second part represents calculation of expected returns of co-integrated $x_{p1}(t_{CV})$ and $x_{p2}(t_{CV})$, $R^i_{CO}(t_{CV})$.

In Table 7 there are 41 blocks whereas the DBKGR and GLEFP stocks are co-integrated. Each block has different numbers of data ranging from the smallest to the highest values, which are 6 to 1040, respectively. We calculate the different expected returns in each block of DBKGR and GLEFP by using the returns of DBKGR and GLEFP multiply by the same probability value of 0.75. As a result, the total expected return of both co-integrated DBKGR and GLEFP to US$ 2003.77.

As regard to the calculation shown in Table 8, the expected returns of co-integrated $x_{p1}(t_{CV})$ and $x_{p2}(t_{CV})$, $R^i_{CO}(t_{CV})$ using Mean Reversion and CV consist of 94 blocks. In each block the number of data points is ranging from 3 to 245, depending on the distribution of CV classes, e.g., in block 1 there are 4 data points at the ranking of 44^{th} to 47^{th}. We omit to calculate the blocks that have the number of data less than 3. It is because the stocks may be highly volatile from the first two observations when the stocks have been reverted. The probabilities of both DBKGR and GLEFP are based on Markov chain, in which represent the smallest value of 0.887955 and the highest value of 0.982759. Apparently, the returns of co-integrated DBKGR and GLEFP, and the expected returns of co-integrated DBKGR and GLEFP using Mean Reversion and CV are demonstrated, given the total expected returns of both equals US$ 2351.84. However, the allocation of each CV class undertakes values of observations. Thus, during the calculation process; each $R^i_{CO}(t_{CV})$ has never been mixed up. For example, calculation of any $R^i_{CO}(t_{CV})$ in CV_1 will use the observation and probability belonging to its class as shown in Table 8.

Table 8: represents the expected returns in US dollars of the co-integrated Pairs Trading using Mean Reversion and CV

Block no.	Ranking	Data points	CV DBKGR	CV GLEFP	Probability DBKGR	Probability GLEFP	Returns of DBKGR and GLEFP returns	Returns of DBKGR and GLEFP expected returns
1	44^{th}–47^{th}	4	4	4	0.887955	0.977044	4.087128	4.087162
2	79^{th}–81^{st}	3	2	2	0.980488	0.977044	3.065304	3.065297
3	83^{rd}–92^{nd}	10	2	2	0.980488	0.977044	10.217159	10.217149
4	141^{st}–153^{rd}	13	2	2	0.980488	0.977044	13.282038	13.282027
5	156^{th}–159^{th}	4	2	2	0.980488	0.900285	4.086895	4.086883
6	162^{nd}–164^{th}	3	2	2	0.980488	0.977044	3.064994	3.064993
7	171^{st}–277^{th}	107	2	2	0.980488	0.977044	109.316171	109.316182
8	282^{nd}–287^{th}	6	2	2	0.980488	0.977044	6.129851	6.129852
9	372^{nd}–385^{th}	14	3	3	0.899522	0.900285	14.303173	14.303167
10	393^{rd}–397^{th}	5	3	3	0.899522	0.900285	5.108259	5.108259
11	468^{th}–480^{th}	13	3	3	0.899522	0.900285	13.281527	13.281521
12	483^{rd}–486^{th}	4	3	3	0.899522	0.891496	4.086565	4.086569
13	534^{th}–545^{th}	12	4	4	0.887955	0.962617	12.259873	12.259869
14	716^{th}–719^{th}	4	3	3	0.899522	0.900285	4.086628	4.086626
15	731^{st}–735^{th}	5	3	3	0.899522	0.891496	5.108277	5.108275
16	742^{nd}–774^{th}	33	3	3	0.899522	0.891496	33.714525	33.714521
17	780^{th}–783^{rd}	4	4	4	0.887955	0.962617	4.086590	4.086593
18	794^{th}–803^{rd}	10	4	4	0.887955	0.962617	10.216485	10.216488
19	806^{th}–810^{th}	5	4	4	0.887955	0.891496	5.108249	5.108249
20	821^{st}–846^{th}	26	4	4	0.887955	0.962617	26.562848	26.562858
21	848^{th}–851^{st}	4	4	4	0.887955	0.962617	4.086619	4.086618
22	873^{rd}–879^{th}	7	4	4	0.887955	0.962617	7.151586	7.151585
23	882^{nd}–886^{th}	5	5	5	0.975659	0.962617	5.108251	5.108251
24	891^{st}–903^{rd}	13	5	5	0.975659	0.962617	13.281491	13.281490
25	906^{th}–913^{th}	8	4	4	0.887955	0.962617	8.173207	8.173206
26	916^{th}–918^{th}	3	5	5	0.975659	0.962617	3.064923	3.064924
27	925^{th}–990^{th}	66	5	5	0.975659	0.962617	67.429052	67.429050
28	993^{rd}–1050^{th}	58	4	4	0.887955	0.962617	59.255850	59.255847
29	1056^{th}–1064^{th}	9	4	4	0.887955	0.962617	9.194876	9.194875
30	1067^{th}–1070^{th}	4	5	5	0.975659	0.962617	4.086599	4.086599
31	1077^{th}–1321^{st}	245	5	5	0.975659	0.962617	250.304549	250.304545
32	1324^{th}–334^{th}	11	5	5	0.975659	0.974293	11.238162	11.238162

33	1339^{th}–1341^{st}	3	5	5	0.975659	0.974293	3.064954	3.064954
34	1344^{th}–1348^{th}	5	6	6	0.978723	0.962617	5.108240	5.108241
35	1351^{st}–1358^{th}	8	5	5	0.975659	0.962617	8.173221	8.173221
36	$1366^{th}-1370^{th}$	5	6	6	0.978723	0.962617	5.108239	5.108240
37	1374^{th}–1378^{th}	5	6	6	0.978723	0.962617	5.108252	5.108252
38	1382^{nd}–1392^{rd}	12	5	5	0.975659	0.962617	12.259858	12.259857
39	1399^{th}–1473^{rd}	75	5	5	0.975659	0.962617	76.623892	76.623891
40	1476^{th}–1497^{th}	22	5	5	0.975659	0.974293	22.476308	22.476308
41	1500^{th}–1503^{rd}	4	5	5	0.975659	0.962617	4.086596	4.086597
42	1506^{th}–1524^{th}	19	5	5	0.975659	0.974293	19.411347	19.411347
43	1527^{th}–1684^{th}	158	6	6	0.978723	0.974293	161.420899	161.420899
44	1688^{th}–1690^{th}	3	5	5	0.975659	0.962617	3.064954	3.064954
45	1694^{th}–1834^{th}	141	6	6	0.978723	0.974293	144.052824	144.052824
46	1841^{st}–1887^{th}	47	5	5	0.975659	0.962617	48.017627	48.017627
47	1890^{th}–1900^{th}	11	5	5	0.975659	0.974293	11.238163	11.238163
48	1903^{rd}–1964^{th}	62	5	5	0.975659	0.962617	63.342294	63.342297
49	1990^{th}–1996^{th}	7	5	5	0.975659	0.962617	7.151573	7.151573
50	2031^{st}–2034^{th}	4	5	5	0.975659	0.891496	4.086613	4.086611
51	2060^{th}–2062^{nd}	3	5	5	0.975659	0.900285	3.064954	3.064954
52	2200^{th}–2203^{rd}	4	4	4	0.887955	0.962617	4.086589	4.086591
53	2207^{th}–2209^{th}	3	4	4	0.887955	0.891496	3.064963	3.064962
54	2215^{th}–2223^{rd}	9	3	3	0.899522	0.891496	9.194882	9.194880
55	2243^{rd}–2250^{th}	8	3	3	0.899522	0.891496	8.173219	8.173218
56	2271^{st}–2274^{th}	4	2	2	0.980488	0.977044	4.086627	4.086627
57	2277^{th}–2286^{th}	10	1	1	0.977011	0.977044	10.216470	10.216471
58	2291^{st}–$s363^{rd}$	73	1	1	0.977011	0.977044	74.580551	74.580551
59	2372^{nd}–2387^{th}	16	1	1	0.977011	0.977044	16.346288	16.346291
60	2390^{th}–2412^{th}	23	1	1	0.977011	0.982759	23.497945	23.497945
61	2483^{rd}–2496^{th}	14	2	2	0.980488	0.977044	14.303130	14.303130
62	2535^{th}–2542^{nd}	8	2	2	0.980488	0.977044	8.173227	8.173227
63	2548^{th}–2553^{rd}	6	2	2	0.980488	0.977044	6.129920	6.129920
64	2579^{th}–2589^{th}	11	2	2	0.980488	0.900285	11.238189	11.238188
65	2592^{nd}–2595^{th}	4	2	2	0.980488	0.977044	4.086590	4.086591
66	2688^{th}–2692^{nd}	5	2	2	0.980488	0.977044	5.108259	5.108259
67	2784^{th}–2787^{th}	4	3	3	0.899522	0.977044	4.086612	4.086611
68	2794^{th}–2812^{th}	19	3	3	0.899522	0.977044	19.411395	19.411393
69	2820^{th}–2829^{th}	7	3	3	0.899522	0.977044	7.151581	7.151579
70	2835^{th}–2917^{th}	83	2	2	0.980488	0.977044	84.797036	84.797036
71	2921^{st}–2923^{rd}	3	3	3	0.899522	0.977044	3.064959	3.064959
72	2925^{th}–2936^{th}	12	3	3	0.899522	0.977044	12.259825	12.259825
73	2942^{nd}–2953^{rd}	12	2	2	0.980488	0.977044	12.259819	12.259819
74	2977^{th}–2979^{th}	3	2	2	0.980488	0.977044	3.064966	3.064966
75	3103^{rd}–3105^{th}	3	2	2	0.980488	0.977044	3.064964	3.064964
76	3148^{th}–3168^{th}	21	2	2	0.980488	0.977044	21.454681	21.454680
77	3171^{st}–3177^{th}	7	2	2	0.980488	0.977044	7.151567	7.151567
78	3288^{th}–3293^{rd}	6	2	2	0.980488	0.977044	6.129953	6.129952
79	3304^{th}–3373^{rd}	70	1	1	0.977011	0.982759	71.515665	71.515664
80	3376^{th}–3378^{th}	3	2	2	0.980488	0.982759	3.064974	3.064974
81	3381^{st}–3441^{st}	61	1	1	0.977011	0.982759	62.320742	62.320742
82	3445^{th}–3457^{th}	13	1	1	0.977011	0.982759	13.281467	13.281467
83	3460^{th}–3573^{rd}	114	2	2	0.980488	0.982759	116.468276	116.468275

84	3576^{th}–3689^{th}	114	1	1	0.977011	0.982759	116.468143	116.468145
85	3692^{nd}–3729^{th}	38	2	2	0.980488	0.982759	38.822765	38.822764
86	3732^{nd}–3735^{th}	4	2	2	0.980488	0.977044	4.086599	4.086600
87	3738^{th}–3763^{rd}	26	2	2	0.980488	0.982759	26.562934	26.562934
88	3766^{th}–3819^{th}	54	2	2	0.980488	0.977044	55.169136	55.169137
89	3822^{nd}–3826^{th}	5	2	2	0.980488	0.977044	5.108259	5.108259
90	3836^{th}–3888^{th}	53	2	2	0.980488	0.977044	54.147539	54.147538
91	3891^{st}–3896^{th}	6	2	2	0.980488	0.982759	6.129900	6.129901
92	3900^{th}–3904^{th}	5	2	2	0.980488	0.982759	5.108263	5.108263
93	3907^{th}–3911^{th}	5	2	2	0.980488	0.977044	5.108251	5.108252
94	3920^{th}–3960^{th}	41	2	2	0.980488	0.977044	41.887700	41.887700

Comparison of the performance of the conventional co-integration (without Mean Reversion and CV) with the co-integration using Mean Reversion and CV can be demonstrated by looking at values of the expected returns of both cases. The expected returns of the conventional co-integration and the proposed model using Mean Reversion and CV are US$ 2351.84 and US$ 2003.77, respectively. AS a result, the returns of co-integration using Mean Reversion and CV are higher than the conventional co-integration (without Mean Reversion and CV). Therefore, we conclude that the proposed co-integrated pairs trading using Mean Reversion and CV outperforms the conventional co-integrated pairs trading model. Therefore, the net premium in 10-year trading with the co-integrated pairs trading using Mean Reversion and CV, which calculated the difference of both cases, yields to US$ 348.07, equalling to 17.37%.

4.5.3 Robustness Test

To compute the expected returns of the cross-paired trading, we assign the contents in Table 2, which are the top ten pairs that have been characterised for the highest correlation as input. Then, we use the same techniques that have been used to calculate the expected returns in Table 7 and 8 for computing the expected returns of the top ten pairs. The results are listed in Table 9 and Table 6b. Whereas, Table 9 represents the expected returns of the conventional co-integrated pairs trading (without Mean Reversion and CV), and Table 10 represents the expected returns of the co-integrated pairs trading using Mean Reversion and CV.

The results of computing the expected returns of the co-integrated Pairs Trading using Mean Reversion and CV are shown in Table 10. Apparently, the average expected returns of the co-integrated Pairs Trading using Mean Reversion and CV are US$ 327051 and US$ 3270.51, respectively. The expected returns of the co-integrated Pairs Trading using Mean Reversion and CV outperforms those of the conventional co-integrated Pairs Trading (without Mean Reversion and CV), see Table 9. It is proven that the benefit of co-integrated Pairs Trading using Mean Reversion and CV, for those top ten cross-paired stocks with the 10-year investment, is US$ 43825, equalling to 15.48%.

Table 7: represents the expected returns of the conventional co-integrated Pairs Trading, in US dollars

Block no.	Ranking	Data points	Probability	Returns of DBKGR and GLEFP	Expected returns of DBKGR and GLEFP
1	$35^{th} - 40^{th}$	6	0.75	6.187233	4.640425
2	$43^{rd} - 49^{th}$	7	0.75	7.158220	5.368665
3	$51^{st} - 56^{th}$	6	0.75	6.136712	4.602534
4	$79^{th} - 81^{st}$	3	0.75	3.066100	2.299575
5	$83^{rd} - 92^{nd}$	10	0.75	10.217934	7.663451
6	$138th - 287^{th}$	150	0.75	153.250577	114.937933
7	$372^{nd} - 385^{th}$	14	0.75	14.303218	10.727413
8	$392^{nd} - 397^{th}$	6	0.75	6.129928	4.597446
9	$463^{rd} - 488^{th}$	26	0.75	26.563032	19.922274
10	$534^{th} - 547^{th}$	14	0.75	14.303178	10.727383
11	$551^{st} - 553^{rd}$	3	0.75	3.065026	2.298769
12	$716^{th} - 719^{th}$	4	0.75	4.086666	3.065000
13	$728^{th} - 846^{th}$	119	0.75	121.576648	91.182486
14	$848^{th} - 851^{st}$	4	0.75	4.086633	3.064975
15	$873^{th} - 886^{th}$	14	0.75	14.303130	10.727347
16	$888^{th} - 918^{th}$	31	0.75	31.671212	23.753409
17	$925^{th} - 1964^{th}$	1040	0.75	1062.517470	796.888103
18	$1990^{th} - 1996^{th}$	7	0.75	7.151577	5.363683
19	$2031^{st} - 2036^{th}$	6	0.75	6.129917	4.597438
20	$2059^{th} - 2064^{th}$	6	0.75	6.129925	4.597444
21	$2197^{th} - 2204^{th}$	8	0.75	8.173250	6.129938
22	$2207^{th} - 2240^{th}$	34	0.75	34.736173	26.052130
23	$2242^{nd} - 2412^{nd}$	171	0.75	174.702397	131.026798
24	$2483^{rd} - 2496^{th}$	14	0.75	14.303136	10.727352
25	$2535^{th} - 2542^{nd}$	8	0.75	8.173234	6.129926
26	$2548^{th} - 2553^{rd}$	6	0.75	6.129925	4.597443
27	$2563^{rd} - 2595^{th}$	34	0.75	34.736189	26.052141
28	$2688^{th} - 2692^{nd}$	5	0.75	5.108261	3.831196
29	$2725^{th} - 2727^{th}$	3	0.75	3.064959	2.298719
30	$2784^{th} - 2787^{th}$	4	0.75	4.086615	3.064961
31	$2794^{th} - 2818^{th}$	25	0.75	25.541302	19.155976
32	$2820^{th} - 2917^{th}$	98	0.75	100.121849	75.091387
33	$2921^{st} - 2923^{rd}$	3	0.75	3.064961	2.298721
34	$2925^{th} - 2953^{rd}$	29	0.75	29.627909	22.220932
35	$2977^{th} - 2979^{th}$	3	0.75	3.064971	2.298728
36	$3103^{rd} - 3105^{th}$	3	0.75	3.064969	2.298726
37	$3148^{th} - 3186^{th}$	21	0.75	21.454683	16.091012
38	$3171^{st} - 3177^{th}$	7	0.75	7.151571	5.363678
39	$3288^{th} - 3819^{th}$	532	0.75	543.518510	407.638882
40	$3822^{nd} - 3826^{th}$	5	0.75	5.108260	3.831195
41	$3836^{th} - 3961^{st}$	126	0.75	128.728079	96.546059

Table 9: The expected returns in US dollars of the conventional co-integrated Pairs Trading

	GLEFP	IBMUS	MCDUS	BHPAU	BHPAU	GLEFP	X8031JP	IBMUS	GLEFP	RIOLN
DBKGR	2004	2612	3007	2487	2487	2004	3035	2612	2004	3035
AMZNUS	2888	2626	2740	3001	3001	2888	3035	2626	2888	3035
NKEUS	2798	3035	3017	2394	2394	2798	3035	3035	2798	3035
VALEBZ	2684	3035	3016	1685	1685	2684	3035	3035	2684	3035
ABBSS	2987	2078	2858	3033	3033	2987	3035	2078	2987	3035
BBVASM	3035	2035	3035	2814	2814	3035	3035	3035	3035	3035
X8058JP	3035	3035	3035	3035	3035	3035	3035	3035	3035	2125
AAPLUS	3002	2788	2980	2831	2831	3002	3035	2788	3002	3035
UBSNVX	3035	3035	2994	2996	2996	3035	3035	3035	3035	3035
FCXUS	2799	3035	2897	1922	1922	2799	3035	3035	2799	3035

Table 10: The expected returns in US dollars of the co-integrated Pairs Trading using Mean Reversion and CV

	GLEFP	IBMUS	MCDUS	BHPAU	BHPAU	GLEFP	X8031JP	IBMUS	GLEFP	RIOLN
DBKGR	2352	2920	3481	2864	2864	2352	3408	2920	2352	3358
AMZNUS	3435	3147	3339	3675	3675	3435	3595	3147	3435	3525
NKEUS	3210	3568	3600	2868	2868	3210	3522	3568	3210	3459
VALEBZ	3075	3535	3559	1978	1978	3075	3495	3535	3075	3434
ABBSS	3449	2464	3373	3629	3629	3449	3496	2464	3449	3445
BBVASM	3421	3430	3494	3286	3286	3421	3414	3430	3421	3343
X8058JP	3399	3402	3456	3478	3478	3399	3399	3402	3399	2221
AAPLUS	3581	3335	3634	3461	3461	3581	3605	3335	3581	3531
UBSNVX	3486	3500	3510	3544	3544	3486	3460	3500	3486	3376
FCXUS	3059	3372	3269	2149	2149	3059	3365	3372	3059	3275

5 Conclusion and Discussion

The concept of Pairs Trading is a market neutral strategy that uses a portfolio of only two securities. A long position is adopted with respect to one safety and a short position with respect to the other. The strategy of pairs trading requires adopting a position when the spread is distant from the mean in anticipation of spread reversion. This thesis introduces a multiclass Pairs Trading model using Mean Reversion and CV that enhances the original approach of Mean Reversion Pairs Trading. The simulation results show that the co-integrated Pairs Trading using the proposed method outperforms those of the conventional co-integrated Pairs Trading. Thus, benefits of the proposed model are to build a new set of risk mitigation and maximise returns of co-integrated stocks. Future research could examine the formation of frequency domain datasets rather than times series as an alternative to correlation coefficient pairing.

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