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An Interior-Point Trust-Region Algorithm for Quadratic Stochastic Symmetric Programming

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Abstract : Stochastic programming is a framework for modeling optimization problems that involve uncertainty. In this paper, we study two-stage stochastic quadratic symmetric programming to handle uncertainty in data defining (Deterministic) symmetric programs in which a quadratic function is minimized over the intersection of an affine set and a symmetric cone with finite event space. Two-stage stochastic programs can be modeled as large deterministic programming and we present an interior point trust region algorithm to solve this problem. Numerical results on randomly generated data are available for stochastic symmetric programs. The complexity of our algorithm is proved.

Keywords : interior-point algorithm; trust-region subproblem; symmetric programming; Stochastic programming.
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1 Introduction

Deterministic optimization problems are formulated to find optimal solutions in problems with certainty in data. In fact, in some applications we cannot specify the model entirely because it depends on information which is not available at the time of formulation but that will be determined at some point in the future.

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Stochastic programming began in the 1950s to handle those problems that involve uncertainty in data. Many real world applications carry an inherent uncertainty within them. In particular, two-stage stochastic programs have been established to formulate many applications to linear, non-linear and integer programming.

Conic programming deals with an important class of tractable convex optimization problems. Symmetric programming is a generalization of linear programming that includes second-order cone programming and semidefinite programming as special cases. All of these problems are treated in a unified theoretical framework for the design of algorithms and the analysis of their complexity. Stochastic symmetric programming may be viewed as an extension of symmetric programming by allowing uncertainty in data.

Interior point methods are considered to be one of the most successful classes of algorithms for solving convex optimization problems. We refer to [1] which developed an interior-point trust-region algorithm for minimizing a quadratic objective function in the intersection of a symmetric cone and an affine subspace by solving sequential trust-region subproblems. Global first-order and second-order convergence results were proved. The techniques and properties in both interior-point algorithms and trust-region methods show the complexity of the algorithm by providing strong theoretical support. Our algorithm is developed as an interior-point trust-region algorithm for minimizing stochastic symmetric programming.

In this paper, we present stochastic symmetric programming with a quadratic function and finite scenarios. And then we will explicitly formulate a problem as a large scale deterministic program and develop an interior-point trust-region algorithm to solve it.

The organization of this paper is as follows. In section 2, we present some concepts and results of symmetric cone and self-concordant barrier function used in the theory of interior-point method. In section 3, we propose a model of two-stage stochastic symmetric programs with linear and quadratic function. We also describe a model in deterministic programming. In section 4, we present an interior point trust-region algorithm for solving stochastic symmetric programming and provide complexity of our algorithm. In section 5, we present computational results on a case study problem. In addition, we compared our results with those of the quadprog function in the MATLAB optimization toolbox in cases of the nonnegative orthant cones. Our algorithm can solve stochastic symmetric programming problems that include variables in second order cone and cones of real symmetric positive semidefinite matrices. The quadprog function is unable to solve problems with these variables. Section 6 has some conclusion and remarks.

2 Symmetric Cone and Self-Concordant Barrier

In this section, we introduce some of the concepts and relevant results that will be useful in section 4. For more detailed exposure to those concepts, see the reference [2] and [3].

Let E be finite-dimensional real Euclidean space with inner product $\langle \cdot\,, \cdot \rangle$.

Definition 2.1 (Cone)**.** A subset $\mathcal{K}$ of E is said to be a *cone* if for every $x \in \mathcal{K}$ and $\lambda > 0$ imply that $\lambda x \in \mathcal{K}$.

Definition 2.2 (Convex cone)**.** A subset $\mathcal{K}$ of E is a *convex cone* if and only if for every $x, y \in \mathcal{K}$ and $\lambda, \mu > 0$ imply that $\lambda x + \mu y \in \mathcal{K}$.

We assume that $\mathcal{K}$ is a convex cone in a E. The dual of $\mathcal{K}$ is defined as

Definition 2.3 (Dual cone)**.** Let $\mathcal{K}$ be a cone. The set

$$\mathcal{K}^* = \{y \in E \mid \langle x, y\rangle \geq 0, \forall x \in \mathcal{K}\}.$$

is called the *dual cone* of $\mathcal{K}$.

As the name suggests, $\mathcal{K}^*$ is a cone, and is always convex, even when the original cone is not. If cone $\mathcal{K}$ and its dual $\mathcal{K}^*$ coincide, we say that $\mathcal{K}$ is self-dual. In particular, this implies that $\mathcal{K}$ has a nonempty interior and does not contain any straight line (i.e., it is pointed).

We denote by $GL(E)$, the group of general linear transformations on E, and by $Aut(\mathcal{K})$, the automorphism group of convex cone $\mathcal{K}$, that is

$$Aut(\mathcal{K}) = \{g \in GL(E) | g(\mathcal{K}) = \mathcal{K}\}.$$

Definition 2.4 (Homogeneity)**.** The convex cone $\mathcal{K}$ is said to be *homogeneous* if for every pair $x, y \in int\mathcal{K}$, there exists an invertible linear operator g for which $g\mathcal{K} = \mathcal{K}$ and $gx = y$.

Definition 2.5 (Symmetric cone)**.** The convex cone $\mathcal{K}$ is said to be *symmetric* if it is self-dual and homogeneous.

Almost all conic optimization problems in real world applications are associated with symmetric cones such as nonnegative orthant cones, second-order cones and cone of positive semi-definite matrices over the real or complex numbers.

The nonnegative orthant cone

$$R^n_+ = \{x \in \mathbb{R}^n | x_i \geq 0, i = 1, 2, ..., n\}.$$

The second-order cone (also known as the quadratic, Lorentz, or the ice cream cone) of dimension n

$$\mathcal{Q}^n = \{x \in \mathbb{R}^n | \sum_{i=1}^{n-1} x_i^2 \leq x_n^2 \text{ and } x_n \geq 0\}.$$

The positive semi-definite cone

$$\mathcal{S}^n_+ = \{X \mid X \in \mathbb{R}^{n\times n}, X \text{ is positive semidefinite matric}\}.$$

A self-concordant barrier function is crucial to an interior-point methods. We introduce some of the fundamentals idea a self-concordant barrier function that will be useful in this paper.

Definition 2.6 (Self-Concordant Function). [3] Let $\mathcal{K}^0 \subset \mathcal{K} \subseteq E$ and $F : \mathcal{K}^0 \to \mathbb{R}$ be a C^3 smooth convex function such that F is called *self-concordant function* if

$$|F'''(x)[h,h,h]| \leq 2\big(F''(x)(h,h)\big)^{3/2} \tag{2.1}$$

for all $x \in \mathcal{K}^0$ and for all $h \in S$.

Definition 2.7 (Self-concordant barrier). [3] A self-concordant function F is a *self-concordant barrier* for a closed convex set $\mathcal{K}$ if

$$\vartheta = \sup_{x \in \mathcal{K}^0} \big\langle F'(x), F''(x)^{-1}F'(x)\big\rangle \tag{2.2}$$

the value ϑ is called a *barrier parameter* of F.

In principle, every convex cone admits a self-concordant barrier(see [3], section 4). We consider a self-concordant barrier of symmetric cones as follows

$$\begin{aligned} F(x) &= -\sum_{i=1}^{n} lnx_i && \text{with } \vartheta = n, \text{ for cone of nonnegative orthant,} \\ F(x) &= -ln(x_n^2 - \sum_{i=1}^{n-1} x_i^2) && \text{with } \vartheta = 2, \text{ for second-order cone,} \\ F(x) &= -lndet(X) && \text{with } \vartheta = n, \text{ for positive semidefinite cone.} \end{aligned}$$

Lemma 2.1. [4] *Let $F(x)$ is a self-concordant barrier for $\mathcal{K}$, then*

$$F''(x)^{-1}F'(x) = -x, \tag{2.3}$$

$$\langle -F'(x), (x)\rangle = \vartheta. \tag{2.4}$$

Lemma 2.2. [4] *If $\mathcal{K}$ $F(x)$ is a symmetric cone and F is a self-concordant barrier for $\mathcal{K}$, then $F''(x)$ is a linear automorphism of $\mathcal{K}$ for each $x \in \mathcal{K}^0$.*

The strictly convex assumption of $F(x)$ implies that $F''(x)$ is positive definite for every $x \in \mathcal{K}^0$. This allows us to define a norm as follows

$$\|h\|_x^2 = \langle h, F''(x)h\rangle \tag{2.5}$$

is a norm on E induced by $F''(x)$. Let $B_x(y, r)$ denote the open ball of radius r centered at y, where the radius is measured with respect to $\|.\|_x$.

Lemma 2.3. [4] *If $F(x)$ is a self-concordant function for $\mathcal{K}$, then we have $B_x(x, 1) \subseteq \mathcal{K}^0$ for all $x \in \mathcal{K}^0$.*

Lemma 2.4. [4] *Assume $F(x)$ is a self-concordant function for $\mathcal{K}$, $x \in \mathcal{K}^0$, and $y \in B_x(x, 1)$, then*

$$\left| F(y) - F(x) - \langle F'(x), y - x\rangle - \frac{\langle y - x, F''(x)(y - x)\rangle}{2} \right| \leq \frac{\|y - x\|_x^3}{3(1 - \|y - x\|_x)}. \tag{2.6}$$

Lemma 2.5. [4] *Assume $F(x)$ is a self-concordant function for $\mathcal{K}$. If $\|n(x)\|_x \leq 1/4$, then $F(x)$ has a minimizer z and*

$$\|z - x\|_x \leq \|n(x)\|_x + \frac{3\|n(x)\|_x^2}{(1 - \|n(x)\|_x)^3}, \tag{2.7}$$

where $n(x) = -F''(x)^{-1}F'(x)$ be the Newton step of $F(x)$.

Lemma 2.6. [4] *Assume $F(x)$ is a self-concordant barrier with barrier parameter ϑ. If $x, y \in \mathcal{K}^0$, then*

$$\langle F'(x), y - x\rangle \leq \vartheta. \tag{2.8}$$

3 The Problem and Its Modeling

In a standard two-stage stochastic programming model, decision variables are divided into two subsets. First-stage variables are groups of variables determined before the realizations of random events are known. Once the uncertain events have unfolded, further design or operational adjustments can be made through values of the second-stage known as recourse variables, which are determined after knowing the realized values of the random events.

A standard formulation of the two-stage stochastic program is

$$\min \quad f_1(x) + E[Q(x,\xi)] \tag{3.1}$$
$$\text{st.} \quad A_0 x = b, \tag{3.2}$$
$$x \in \mathcal{K}_1, \tag{3.3}$$

where x is the first-stage decision variable, $Q(x,\xi)$ is the optimal value of the second-stage problem:

$$\min \quad f_2(y,\xi) \tag{3.4}$$
$$\text{st.} \quad T(\xi)x + W(\xi)y = h(\xi), \tag{3.5}$$
$$y \in \mathcal{K}_2 \tag{3.6}$$

where y is the second-stage variable and the cones $\mathcal{K}_1$ and $\mathcal{K}_2$ are symmetric cones. The matrix A_0 and the vectors b and c are deterministic data, the matrices $W(\xi)$ and $T(\xi)$ and the vectors $h(\xi)$ and $d(\xi)$ are random data whose realizations depend on an underlying outcome ξ with a known probability function p.

3.1 The Stochastic Linear Symmetric Programs (SLSP)

The stochastic linear symmetric programs can be stated as:

$$\min \quad \langle c, x\rangle + E[Q(x,\xi)] \tag{3.7}$$
$$\text{st.} \quad A_0 x = b, \tag{3.8}$$
$$x \in \mathcal{K}_1, \tag{3.9}$$

where x is the first-stage decision variable, $Q(x,\xi)$ is the minimum of the problem

$$\min \quad \langle g(\xi), y\rangle \tag{3.10}$$
$$\text{st.} \quad T(\xi)x + W(\xi)y = h(\xi), \tag{3.11}$$
$$y \in \mathcal{K}_2 \tag{3.12}$$

where y is the second-stage variable and the cones $\mathcal{K}_1$ and $\mathcal{K}_2$ are symmetric cones. The formulation of the above problem assumes that the second-stage data ξ can be modeled as a random vector with a known probability distribution. The random vector ξ has a finite number of possible realizations, called scenarios, say $\xi_1,\ldots,\xi_K$ with respective probability masses $p_1,\ldots,p_K$. Then the expectation in the first-stage problem's objective function can be written as the summation

$$E[Q(x,\xi)] = \sum_{k=1}^{K} p_k Q(x,\xi_k). \tag{3.13}$$

A SLSP can be written as a deterministic program and represented as follows:

$$\min \quad q(x, y^{(1)}, y^{(2)}, ..., y^{(K)}) = \langle c, x\rangle + \sum_{k=1}^{K} p_k \langle g^{(k)}, y^{(k)}\rangle \tag{3.14}$$
$$\text{st.} \quad A_0 x = b, \tag{3.15}$$
$$T^{(k)}x + W^{(k)}y^{(k)} = h^{(k)}, k = 1,2,...,K, \tag{3.16}$$
$$x \in \mathcal{K}_1, y^{(k)} \in \mathcal{K}_2, k = 1,2,...,K, \tag{3.17}$$

where $x \in \mathbb{R}^{n_1}$ is the first-stage decision variable and $y^{(k)} \in \mathbb{R}^{2_k}$ are the second stage variable. The matrix $A_0 \in \mathbb{R}^{m_1\times n_1}$, the vectors $b \in \mathbb{R}^{m_1}$ and $c \in \mathbb{R}^{n_1}$ are deterministic data. The matrices $W^{(k)} \in \mathbb{R}^{m_{2_k}\times n_{2_k}}$, $T^{(k)} \in \mathbb{R}^{m_{2_k}\times n_1}$ and vectors $h^{(k)} \in \mathbb{R}^{n_{2_k}}$, $g^{(k)} \in \mathbb{R}^{n_{2_k}}$ are random data associated with probability for $k = 1,2,\ldots,K$. The cones $\mathcal{K}_1$ and $\mathcal{K}_2$ are symmetric cone in $\mathbb{R}^{n_1}$ and $\mathbb{R}^{n_{2_k}}$ for $k = 1,2,\ldots,K$ (n_1, n_{2_k} are positive integers).

3.2 The Stochastic Quadratic Symmetric Programs (SQSP)

The stochastic quadratic symmetric programs can also be cast as SLSP. The matrix $H_0 \in \mathbb{R}^{n_1\times n_1}, H_0 \succ 0$ is deterministic data and the matrices $H_1(\xi)$ is random data whose realizations depend on an underlying outcome ξ with a known probability function p. Let $H_1^{(k)} \in \mathbb{R}^{n_{2_k}\times n_{2_k}}, H_1^{(k)} \succ 0$ be random data associated with probability for $k = 1,2,\ldots,K$. A SQSP can be formulated as

$$\min \frac{1}{2}\langle x, H_0 x\rangle + \langle c, x\rangle + E[Q(x,\xi)] \tag{3.18}$$
$$\text{st.} A_0 x = b, \tag{3.19}$$
$$x \in \mathcal{K}_1, \tag{3.20}$$

where x is the first-stage decision variable, and $Q(x,\xi)$ is the minimum of the problem

$$\min \frac{1}{2}\langle y, H_1(\xi)y\rangle + \langle g(\xi), y\rangle \tag{3.21}$$
$$\text{st.} T(\xi)x + W(\xi)y = h(\xi), \tag{3.22}$$
$$y \in \mathcal{K}_2, \tag{3.23}$$

where y is the second-stage variable and the cones $\mathcal{K}_1$ and $\mathcal{K}_2$ are symmetric cones.

A SQSP can be equivalently written as a deterministic symmetric programs as:

$$\begin{aligned}\min \quad & q\big(x, y^{(1)}, y^{(2)}, \ldots, y^{(K)}\big) = \frac{1}{2}\langle x, H_0x\rangle + \langle c, x\rangle \\ & + \sum_{k=1}^{K} p_k\big(\frac{1}{2}\langle y^{(k)}, H_1^{(k)}y^{(k)}\rangle + \langle g^{(k)}, y^{(k)}\rangle\big) \quad (3.24)\\ \text{st.} \quad & A_0x = b, \quad (3.25)\\ & T^{(k)}x + W^{(k)}y^{(k)} = h^{(k)}, k = 1, 2, \ldots, K, \quad (3.26)\\ & x \in \mathcal{K}_1, y^{(k)} \in \mathcal{K}_2, k = 1, 2, \ldots, K. \quad (3.27)\end{aligned}$$

Define

$$\mathcal{K} = \mathcal{K}_1 \times \mathcal{K}_2^{(1)} \times \mathcal{K}_2^{(2)} \times \ldots, \mathcal{K}_2^{(K)} \text{ is the symmetric cone,} \tag{3.28}$$

$$X = [x, y^{(1)}, y^{(2)}, \cdots, y^{(K)}]^T \in \mathbb{R}^{(n_1+\sum_{k=1}^{K} n_{2_k})}, x \in \mathcal{K}_1, y^{(k)} \in \mathcal{K}_2, \text{for } k = 1, 2, \ldots, K, \tag{3.29}$$

$$B = \big[b, \quad h^{(1)}, \quad h^{(2)}, \quad \cdots, \quad h^{(K)}\big]^T \in \mathbb{R}^{(m_1+\sum_{k=1}^{K} m_{2_k})}, \tag{3.30}$$

$$C = \big[c, p_1g^{(1)}, p_2g^{(2)}, \cdots, p_Kg^{(K)}\big]^T \in \mathbb{R}^{(n_1+\sum_{k=1}^{K} n_{2_k})}, \tag{3.31}$$

$$A = \begin{bmatrix} A_0 & 0 & 0 & \cdots & 0 \\ T^{(1)} & W^{(1)} & 0 & \cdots & 0 \\ T^{(2)} & 0 & W^{(2)} & \cdots & 0 \\ \vdots & \vdots & & \ddots & \\ T^{(K)} & 0 & 0 & \cdots & W^{(K)} \end{bmatrix} \in \mathbb{R}^{(m_1+\sum_{k=1}^{K} m_{2_k})\times(n_1+\sum_{k=1}^{K} n_{2_k})}, \tag{3.32}$$

$$Q = \begin{bmatrix} H_0 & 0 & 0 & \cdots & 0 \\ 0 & p_1H_1^{(1)} & 0 & \cdots & 0 \\ 0 & 0 & p_2H_1^{(2)} & \cdots & 0 \\ \vdots & \vdots & & \ddots & \\ 0 & 0 & 0 & \cdots & p_KH_1^{(K)} \end{bmatrix} \in \mathbb{R}^{(n_1+\sum_{k=1}^{K} n_{2_k})\times(n_1+\sum_{k=1}^{K} n_{2_k})}, \tag{3.33}$$

we can rewrite a SQSP as a large deterministic program as follow

$$\min q(X) = \frac{1}{2}\langle X, QX\rangle + \langle C, X\rangle \tag{3.34}$$

$$st. AX = B, \tag{3.35}$$

$$X \in \mathcal{K}. \tag{3.36}$$

4 The Interior-Point Trust-Region Algorithm

In this section, we present our interior-point trust-region algorithm for solving (3.34)-(3.36). First, we define the function as

$$\begin{aligned} f_{\eta_i}(X) &= \eta_i q(X) + F(X) \\ &= \eta_i\Big(\frac{1}{2}\langle x, H_0x\rangle + \langle c, x\rangle + \sum_{k=1}^{K} p_k\big(\frac{1}{2}\langle y^{(k)}, H_1^{(k)}y^{(k)}\rangle + \langle g^{(k)}, y^{(k)}\rangle\big)\Big) \\ &\quad + F_1(x) + \sum_{k=1}^{K} F_2(y^{(k)}) \end{aligned} \tag{4.1}$$

where $F(X) : \mathcal{K} \to \mathbb{R}$ is a self-concordant barrier for the symmetric cone $\mathcal{K}$ with a barrier parameter ϑ. A self-concordant barrier define by $F(X) = F_1(x) + \sum\limits_{k=1}^{K} F_2(y^{(k)})$ where $F_1(x)$ is a self-concordant barrier function for cone $\mathcal{K}_1$ with a barrier parameter ϑ_1 and $F_2(y^{(k)})$ is a self-concordant barrier function for cone $\mathcal{K}_2^{(k)}$ with a barrier parameter $\vartheta_2^{(k)}$ for $k = 1, 2, 3, \ldots, K$ and $\vartheta = \vartheta_1 + \sum\limits_{k=1}^{K} \vartheta_2^{(k)}$.

Since $q(X)$ is quadratic and from definition 2.6 then f_{η_i} is also a self-concordant barrier function. Consider interior-point trust-region algorithm in each inner iteration for a fix η_i we need to find the central path for decreased the value of $f_{\eta_i}(X)$. For outer iterations we need increase the value of η_i to positive infinity, which implies that the central path converges to a solution that is optimal for the original problem.

Let $d = \big(d_1, d_2^{(1)}, d_2^{(2)}, d_2^{(3)}, \cdots, d_2^{(K)}\big) \in E$ be the trail step of f_{η_i}. Consider function f_{η_i} in each inner iteration

$$\begin{aligned} &f_{\eta_i}(x_{i,j}+d_1, y_{i,j}^{(1)} + d_2^{(1)}, y_{i,j}^{(2)} + d_2^{(2)}, \ldots, y_{i,j}^{(K)} + d_2^{(K)}) - f_{\eta_i}(x_{i,j}, y_{i,j}^{(1)}, y_{i,j}^{(2)}, \ldots, y_{i,j}^{(K)}) \\ &= \Big(\eta_i q(x_{i,j} + d_1, y_{i,j}^{(1)} + d_2^{(1)}, y_{i,j}^{(2)} + d_2^{(2)}, \ldots, y_{i,j}^{(K)} + d_2^{(K)}) + F_1(x_{i,j} + d_1) \\ &+ \sum_{k=1}^{K} F_2(y_{i,j}^{(k)} + d_2^{(k)})\Big) - \Big(\eta_i q(x_{i,j}, y_{i,j}^{(1)}, y_{i,j}^{(2)}, \ldots, y_{i,j}^{(k)}) + F_1(x_{i,j}) + \sum_{k=1}^{K} F_2(y_{i,j}^{(k)})\Big) \end{aligned}$$

$$= \left(\frac{1}{2}\eta_i\big\langle x_{i,j}+d_1, H_0(x_{i,j}+d_1)\big\rangle + \eta_i\langle c, x_{i,j}+d_1\rangle + F_1(x_{i,j}+d_1)\right.$$

$$\left.+\sum_{k=1}^{K}\Big(\tfrac{1}{2}\eta_i p_k\big\langle y_{i,j}^{(k)}+d_2^{(k)}, H_1^{(k)}(y_{i,j}^{(k)}+d_2^{(k)})\big\rangle+\eta_i p_k\langle g^{(k)}, y_{i,j}^{(k)}+d_2^{(k)}\rangle+F_2^{(k)}(y_{i,j}^{(k)}+d_2^{(k)})\Big)\right)$$

$$-\left(\frac{1}{2}\eta_i\big\langle x_{i,j}, H_0 x_{i,j}\big\rangle + \eta_i\langle c, x_{i,j}\rangle + F_1(x_{i,j}) + \sum_{k=1}^{K}\Big(\frac{1}{2}\eta_i p_k\big\langle y_{i,j}^{(k)}, H_1^{(k)} y_{i,j}^{(k)}\big\rangle\right.$$

$$\left.+\,\eta_i p_k\langle g^{(k)}, y_{i,j}^{(k)}\rangle + F_2^{(k)}(y_{i,j}^{(k)})\Big)\right)$$

$$= \frac{1}{2}\langle d_1, \eta_i H_0 d_1\rangle + \Big\langle(\eta_i H_0 x_{i,j}+c), d_1\Big\rangle + \sum_{k=1}^{K}\frac{1}{2}\langle d_2^{(k)}, \eta_i p_k H_1^{(k)} d_2^{(k)}\rangle$$

$$+\sum_{k=1}^{K}\big\langle\eta_i p_k(H_1^{(k)} y_{i,j}^{(k)} + g^{(k)}), d_2^{(k)}\big\rangle + F_1(x_{i,j}+d_1) - F_1(x_{i,j}) +$$

$$\sum_{k=1}^{K}\big(F_2(y_{i,j}^{(k)}+d_2^{k}) - F_2(y_{i,j}^{(k)})\big). \tag{4.2}$$

We want to find the bound of $f_{\eta_i}(x_{i,j}+d_1, y_{i,j}^{(1)}+d_2^{(1)}, y_{i,j}^{(2)}+d_2^{(2)}, \ldots, y_{i,j}^{(K)}+d_2^{(K)})$ then we consider the self-concordant barrier in above equation. From Lemma 2.3, for any $X_{i,j}\in\mathcal{K}^0$, we have $X_{i,j}+d\in\mathcal{K}^0$. Given $\|F_1''(x_{i,j})^{\frac{1}{2}}d_1\|\le\alpha_{1_{i,j}}<1$, $\|F_2''(y_{i,j}^{(k)})^{\frac{1}{2}}d_2^{(k)}\|\le\alpha_{2_{i,j}}^{(k)}<1$ for $k=1,2,\ldots,K$ and $\alpha_{1_{i,j}}+\sum_{k=1}^{K}\alpha_{2_{i,j}}^{(k)}<1$. Implied that $\|F_1''(x_{i,j})^{\frac{1}{2}}d_1\|^2+\sum_{k=1}^{K}\|F_2''(y_{i,j}^{(k)})^{\frac{1}{2}}d_2^{(k)}\|^2\le\alpha_{1_{i,j}}^2+\sum_{k=1}^{K}{\alpha_{2_{i,j}}^{(k)}}^2<1$.

Let $F''(X)=F_1''(x)+\sum_{k=1}^{K}F_2''(y^{(k)})$ denote the Hessian of a self-concordant function $F(X)$. Since it is positive definite for every $X\in\mathcal{K}^0$ implies that $F''(X)$ is positive definite. For $h=[h_1, h_2^{(1)}, h_2^{(2)}, \cdots, h_2^{(K)}]\in E$. Let further we define a norm on E induced by $F''(X)$ as $\|h\|_X^2=\langle h, F''(X)h\rangle=\langle h_1, F_1''(x)h_1\rangle+\sum_{k=1}^{K}\langle h_2^{(k)}, F_2''(y^{(k)})h_2^{(k)}\rangle$

$$F_1(x_{i,j}+d_1)-F_1(x_{i,j})+\sum_{k=1}^{K}\big(F_2(y_{i,j}^{(k)}+d_2^{(k)})-F_2(y_{i,j}^{(k)})\big)$$

$$\le\big\langle F_1'(x_{i,j}), d_1\big\rangle+\frac{1}{2}\big\langle d_1, F_1''(x_{i,j})d_1\big\rangle+\frac{\|d_1\|_{x_{i,j}}^3}{3\big(1-\|d_1\|_{x_{i,j}}\big)}+\sum_{k=1}^{K}\big\langle F_2'(y_{i,j}^{(k)}), d_2^{(k)}\big\rangle$$

$$+\sum_{k=1}^{K}\frac{1}{2}\big\langle d_2^{(k)}, F_2''(y_{i,j}^{(k)})d_2^{(k)}\big\rangle+\sum_{k=1}^{K}\frac{\|d_2^{(k)}\|_{y_{i,j}^{(k)}}^3}{3\big(1-\|d_2^{(k)}\|_{y_{i,j}^{(k)}}\big)}$$

$$\leq \big\langle F_1'(x_{i,j}), d_1\big\rangle + \frac{1}{2}\big\langle d_1, F_1''(x_{i,j})d_1\big\rangle + \sum_{k=1}^{K}\langle F_2'(y_{i,j}^{(k)}), d_2^{(k)}\rangle$$

$$+ \sum_{k=1}^{K}\frac{1}{2}\big\langle d_2^{(k)}, F_2''(y_{i,j}^{(k)})d_2^{(k)}\big\rangle + \frac{1}{3}\Big(\frac{\alpha_{1_{i,j}}^3}{1-\alpha_{1_{i,j}}} + \sum_{k=1}^{K}\frac{\alpha_{2_{i,j}}^{(k)\,3}}{1-\alpha_{2_{i,j}}^{(k)}}\Big)$$

$$\leq \big\langle F_1'(x_{i,j}), d_1\big\rangle + \frac{1}{2}\big\langle d_1, F_1''(x_{i,j})d_1\big\rangle + \sum_{k=1}^{K}\langle F_2'(y_{i,j}^{(k)}), d_2^{(k)}\rangle$$

$$+ \sum_{k=1}^{K}\frac{1}{2}\big\langle d_2^{(k)}, F_2''(y_{i,j}^{(k)})d_2^{(k)}\big\rangle + \frac{1}{3}\Big(\frac{(\alpha_{1_{i,j}} + \sum_{k=1}^{K}\alpha_{2_{i,j}}^{(k)})^3}{1-(\alpha_{1_{i,j}} + \sum_{k=1}^{K}\alpha_{2_{i,j}}^{(k)})}\Big). \tag{4.3}$$

The third inequality of (4.3) follows from Lemma 2.4. From (4.3) we can rewrite (4.2) as

$$f_{\eta_i}(x_{i,j}+d_1, y_{i,j}^{(1)}+d_2^{(1)}, y_{i,j}^{(2)}+d_2^{(2)}, \ldots, y_{i,j}^{(K)}+d_2^{(K)}) - f_{\eta_i}(x_{i,j}, y_{i,j}^{(1)}, y_{i,j}^{(2)}, \ldots, y_{i,j}^{(K)})$$

$$\leq \frac{1}{2}\langle d_1, \eta_i H_0 d_1\rangle + \Big\langle (\eta_i H_0 x_{i,j} + c), d_1\Big\rangle + \sum_{k=1}^{K}\frac{1}{2}\langle d_2^{(k)}, \eta_i p_k H_1^{(k)} d_2^{(k)}\rangle$$

$$+ \big\langle F_1'(x_{i,j}), d_1\big\rangle + \frac{1}{2}\big\langle d_1, F_1''(x_{i,j})d_1\big\rangle + \sum_{k=1}^{K}\langle F_2'(y_{i,j}^{(k)}), d_2^{(k)}\rangle$$

$$+ \sum_{k=1}^{K}\frac{1}{2}\big\langle d_2^{(k)}, F_2''(y_{i,j}^{(k)})d_2^{(k)}\big\rangle + \frac{1}{3}\Big(\frac{(\alpha_{1_{i,j}} + \sum_{k=1}^{K}\alpha_{2_{i,j}}^{(k)})^3}{1-(\alpha_{1_{i,j}} + \sum_{k=1}^{K}\alpha_{2_{i,j}}^{(k)})}\Big)$$

$$= \frac{1}{2}\Big\langle d_1, \big(\eta_i H_0 + F_1''(x_{i,j})\big)d_1\Big\rangle + \big\langle \eta_i(H_0 x_{i,j} + c) + F_1'(x_{i,j}), d_1\big\rangle$$

$$+ \sum_{k=1}^{K}\tfrac{1}{2}\Big\langle d_2^{(k)}, \big(\eta_i p_k H_1^{(k)} + F_2''(y_{i,j}^{(k)})\big)d_2^{(k)}\Big\rangle + \sum_{k=1}^{K}\Big\langle \eta_i p_k\big(H_1^{(k)} y_{i,j}^{(k)} + g^{(k)}\big) + F_2'(y_{i,j}^{(k)}), d_2^{(k)}\Big\rangle$$

$$+ \frac{1}{3}\Big(\frac{(\alpha_{1_{i,j}} + \sum_{k=1}^{K}\alpha_{2_{i,j}}^{(k)})^3}{1-(\alpha_{1_{i,j}} + \sum_{k=1}^{K}\alpha_{2_{i,j}}^{(k)})}\Big). \tag{4.4}$$

Now the inequality (4.4) gives an upper bound for $f_{\eta_i}(x_{i,j}+d_1, y_{i,j}^{(1)}+d_2^{(1)}, y_{i,j}^{(2)}+d_2^{(2)}, \ldots, y_{i,j}^{(K)}+d_2^{(K)})$. We can try to minimize this bound given in the right-hand side of inequality. This lead to the following trust-region subproblem

$$\min \frac{1}{2}\Big\langle d_1, \big(\eta_i H_0 + F_1''(x_{i,j})\big) d_1 \Big\rangle + \big\langle \eta_i (H_0 x_{i,j} + c) + F_1'(x_{i,j}), d_1 \big\rangle$$
$$+ \sum_{k=1}^{K} \frac{1}{2} \Big\langle d_2^{(k)}, \big(\eta_i p_k H_1^{(k)} + F_2''(y_{i,j}^{(k)})\big) d_2^{(k)} \Big\rangle$$
$$+ \sum_{k=1}^{K} \Big\langle \eta_i p_k \big(H_1^{(k)} y_{i,j}^{(k)} + g^{(k)}\big) + F_2'(y_{i,j}^{(k)}), d_2^{(k)} \Big\rangle$$
$$= m_{i,j}(d_1, d_2^{(1)}, d_2^{(2)}, \ldots, d_2^{(K)}) \tag{4.5}$$
$$\text{st.} \|F_1''(x_{i,j})^{\frac{1}{2}} d_1\|^2 + \sum_{k=1}^{K} \|F_2''(y_{i,j}^{(k)})^{\frac{1}{2}} d_2^{(k)}\|^2 \leq \alpha_{1_{i,j}}^2 + \sum_{k=1}^{K} {\alpha_{2_{i,j}}^{(k)}}^2. \tag{4.6}$$

Let the transformation

$$d_1' = F_1''(x_{i,j})^{\frac{1}{2}} d_1, \tag{4.7}$$

$${d_2'}^{(k)} = F_2''(y_{i,j}^{(k)})^{\frac{1}{2}} d_2^{(k)}, \text{for } k = 1, 2, \ldots, K, \tag{4.8}$$

and define

$$Q_{1_{i,j}} = \eta_i F_1''(x_{i,j})^{-\frac{1}{2}} H_0 F_1''(x_{i,j})^{-\frac{1}{2}} + I, \tag{4.9}$$

$$C_{1_{i,j}} = F_1''(x_{i,j})^{-\frac{1}{2}} \big(\eta_i (H_0 x_{i,j} + c) + F_1'(x_{i,j})\big), \tag{4.10}$$

$$Q_{2_{i,j}}^{(k)} = \eta_i p_k F_2''(y_{i,j}^{(k)})^{-\frac{1}{2}} H_1^{(k)} F_2''(y_{i,j}^{(k)})^{-\frac{1}{2}} + I^{(k)}, \text{for } k = 1, 2, \ldots, K, \tag{4.11}$$

$$C_{2_{i,j}}^{(k)} = F_2''(y_{i,j}^{(k)})^{-\frac{1}{2}} \big(\eta_i p_k (H_1^{(k)} y_{i,j}^{(k)} + g^{(k)}) + F_2'(y_{i,j}^{(k)})\big), \text{for } k = 1, 2, \ldots, K. \tag{4.12}$$

We can rewrite equations (4.5) and (4.6) as follow

$$\min q_{i,j}'(d_1', {d_2'}^{(1)}, {d_2'}^{(2)}, \ldots, {d_2'}^{(K)})$$
$$= \frac{1}{2} \langle d_1', Q_{1_{i,j}} d_1' \rangle + \langle C_{1_{i,j}}, d_1' \rangle + \sum_{k=1}^{K} \big(\frac{1}{2} \langle {d_2'}^{(k)}, Q_{2_{i,j}}^{(k)} {d_2'}^{(k)} \rangle + \langle C_{2_{i,j}}^{(k)}, {d_2'}^{(k)} \rangle \big) \tag{4.13}$$
$$\text{st.} \|d_1'\|^2 + \sum_{k=1}^{K} \|{d_2'}^{(k)}\|^2 \leq \alpha_{1_{i,j}}^2 + \sum_{k=1}^{K} {\alpha_{2_{i,j}}^{(k)}}^2. \tag{4.14}$$

Once $d_{i,j}'$ is computed then we obtain the trail step

$$d_{1_{i,j}} = F_1''(x_{i,j})^{-\frac{1}{2}} d_{1_{i,j}}', \tag{4.15}$$

$$d^{(k)}_{2_{i,j}} = F''_2(y^{(k)}_{i,j})^{-\frac{1}{2}} {d'}^{(k)}_{2_{i,j}} \text{for } k = 1,2,\dots,K \tag{4.16}$$

and from inequality (4.4) that we have

$$f_{\eta_i}(x_{i,j} + d_{1_{i,j}}, y^{(1)}_{i,j} + d^{(1)}_{2_{i,j}}, \dots, y^{(K)}_{i,j} + d^{(K)}_{2_{i,j}}) - f_{\eta_i}(x_{i,j}, y^{(1)}_{i,j}, y^{(2)}_{i,j}, \dots, y^{(K)}_{i,j})$$
$$\leq q'_{i,j}(d'_{1_{i,j}}, {d'}^{(1)}_{2_{i,j}}, \dots, {d'}^{(K)}_{2_{i,j}}) + \frac{1}{3}\Big(\frac{(\alpha_{1_{i,j}} + \sum_{k=1}^{K} \alpha^{(k)}_{2_{i,j}})^3}{1-(\alpha_{1_{i,j}} + \sum_{k=1}^{K} \alpha^{(k)}_{2_{i,j}})}\Big). \tag{4.17}$$

Let $n_{\eta_i}(X_{i,j}) = n_{\eta_i}(x_{i,j}, y^{(1)}_{i,j}, \dots, y^{(K)}_{i,j})$ be the Newton step of $f_{\eta_i}(x, y^{(1)}, \dots, y^{(K)})$ at the point $(x_{i,j}, y^{(1)}_{i,j}, y^{(2)}_{i,j}, \dots, y^{(K)}_{i,j})$. We should point out that

$$\begin{aligned}
&\|n_{\eta_i}(X_{i,j})\|^2_{X_{i,j}}\\
&= \Big\langle -f''_{\eta_i}(X_{i,j})^{-1} f'_{\eta_i}(X_{i,j}), f''_{\eta_i}(X_{i,j})\big(-f''_{\eta_i}(X_{i,j})^{-1} f'_{\eta_i}(X_{i,j})\big)\Big\rangle\\
&= \big\langle f'_{\eta_i}(X_{i,j}), f''_{\eta_i}(X_{i,j})^{-1} f'_{\eta_i}(X_{i,j})\big\rangle\\
&= \Big\langle \eta_i(H_0 x_{i,j} + c) + F'_1(x_{i,j}), (\eta_i H_0 + F''_1(x_{i,j}))^{-1} \times \big(\eta_i(H_0 x_{_{i,j}} + c) + F'_1(x_{i,j})\big)\Big\rangle\\
&+ \sum_{k=1}^{K} \Big\langle \eta_i p_k(H^{(k)}_1 y^{(k)}_{i,j} + g^{(k)}) + F'_2(y^{(k)}_{i,j}), \big(\eta_i p_k H^{(k)}_1 + F''_2(y^{(k)}_{i,j})\big)^{-1}\\
&\times \big(\eta_i p_k(H^{(k)}_1 y^{(k)}_{i,j} + g^{(k)}) + F'_2(y^{(k)}_{i,j})\big)\Big\rangle\\
&= \langle C_{1_{i,j}}, Q^{-1}_{1_{i,j}} C_{1_{i,j}}\rangle + \sum_{k=1}^{K} \langle C^{(k)}_{2_{i,j}}, {Q^{(k)}}^{-1}_{2_{i,j}} C^{(k)}_{2_{i,j}}\rangle
\end{aligned} \tag{4.18}$$

where the last equality follows equalities (4.9) - (4.12).

Now we are ready to present our algorithm. From what we defined in (3.29), (3.30) and (3.32), we define the feasible set by $\mathcal{F} = \{X \in E \mid AX = B, x \in \mathcal{K}_1, y^{(k)} \in \mathcal{K}_2, \text{ for } k = 1,2,\dots,K\}$

	Algorithm : An interior-point trust-region algorithm
Step 0	**Initialization**
	set$i = 0, j = 0,$ choose starting point $(x_{0,0}, y^{(1)}_{0,0}, y^{(2)}_{0,0}, \dots, y^{(K)}_{0,0}) \in \{\mathcal{F}\}$, an initial trust-region radius $\alpha_{0,0} \in (0,1),$ and an initial parameter η_0 are given.
	Until convergence repeat.
Step 1	**Test inner iteration termination** compute

$$\|n_{\eta_i}(X_{i,j})\|^2_{X_{i,j}} \leq \frac{1}{36} \tag{4.19}$$

if $\|n_{\eta_i}(X_{i,j})\|^2_{X_{i,j}} \leq \dfrac{1}{36}$ go to step 2, otherwise set
$(x_{i+1,j}, y^{(1)}_{i+1,j}, y^{(2)}_{i+1,j}, \ldots, y^{(K)}_{i+1,j}) = (x_{i,j}, y^{(1)}_{i,j}, y^{(2)}_{i,j}, \ldots, y^{(K)}_{i,j})$
and go to step 3.

Step 2 **Step calculation**
solve (4.13)and (4.14) to obtain $(d'_{1_{i,j}}, d'^{(1)}_{2_{i,j}}, d'^{(2)}_{2_{i,j}}, \ldots, d'^{(K)}_{2_{i,j}})$
and compute (4.15)and (4.16) to obtain $(d_{1_{i,j}}, d^{(1)}_{2_{i,j}}, d^{(2)}_{2_{i,j}}, \ldots, d^{(K)}_{2_{i,j}})$,
update $(x_{i,j+1}, y^{(1)}_{i,j+1}, \ldots, y^{(K)}_{i,j+1}) = (x_{i,j} + d_{1_{i,j}}, y^{(1)}_{i,j} + d^{(1)}_{2_{i,j}}, \ldots, y^{(K)}_{i,j} + d^{(K)}_{2_{i,j}})$
and go to step 3.

Step 3 **Update parameter η**
Set $\eta_{i+1} = \theta\eta_i$ for some $\theta > 1$, increase i by 1 and go to *step 1.*

Lemma 4.1. *Any global minimizer* $(d'_{1_{i,j}}, d'^{(1)}_{2_{i,j}}, d'^{(2)}_{2_{i,j}}, \ldots, d'^{(K)}_{2_{i,j}})$ *of the problems (4.13) and (4.14)satisfies the equation*

$$(Q_{1_{i,j}} + \mu_{i,j} I) d'_{1_{i,j}} = -C_{1_{i,j}}, \tag{4.20}$$

$$(Q^{(k)}_{2_{i,j}} + \mu_{i,j} I^{(k)}) d'^{(k)}_{2_{i,j}} = -C^{(k)}_{2_{i,j}}, \ \textit{for } k = 1, 2, \ldots, K, \tag{4.21}$$

which $Q_{1_{i,j}} + \mu_{i,j} I$ *and* $Q^{(k)}_{2_{i,j}} + \mu_{i,j} I^{(k)}$ *for* $k = 1, 2, \ldots, K$ *are positive semi-definite* $\mu_{i,j} \geq 0$ *and* $\mu_{i,j}\big(\|d'_{1_{i,j}}\| + \sum_{k=1}^{K} \|d'^{(k)}_{2_{i,j}}\| - \alpha_{1_{i,j}} - \sum_{k=1}^{K} \alpha^{(k)}_{2_{i,j}}\big) = 0.$

This Lemma is well-known in the trust-region literature. For proof see e.g. Section 7.2 of [5].

Theorem 4.2. *If we choose* $\alpha_{1_{i,j}} + \sum_{k=1}^{K} \alpha^{(k)}_{2_{i,j}} = \dfrac{1}{4}$, *then we have*

$$f_{\eta_i}(x_{i,j+1}, y^{(1)}_{i,j+1}, y^{(2)}_{i,j+1}, \ldots, y^{(K)}_{i,j+1}) - f_{\eta_i}(x_{i,j}, y^{(1)}_{i,j}, y^{(2)}_{i,j}, \ldots, y^{(K)}_{i,j}) < -\frac{1}{145} \tag{4.22}$$

which is independent of i and j.

Proof. If the solution of equation (4.13) and (4.14) line on to boundary of the trust-region, i.e., $\|d'_{1_{i,j}}\| + \sum_{k=1}^{K} \|d'^{(k)}_{2_{i,j}}\| = \alpha_{1_{i,j}} + \sum_{k=1}^{K} \alpha^{(k)}_{2_{i,j}}$ we have

$$\begin{aligned} &q'_{i,j}(d'_{1_{i,j}}, d'^{(1)}_{2_{i,j}}, d'^{(2)}_{2_{i,j}}, \ldots, d'^{(K)}_{2_{i,j}}) \\ &= \frac{1}{2}\langle d'_{1_{i,j}}, Q_{1_{i,j}} d'_{1_{i,j}}\rangle + \langle C_{1_{i,j}}, d'_{1_{i,j}}\rangle + \sum_{k=1}^{K} \big(\frac{1}{2}\langle d'^{(k)}_{2_{i,j}}, Q^{(k)}_{2_{i,j}} d'^{(k)}_{2_{i,j}}\rangle + \langle C^{(k)}_{2_{i,j}}, d'^{(k)}_{2_{i,j}}\rangle\big), \end{aligned}$$

$$= \big\langle {d'}_{1_{i,j}}, (Q_{1_{i,j}} d'_{1_{i,j}} + C_{1_{i,j}}) \big\rangle - \frac{1}{2}\langle {d'}_{1_{i,j}}, Q_{1_{i,j}} d'_{1_{i,j}} \rangle$$
$$+ \sum_{k=1}^{K} \Big(\big\langle {d'}^{(k)}_{2_{i,j}}, (Q^{(k)}_{2_{i,j}} {d'}^{(k)}_{2_{i,j}} + C^{(k)}_{2_{i,j}}) \big\rangle - \frac{1}{2}\langle {d'}^{(k)}_{2_{i,j}}, Q^{(k)}_{2_{i,j}} {d'}^{(k)}_{2_{i,j}} \rangle \Big)$$
$$= -\langle {d'}_{1_{i,j}}, \mu_{i,j} d'_{1_{i,j}} \rangle - \frac{1}{2}\Big\langle {d'}_{1_{i,j}}, \big(\eta_i F''_1(x_{i,j})^{-\frac{1}{2}} H_0 F''_1(x_{i,j})^{-\frac{1}{2}} + I\big) d'_{1_{i,j}} \Big\rangle$$
$$+ \sum_{k=1}^{K} \Big(\langle -{d'}^{(k)}_{2_{i,j}}, \mu_{i,j} {d'}^{(k)}_{2_{i,j}} \rangle - \frac{1}{2}\langle {d'}^{(k)}_{2_{i,j}}, \big(\eta_i p_k F''_2(y^{(k)}_{i,j})^{-\frac{1}{2}} H^{(k)}_1 F''_2(y^{(k)}_{i,j})^{-\frac{1}{2}} + I^{(k)}\big) {d'}^{(k)}_{2_{i,j}} \rangle \Big)$$
$$= -\mu_{i,j}\big(\|d'_{1_{i,j}}\|^2 + \sum_{k=1}^{K} \|{d'}^{(k)}_{2_{i,j}}\|^2\big) - \frac{1}{2}\Big\langle {d'}_{1_{i,j}}, \big(\eta_i F''_1(x_{i,j})^{-\frac{1}{2}} H_0 F''_1(x_{i,j})^{-\frac{1}{2}}\big) d'_{1_{i,j}} \Big\rangle$$
$$-\tfrac{1}{2}\sum_{k=1}^{K} \Big\langle {d'}^{(k)}_{2_{i,j}}, \big(\eta_i p_k F''_2(y^{(k)}_{i,j})^{-\frac{1}{2}} H^{(k)}_1 F''_2(y^{(k)}_{i,j})^{-\frac{1}{2}}\big) {d'}^{(k)}_{2_{i,j}} \Big\rangle - \frac{1}{2}\big(\|d'_{1_{i,j}}\|^2 + \sum_{k=1}^{K} \|{d'}^{(k)}_{2_{i,j}}\|^2\big)$$
$$= -\mu_{i,j}(\alpha^2_{1_{i,j}} + \sum_{k=1}^{K} {\alpha^{(k)}_{2_{i,j}}}^2) - \frac{1}{2}\big\langle {d'}_{1_{i,j}}, \big(\eta_i F''_1(x_{i,j})^{-\frac{1}{2}} H_0 F''_1(x_{i,j})^{-\frac{1}{2}}\big) d'_{1_{i,j}} \big\rangle$$
$$-\frac{1}{2}\sum_{k=1}^{K} \Big\langle {d'}^{(k)}_{2_{i,j}}, \big(\eta_i p_k F''_2(y^{(k)}_{i,j})^{-\frac{1}{2}} H^{(k)}_1 F''_2(y^{(k)}_{i,j})^{-\frac{1}{2}}\big) {d'}^{(k)}_{2_{i,j}} \Big\rangle - \frac{1}{2}(\alpha^2_{1_{i,j}} + \sum_{k=1}^{K} {\alpha^{(k)}_{2_{i,j}}}^2)$$
$$\leq -\frac{1}{2}(\alpha^2_{1_{i,j}} + \sum_{k=1}^{K} {\alpha^{(k)}_{2_{i,j}}}^2) = -\frac{1}{32}. \tag{4.23}$$

In the above, the third equality follows from the equalities (4.9), (4.11), (4.20) and (4.21). The inequality follows from the fact that $\eta_i F''_1(x_{i,j})^{-\frac{1}{2}} H_0 F''_1(x_{i,j})^{-\frac{1}{2}}$ and
$\eta_i p_k F''_2(y^{(k)}_{i,j})^{-\frac{1}{2}} H^{(k)}_1 F''_2(y^{(k)}_{i,j})^{-\frac{1}{2}}$ for $k = 1, 2, \ldots, K$ are positive definite or positive semidefinite. Therefore we get the last inequality. From (4.17) and (4.23) we get

$$\begin{aligned} &f_{\eta_i}(x_{i,j+1}, y^{(1)}_{i,j+1}, y^{(2)}_{i,j+1}, \ldots, y^{(K)}_{i,j+1}) - f_{\eta_i}(x_{i,j}, y^{(1)}_{i,j}, y^{(2)}_{i,j}, \ldots, y^{(K)}_{i,j}) \\ &\leq -\frac{1}{32} + \frac{(1/4)^3}{3(1-1/4)} < -\frac{1}{145}. \end{aligned} \tag{4.24}$$

If the solution of (4.13) and (4.14) lies in the interior of the trust-region, i.e., $\|d'_{1_{i,j}}\|^2 + \sum_{k=1}^{K} \|{d'}^{(k)}_{2_{i,j}}\|^2 < \alpha^2_{1_{i,j}} + \sum_{k=1}^{K} {\alpha^{(k)}_{2_{i,j}}}^2$. From Lemma 4.2, we know $\mu_{i,j} = 0$ and consequently $d'_{1_{i,j}} = -Q^{-1}_{1_{i,j}} C_{1_{i,j}}$, ${d'}^{(k)}_{2_{i,j}} = -{Q^{(k)}}^{-1}_{2_{i,j}} C^{(k)}_{2_{i,j}}$ for $k = 1, 2, \ldots, K$ which gives

$$q'_{i,j}(d'_{1_{i,j}}, {d'}^{(1)}_{2_{i,j}}, {d'}^{(2)}_{2_{i,j}}, \ldots, {d'}^{(K)}_{2_{i,j}})$$
$$= \frac{1}{2}\langle {d'}_{1_{i,j}}, Q_{1_{i,j}} {d'}_{1_{i,j}} \rangle + \langle C_{1_{i,j}}, {d'}_{1_{i,j}} \rangle + \sum_{k=1}^{K} \big(\frac{1}{2}\langle {d'}^{(k)}_{2_{i,j}}, Q^{(k)}_{2_{i,j}} {d'}^{(k)}_{2_{i,j}} \rangle + \langle C^{(k)}_{2_{i,j}}, {d'}^{(k)}_{2_{i,j}} \rangle \big)$$

$$= -\frac{1}{2}\langle C_{1,i,j}, Q_{1_{i,j}}^{-1} C_{1,i,j}\rangle - \sum_{k=1}^{K}\frac{1}{2}\langle C_{2_{i,j}}^{(k)}, {Q^{(k)}}_{2_{i,j}}^{-1} C_{2_{i,j}}^{(k)}\rangle$$
$$= -\frac{1}{2}\Big(\langle C_{1,i,j}, Q_{1_{i,j}}^{-1} C_{1,i,j}\rangle + \sum_{k=1}^{K}\langle C_{2_{i,j}}^{(k)}, {Q^{(k)}}_{2_{i,j}}^{-1} C_{2_{i,j}}^{(k)}\rangle\Big)$$
$$= -\frac{1}{2}\|n_{\eta_i}(X_{i,j})\|_{X_{i,j}}^2. \tag{4.25}$$

By (4.25) and the mechanism of our algorithm, we know that $\|n_{\eta_i}(X_{i,j})\|_{X_{i,j}}^2 > \frac{1}{36}$ for all i and j we can rewrite (4.17) as

$$\begin{aligned} & f_{\eta_i}(x_{i,j+1}, y_{i,j+1}^{(1)}, y_{i,j+1}^{(2)}, \ldots, y_{i,j+1}^{(K)}) - f_{\eta_i}(x_{i,j}, y_{i,j}^{(1)}, y_{i,j}^{(2)}, \ldots, y_{i,j}^{(K)}) \\ & \leq -\frac{1}{2}\|n_{\eta_i}(X_{i,j})\|_{X_{i,j}}^2 + \frac{1}{3}\Big(\frac{(\alpha_{1_{i,j}} + \sum_{k=1}^{K}\alpha_{2_{i,j}}^{(k)})^3}{1-(\alpha_{1_{i,j}} + \sum_{k=1}^{K}\alpha_{2_{i,j}}^{(k)})}\Big) \\ & \leq -\frac{1}{72} + \frac{(1/4)^3}{3(1-1/4)} < -\frac{1}{145}. \end{aligned} \tag{4.26}$$

The proof is completed □

Now, we consider the number of iteration of our algorithm that we can stop the iteration when the reduction of objective function is smaller than some constant.

Lemma 4.3. *Let* $X^* = argmin_{X\in\mathcal{K}} q(X)$ *and* $X^* = [x^*, {y^{(1)}}^*, {y^{(2)}}^*, \cdots, {y^{(K)}}^*]^T \in \mathbb{R}^{(n_1 + \sum_{k=1}^{K} n_{2_k})}, x^* \in \mathcal{K}_1, {y^{(k)}}^* \in \mathcal{K}_2$ *for* $k = 1, 2, \ldots, K$. *If* $\|n_\eta(X)\|_X \leq \frac{1}{6}$, *then*

$$q(X) - q(X^*) \leq \frac{\vartheta + \sqrt{\vartheta}}{\eta}. \tag{4.27}$$

Proof. Let $X(\eta) = \text{arg}min_{X\in\mathcal{K}} f_\eta(X)$. From Lemma 2.5

$$\|X - X(\eta)\|_X \leq \frac{1}{6} + \frac{3(\frac{1}{6})^2}{(1-(\frac{1}{6}))^3} < \frac{1}{3} \tag{4.28}$$

and from Lemma 2.3 we have

$$\|X - X(\eta)\|_{X(\eta)} \leq \frac{\|X - X(\eta)\|_X}{1 - \|X - X(\eta)\|_X} < \frac{1}{2}. \tag{4.29}$$

Consider

$$\begin{aligned} q(X(\eta)) - q(X^*) & \leq \big\langle q'(X(\eta)), X(\eta) - X^*\big\rangle \\ & = \big\langle \frac{-F'(X(\eta))}{\eta}, X(\eta) - X^*\big\rangle \\ & \leq \frac{\vartheta}{\eta}. \end{aligned} \tag{4.30}$$

The first inequality follows from the convexity of $q(X)$. The equality follows from the fact that $f'(X(\eta)) = 0$ and the last inequality follows from Lemma 2.6. From the inequality (4.28) and (4.29) Then, we have

$$\begin{aligned}
&q(X) - q(X(\eta))\\
&= \Big\langle q'\big(X(\eta)\big), X - X(\eta)\Big\rangle + \frac{1}{2}\Big\langle X - X(\eta), Q\big(X - X(\eta)\big)\Big\rangle\\
&= \Big\langle \frac{-F'(X(\eta))}{\eta}, X - X(\eta)\Big\rangle + \frac{1}{2\eta}\Big\langle X - X(\eta), \eta Q\big(X - X(\eta)\big)\Big\rangle\\
&\le \frac{1}{\eta}\Big\langle -F''\big(X(\eta)\big)^{-\frac{1}{2}}F'\big(X(\eta)\big), F''\big(X(\eta)\big)^{-\frac{1}{2}}\big(X - X(\eta)\big)\Big\rangle\\
&+ \frac{1}{2\eta}\|X - X(\eta)\|_X^2\\
&\le \|F''\big(X(\eta)\big)^{-\frac{1}{2}}F'\big(X(\eta)\big)\|\|F''\big(X(\eta)\big)^{-\frac{1}{2}}\big(X - X(\eta)\big)\| + \frac{1}{18\eta}\\
&= \Big\langle F'\big(X(\eta)\big), F''\big(X(\eta)\big)^{-1}F'\big(X(\eta)\Big\rangle^{\frac{1}{2}}\Big\langle \big(X - X(\eta)\big), F''\big(X(\eta)\big)^{-1}\big(X - X(\eta)\big)\Big\rangle^{\frac{1}{2}}\\
&+ \frac{1}{18\eta}\\
&\le \frac{\sqrt{\vartheta}\|X - X(\eta)\|_{X(\eta)}}{\eta} + \frac{1}{18\eta}\\
&\le \frac{\sqrt{\vartheta}}{2\eta} + \frac{1}{18\eta} \le \frac{\sqrt{\vartheta}}{\eta} \qquad (4.31)
\end{aligned}$$

where the third last inequality uses the Lemma 2.6 and from the fact that ϑ is always greater than 1. By adding the inequalities (4.30) and (4.31), then we have

$$q(X) - q(X^*) \le \frac{\vartheta + \sqrt{\vartheta}}{\eta}.$$

The proof is completed □

The above Lemma tells us that to get an ϵ solution. Let $\eta_{i+1} = \theta\eta_i$ for some $\theta > 1$ and, we only need

$$\eta_i = \eta_0\theta^i \ge \frac{\vartheta + \sqrt{\vartheta}}{\epsilon}, \qquad (4.32)$$

provided that the number of outer iterations i satisfies

$$i \ge \frac{ln(\vartheta + \sqrt{\vartheta}/\epsilon\eta_0)}{ln\theta}. \qquad (4.33)$$

Lemma 4.4. *If* $\|n_{\eta i}(X)\|_X \le \frac{1}{6}$, *then*

$$f_{\eta_{i+1}}(X) - f_{\eta_{i+1}}\big(X(\eta_{i+1})\big) \le \theta(\vartheta + \sqrt{\vartheta}). \qquad (4.34)$$

Proof. From the convexity of $f_{\eta_{i+1}}(X)$ and the inequality (4.28), we can show that

$$\begin{aligned}
&f_{\eta_{i+1}}(X) - f_{\eta_{i+1}}(X(\eta_i))\\
&\le \big\langle f'_{\eta_{i+1}}(X), X - X(\eta_i)\big\rangle\\
&= \big\langle \eta_{i+1}(QX + C) + F'(X), X - X(\eta_i)\big\rangle\\
&= \frac{\eta_{i+1}}{\eta_i}\big\langle \eta_i(QX + C) + F'(X), X - X(\eta_i)\big\rangle\\
&+ \big(\frac{\eta_{i+1}}{\eta_i} - 1\big)\big\langle F'(X), X(\eta_i) - X\big\rangle\\
&= \theta\Big\langle {f''_{\eta_i}}^{-\frac{1}{2}}(X)\big(\eta_i(QX + C) + F'(X)\big), {f''_{\eta_i}}^{\frac{1}{2}}(X)(X - X(\eta_i))\Big\rangle\\
&+ (\theta - 1)\Big\langle F''(X)^{-\frac{1}{2}}F'(X), F''(X)^{-\frac{1}{2}}\big(X(\eta_i) - X\big)\Big\rangle\\
&\le \theta\|{f''_{\eta_i}}^{-\frac{1}{2}}(X)\big(\eta_i(QX + C) + F'(X)\big)\|\|{f''_{\eta_i}}^{\frac{1}{2}}(X)\big(X - X(\eta_i)\big)\|\\
&+ (\theta - 1)\|F''(X)^{-\frac{1}{2}}F'(X)\|\|F''(X)^{\frac{1}{2}}\big(X(\eta_i) - X\big)\|\\
&\le \theta\|n_{\eta_i}(X)\|_X\|X(\eta_i) - X\|_X + (\theta - 1)\sqrt{\vartheta}\|X(\eta_i) - X\|_X\\
&\le \theta(\frac{1}{6})(\frac{1}{3}) + (\theta - 1)\sqrt{\vartheta}\frac{1}{3}\\
&\le \theta\sqrt{\vartheta}.
\end{aligned} \tag{4.35}$$

Consider

$$\begin{aligned}
&f_{\eta_{i+1}}\big(X(\eta_i)\big) - f_{\eta_{i+1}}(X(\eta_{i+1}))\\
&\le \Big\langle f'_{\eta_{i+1}}(X(\eta_i)), X(\eta_i) - X(\eta_{i+1})\Big\rangle\\
&= \langle \eta_{i+1}(QX(\eta_i) + C) + F'(X(\eta_i)), X(\eta_i) - X(\eta_{i+1})\rangle\\
&= \frac{\eta_{i+1}}{\eta_i}\Big\langle \eta_i(QX(\eta_i) + C) + F'(X(\eta_i)), X(\eta_i) - X(\eta_{i+1})\Big\rangle\\
&+ \big(\frac{\eta_{i+1}}{\eta_i} - 1\big)\Big\langle F'\big(X(\eta_i)\big), X(\eta_{i+1}) - X(\eta_i)\Big\rangle\\
&= \theta\Big\langle f'_{\eta_i}\big(X(\eta_i)\big), X(\eta_i) - X(\eta_{i+1})\Big\rangle\\
&+ (\theta - 1)\Big\langle F'\Big(X(\eta_i)\Big), X(\eta_{i+1}) - X(\eta_i)\Big\rangle\\
&= (\theta - 1)\Big\langle F'\big(X(\eta_i)\big), X(\eta_{i+1}) - X(\eta_i)\Big\rangle < \theta\vartheta,
\end{aligned} \tag{4.36}$$

where the last equality follows from the fact that $X(\eta_i)$ minimizes $f_{\eta_i}(X)$ (which implies that $f'_{\eta_i}(X) = 0$), the last inequality follows from Lemma 2.6. By adding inequalities (4.35) and (4.36), then we have

$$f_{\eta_{i+1}}(X) - f_{\eta_{i+1}}\big(X(\eta_{i+1})\big) \le \theta(\vartheta + \sqrt{\vartheta}).$$

The proof is completed □

Theorem 4.2 and Lemma 4.4 tell us that the steps in each inner iteration in at most

$$145\theta(\vartheta + \sqrt{\vartheta}) \tag{4.37}$$

steps in each inner iteration.

Theorem 4.5. *If the initial point $X_{0,0}$ satisfies the conditions (4.19), for any $\epsilon > 0$, our algorithm obtains the solution X which satisfies $q(X) - q(X^*) < \epsilon$ in at most*

$$\frac{145\theta(\vartheta - \sqrt{\vartheta}) ln(\dfrac{\vartheta + \sqrt{\vartheta}}{\epsilon\eta_0})}{ln\theta} \tag{4.38}$$

steps, here $X^* = argmin_{X\in\mathcal{K}} q(X)$.

Proof. Inequality (4.33) provides us with the number of outer iterations i and Theorem 4.2 and Lemma 4.4 provides us with the number of steps in each iteration j. The number of iterations is at most the number of outer iterations multiply the number of inner iterations then we have the bound of number of iteration in at most

$$145\theta(\vartheta - \sqrt{\vartheta}) \times \frac{ln(\dfrac{\vartheta + \sqrt{\vartheta}}{\epsilon\eta_0})}{ln\theta}$$

steps. □

Consider the case when the initial point $X_{0,0}$ does not satisfy condition (4.19). We can start from the analytic center of the feasible set $\mathcal{K}$, since $\mathcal{K}$ is a bounded convex set. Let $X(\eta_0) = argmin_{X\in\mathcal{K}} f_{\eta_0}(X)$ and $X^* = \text{arg}min_{X\in\mathcal{K}} q(X)$. Since $X_{0,0} = \text{arg}min_{X\in\mathcal{K}} F(X)$, we have

$$f_{\eta_0}(X_{0,0}) - f_{\eta_0}\big(X(\eta_0)\big) \le \eta_0\big(q(X_{0,0}) - q(X^*)\big) \tag{4.39}$$

and we choose $\eta_0 \le 1/\big(q(X_{0,0}) - q(X^*)\big)$, from Theorem 4.2 implies that condition (4.19) will be satisfied after at most 145 steps.

5 Implementing the Algorithm

In this section we discuss the performance of our algorithm when the domain of the problem is in nonnegative orthant cones and second-order cones. These cones are all well known examples of symmetric cone. We consider multi dimension real sets. We compare our results with those of the MATLAB optimization toolbox is it called "quadprog function"in cases of the nonnegative orthant cones. Our algorithm can solve stochastic symmetric programming problems that include variables in second- order cone and cones of real symmetric positive semidefinite matrices. The quadprog function is unable to solve problems with these variables.

5.1 Problem Statement

The parameter chosen for deterministic data of SQSP (3.24)-(3.27) are shown below.

$$H_0 = \begin{bmatrix} 64 & 1 & 4 & 9 \\ 1 & 81 & 25 & 16 \\ 4 & 25 & 100 & 9 \\ 9 & 16 & 9 & 144 \end{bmatrix} \quad c = \begin{bmatrix} -1 \\ 2 \\ -3 \\ 5 \end{bmatrix} \quad A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 1 & 2 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 33 \\ 13 \end{bmatrix}$$

The random variables $H_1^{(k)}, W^{(k)}, T^{(k)}, h^{(k)}, d^{(k)}$, $k = 1, 2, ..., K$. For simplicity, we consider the simple case $K = 4$ scenarios but the procedure can be easily extended to very large K.

$H_1^{(1)}$	$H_1^{(2)}$	$H_1^{(3)}$
$\begin{bmatrix} 25 & 1 & 4 \\ 1 & 33 & 25 \\ 4 & 25 & 40 \end{bmatrix}$	$\begin{bmatrix} 64 & 1 & 4 & 9 & 2 \\ 1 & 81 & 25 & 16 & 4 \\ 4 & 25 & 100 & 9 & 6 \\ 9 & 16 & 9 & 144 & 8 \\ 2 & 4 & 6 & 8 & 36 \end{bmatrix}$	$\begin{bmatrix} 24 & 1 & 4 & 9 & 2 & 3 \\ 1 & 51 & 25 & 16 & 2 & 4 \\ 4 & 25 & 100 & 9 & 5 & 6 \\ 9 & 16 & 9 & 144 & 3 & 6 \\ 2 & 2 & 5 & 3 & 49 & 1 \\ 3 & 4 & 6 & 6 & 1 & 64 \end{bmatrix}$

$H_1^{(4)}$
$\begin{bmatrix} 20 & 1 & 4 & 9 & 2 & 3 & 4 \\ 1 & 36 & 25 & 16 & 2 & 4 & 5 \\ 4 & 25 & 100 & 9 & 5 & 6 & 6 \\ 9 & 16 & 9 & 144 & 3 & 6 & 7 \\ 2 & 2 & 5 & 3 & 49 & 1 & 8 \\ 3 & 4 & 6 & 6 & 1 & 64 & 9 \\ 4 & 5 & 6 & 7 & 8 & 9 & 40 \end{bmatrix}$

$W^{(1)}$	$W^{(2)}$	$W^{(3)}$	$W^{(4)}$
$\begin{bmatrix} 1 & 1 & 3 \\ 2 & 3 & 3 \end{bmatrix}$	$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 2 & 1 & 4 & 2 \\ 1 & 1 & 2 & 1 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 2 & 3 & 6 & 5 & 4 \\ 2 & 1 & 1 & 1 & 2 & 1 \\ 3 & 4 & 5 & 1 & 2 & 3 \\ 1 & 1 & 2 & 1 & 2 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 1 & 1 & 2 & 3 & 4 & 5 \\ 2 & 2 & 2 & 3 & 2 & 1 & 1 \\ 5 & 2 & 1 & 1 & 4 & 1 & 1 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 1 & 2 & 3 & 5 & 8 & 9 \end{bmatrix}$

$h^{(1)}$	$h^{(2)}$	$h^{(3)}$	$h^{(4)}$
$\begin{bmatrix} 33 \\ 42 \end{bmatrix}$	$\begin{bmatrix} 77 \\ 37 \\ 22 \end{bmatrix}$	$\begin{bmatrix} 91 \\ 49 \\ 78 \\ 40 \end{bmatrix}$	$\begin{bmatrix} 91 \\ 65 \\ 60 \\ 140 \\ 163 \end{bmatrix}$

$T^{(1)}$	$T^{(2)}$	$T^{(3)}$	$T^{(4)}$
$\begin{bmatrix} 3 & 2 & 1 & 0 \\ 1 & 2 & 3 & 0 \end{bmatrix}$	$\begin{bmatrix} 3 & 3 & 4 & 5 \\ 2 & 1 & 1 & 2 \\ 3 & 0 & 2 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 3 & 2 & 1 & 4 \\ 1 & 1 & 3 & 5 \\ 2 & 2 & 2 & 2 \end{bmatrix}$	$\begin{bmatrix} 2 & 1 & 2 & 1 \\ 3 & 2 & 1 & 4 \\ 2 & 1 & 3 & 2 \\ 1 & 1 & 4 & 2 \\ 2 & 4 & 3 & 1 \end{bmatrix}$

$g^{(1)}$	$g^{(2)}$	$g^{(3)}$	$g^{(4)}$
$\begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \\ 2 \end{bmatrix}$	$\begin{bmatrix} 3 \\ 4 \\ 1 \\ 3 \\ 4 \\ 5 \end{bmatrix}$	$\begin{bmatrix} 1 \\ -2 \\ 3 \\ 4 \\ -5 \\ 1 \\ 1 \end{bmatrix}$

we randomly generate probability function p in 6 cases as shown below.

	p_1	p_2	p_3	p_4
case1	0.1	0.2	0.3	0.4
case2	0.4	0.3	0.2	0.1
case3	0.25	0.25	0.25	0.25
case4	0.5	0.2	0.2	0.1
case5	0.9	0.1	0	0
case6	0.3	0.3	0.3	0.1

5.2 Results of an Interior-Point Trust-Region Algorithm

In order to make a comparison, the problems (3.24)-(3.27) are described in chapter 4. Our algorithm is then implemented under MATLAB environment release 2015a. The results are in table (1) - (2), for case nonnegative orthant cones, we compare our results with those of the quadprog function in MATLAB optimization toolbox and our algorithm as in table in table (1) and (2). In particular, we present our results of stochastic second order cone programming as in Table (3). From the numerical results example, we have the same optimal solutions with those of the quadprog function and our algorithm. The numerical results for stochastic second order cone programming, some case of problem are same optimal solutions and some case are difference because the restrictions of cones.

Table 1: The solution of SQSP when $\mathcal{K}_1$ and $\mathcal{K}_2$ are nonnegative cone by quadprog function for R^n_+.

	Case1	Case2	Case3	Case4	Case5	Case6
x_1	1.0676	0.91286	1.0212	0.87326	0.90544	0.82258
x_2	2.557	2.354	2.4921	2.4486	3.0439	2.1565
x_3	2.1366	2.3107	2.1904	2.2966	2.0392	2.4438
x_4	5.1021	5.1118	5.1058	5.0849	4.9723	5.1332
$y_1^{(1)}$	4.4744	4.4389	4.464	4.4208	4.3938	4.4231
$y_2^{(1)}$	1.1938	0.8827	1.0988	0.86628	1.1694	0.66718
$y_3^{(1)}$	5.6261	5.9737	5.733	5.9665	5.5312	6.2284
$y_1^{(2)}$	1.547	1.5907	1.5593	1.631	1.7314	1.6006
$y_2^{(2)}$	0.77638	0.79472	0.7818	0.80212	0.80853	0.80395
$y_3^{(2)}$	0.79228	0.78553	0.78751	0.88298	1.2586	0.72812
$y_4^{(2)}$	0.97662	0.9977	0.98368	0.9761	0.86986	1.0246
$y_5^{(2)}$	4.5372	4.5741	4.5498	4.527	4.306	4.626
$y_1^{(3)}$	4.51E-01	7.53E-01	5.43E-01	7.96E-01	1.8008	0.94799
$y_2^{(3)}$	3.5633	3.3365	3.4927	3.3736	3.1436	3.1529
$y_3^{(3)}$	0.58643	0.5789	0.58513	0.54237	1.6117	0.59316
$y_4^{(3)}$	2.2359	2.1789	2.2182	2.1884	4.2732	2.1327
$y_5^{(3)}$	2.7175	3.0054	2.8069	2.9676	1.0372	3.2335
$y_6^{(3)}$	5.4149	5.1841	5.3432	5.2154	3.5629	5.0007
$y_1^{(4)}$	1.2744	1.8526	1.4545	1.7549	0.44043	2.3223
$y_2^{(4)}$	1.3673	1.4487	1.387	1.6402	5.7409	1.4042
$y_3^{(4)}$	1.5263	1.5039	1.5222	1.4023	1.3403	1.5427
$y_4^{(4)}$	2.1789	2.472	2.2693	2.4529	0.19677	2.6938
$y_5^{(4)}$	3.3081	2.3796	3.0207	2.4709	2.6141	1.66E+00
$y_6^{(4)}$	4.7635	4.8505	4.7931	4.7462	5.692	4.9694
$y_7^{(4)}$	7.8856	8.1594	7.9684	8.2052	13.714	8.3322
Optimal value	5488.1	4489.4	4991.5	4539.2	4084	4581.5

Table 2: The solution of SQSP when $\mathcal{K}_1$ and $\mathcal{K}_2$ are nonnegative con by an interior-point trust-region algorithm for R^n_+.

	Case1	Case2	Case3	Case4	Case5	Case6
x_1	1.0676	0.91286	1.0212	0.87326	0.90543	0.82258
x_2	2.557	2.354	2.4921	2.4486	3.0439	2.1565
x_3	2.1366	2.3107	2.1904	2.2966	2.0392	2.4438
x_4	5.1021	5.1118	5.1058	5.0849	4.9723	5.1332
$y_1^{(1)}$	4.4744	4.4389	4.464	4.4208	4.3938	4.4231
$y_2^{(1)}$	1.1938	0.88271	1.0988	0.86628	1.1693	0.66719
$y_3^{(1)}$	5.6261	5.9737	5.733	5.9665	5.5312	6.2284
$y_1^{(2)}$	1.547	1.5907	1.5593	1.6309	1.7314	1.6006
$y_2^{(2)}$	0.77639	0.79473	0.78182	0.80213	0.80859	0.80396
$y_3^{(2)}$	0.7923	0.78554	0.78751	0.88298	1.2586	0.72812
$y_4^{(2)}$	0.97665	0.9977	0.98369	0.9761	0.86987	1.0246
$y_5^{(2)}$	4.5372	4.574	4.5498	4.527	4.306	4.626
$y_1^{(3)}$	0.45125	0.7532	0.54272	0.79587	1.7076	0.948
$y_2^{(3)}$	3.5633	3.3365	3.4927	3.3736	3.1237	3.1529
$y_3^{(3)}$	0.58646	0.57891	0.58515	0.54238	1.5184	0.59317
$y_4^{(3)}$	2.2359	2.1789	2.2182	2.1884	4.0644	2.1327
$y_5^{(3)}$	2.7175	3.0054	2.8069	2.9676	1.16	3.2335
$y_6^{(3)}$	5.4149	5.1841	5.3432	5.2154	3.8259	5.0007
$y_1^{(4)}$	1.2744	1.8526	1.4545	1.7549	1.0404	2.3222
$y_2^{(4)}$	1.3674	1.4488	1.387	1.6402	2.4054	1.4042
$y_3^{(4)}$	1.5263	1.5039	1.5222	1.4023	2.1616	1.5427
$y_4^{(4)}$	2.1789	2.472	2.2693	2.4529	1.3539	2.6938
$y_5^{(4)}$	3.3081	2.3796	3.0207	2.4709	3.2819	1.6607
$y_6^{(4)}$	4.7636	4.8505	4.7931	4.7462	3.2029	4.9694
$y_7^{(4)}$	7.8856	8.1594	7.9684	8.2052	9.2244	8.3322
Optimal value	5488.1	4489.4	4991.5	4539.2	4084	4581.5

Table 3: The solution of SQSP when $\mathcal{K}_1$ and $\mathcal{K}_2$ are second-order cone by an interior-point trust-region algorithm for SOCP.

	Case1	Case2	Case3	Case4	Case5	Case6
x_1	1.0658	0.91295	1.0212	0.87326	0.90544	0.87815
x_2	2.5575	2.3541	2.4921	2.4486	3.0439	2.2279
x_3	2.1375	2.3106	2.1904	2.2966	2.0392	2.3819
x_4	5.1017	5.1118	5.1058	5.0849	4.9723	5.13
$y_1^{(1)}$	4.3596	4.4387	4.464	4.4207	4.3938	3.837
$y_2^{(1)}$	1.2485	0.88299	1.0988	0.8663	1.1694	1.0777
$y_3^{(1)}$	5.6473	5.9735	5.733	5.9664	5.5312	6.2044
$y_1^{(2)}$	1.5532	1.5907	1.5593	1.631	1.7314	1.5657
$y_2^{(2)}$	0.77621	0.7947	0.7818	0.80212	0.80856	0.82513
$y_3^{(2)}$	0.79222	0.78552	0.78751	0.88298	1.2586	0.72748
$y_4^{(2)}$	0.97088	0.99765	0.98368	0.97609	0.86985	1.0236
$y_5^{(2)}$	4.5412	4.5741	4.5498	4.527	4.306	4.6022
$y_1^{(3)}$	0.47082	0.75396	0.54272	0.79593	-18.313	0.88651
$y_2^{(3)}$	3.5008	3.3332	3.4926	3.3733	-88.92	3.1116
$y_3^{(3)}$	0.60241	0.57985	0.58517	0.54246	-18.502	0.6428
$y_4^{(3)}$	2.2219	2.1782	2.2181	2.1883	-109.73	2.1538
$y_5^{(3)}$	2.706	3.0045	2.8069	2.9675	40.049	3.0888
$y_6^{(3)}$	5.4647	5.1871	5.3433	5.2157	191.94	5.1488
$y_1^{(4)}$	1.28	1.8524	1.4545	1.7549	4.0762	2.2052
$y_2^{(4)}$	1.3638	1.4483	1.387	1.6402	-0.69272	0.95488
$y_3^{(4)}$	1.5334	1.5043	1.5223	1.4023	10.911	1.6957
$y_4^{(4)}$	2.1791	2.4718	2.2693	2.4529	-1.9777	2.712
$y_5^{(4)}$	3.3016	2.3801	3.0207	2.4709	-0.22866	2.0532
$y_6^{(4)}$	4.7489	4.85	4.7931	4.7462	-7.0953	4.5863
$y_7^{(4)}$	7.8997	8.1596	7.9684	8.2052	19.165	8.4675
Optimal value	5488.2	4489.4	4991.5	4539.2	4084	4585.5

6 Conclusion and Remarks

In this paper, we present stochastic symmetric programming with linear function and quadratic function and finite scenarios. We can explicitly formulate the problem as a large scale deterministic programs. The complexity of our algorithm is proved to be as good as the interior-point polynomial algorithm. We have verified our performance of the algorithm on simple case study problems. Numerical results show the effectiveness of our algorithm.

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