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Some Common Fixed Point Theorems in Fuzzy Metric Space with Property (E.A.)

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Abstract : The aim of the present paper is to prove some common fixed point theorems for weakly compatible self mappings in a fuzzy metric space using property (E.A.) which generalize and improve various well-known comparable results.

Keywords : common fixed point; fuzzy metric space; property (E.A.); weakly compatible maps.
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1 Introduction

The study of common fixed points of mappings in a fuzzy metric space satisfying certain contractive conditions has been at the center of vigorous research activity. The concept of fuzzy sets was initiated by Zadeh [1] in 1965. With the concept of fuzzy sets, the fuzzy metric space was introduced by Kramosil and Michalek [2]. Also, Grabiec [3] proved the contraction principle in the setting of the fuzzy metric space which was further generalization of results by Subrahmanyam [4] for a pair of commuting mappings. Also, George and Veeramani [5] modified the notion of fuzzy metric spaces with the help of continuous t-norm, by

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generalizing the concept of probabilistic metric space to fuzzy situation. In 1999, Vasuki [6] introduced the concept of R-weak commutativity of mappings in fuzzy metric space. Mishra et al. [7] introduced the notion of compatible maps under the name of asymptotically commuting maps in fuzzy metric spaces. Singh and Jain [8] studied the notion of weakly compatibility in fuzzy metric space that was introduced by Jungck and Rhoades [9] in metric space. Balasubramaniam et al. [10] proved a common fixed point theorem for reciprocally continuous mappings in fuzzy metric space.

Pant and Jha [11] proved a fixed point theorem that gives an analogue of the results by Balasubramaniam et al. [10] by obtaining a connection between the continuity and reciprocal continuity for four mappings in fuzzy metric space. Recently, Kutukcuet al. [12] has established a common fixed point theorem in a fuzzy metric space by studying the relationship between the continuity and reciprocal continuity which is a generalization of the results of Mishra [13] and also gives an answer to the open problem of Rhoades [14] in fuzzy metric space. Also, Regan and Abbas [15] obtained some necessary and sufficient conditions for the existence of common fixed point theorem for mappings in fuzzy metric space. Cho et al. [16] established some fixed point theorems for mappings satisfying contractive condition in fuzzy metric space.

On the other hand, Aamri and Moutawakil [17], in 2002, studied a new property for pair of maps, that is, so called property (E.A.) which is a generalization of the concept of non-compatible maps in metric space. Also, Pant and Pant [18] studied the common fixed points of a pair of non-compatible maps and the property (E.A.) in fuzzy metric space. Now a days, implicit relations are used as a tool for finding common fixed point of contraction maps (for details, please do refer [19–23]). These implicit relations guarantee coincidence point for pair of maps that ultimately leads to the existence of common fixed points of a quadruple of mappings satisfying weak compatibility criterion. In 2008, Altun and Turkoghu [19] proved two common fixed point theorems on complete fuzzy metric space with an implicit relation for continuous compatible maps of types (α) or (β). Recently, Abbas et al. [20] proved some common fixed point theorems for non-compatible maps in fuzzy metric space using implicit relation. Also, Kumar and Fisher [21] obtained a common fixed point theorem for weakly compatible maps in fuzzy metric space using property (E.A.).

The purpose of this paper is to prove some common fixed point theorems for sequence of self mappings in fuzzy metric space under the weak contractive conditions using the implicit relation, by relaxing the continuity of mappings and even the completeness. Our results generalize and improve various other similar results of fixed points. We also give an example to illustrate our main theorems.

We have used the following notions:

Definition 1.1 ([1])**.** Let X be any set. A *fuzzy set* A in X is a function with domain X and values in $[0, 1]$.

Definition 1.2 ([22])**.** A binary operation $* : [0, 1] \times [0, 1] \to [0, 1]$ is called a *continuous* $t-$*norm* if, $([0, 1], *)$ is an abelian topological monoid with unit 1 such that $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$, for all a, b, c, d in $[0, 1]$.

Example 1.3. $a * b = ab$, $a * b = \min\{a, b\}$.

Definition 1.4 ([2]). The triplet $(X, M, *)$ is called a *fuzzy metric space* (shortly, a *FM-space*) if, X is an arbitrary set, $*$ is a continuous $t-$norm and M is a fuzzy set on $X^2 \times [0, \infty)$ satisfying the following conditions: for all x, y, z in X, $s, t > 0$,

(i) $M(x, y, 0) = 0$,

(ii) $M(x, y, t) = 1$ for all $t > 0$ if and only if $x = y$,

(iii) $M(x, y, t) = M(y, x, t)$,

(iv) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$,

(v) $M(x, y, \cdot) : [0, \infty) \to [0, 1]$ is left continuous and $s, t > 0$.

In this case, M is called a *fuzzy metric* on X and the function $M(x, y, t)$ denotes the degree of nearness between x and y with respect to t. Also, we consider the following condition in the fuzzy metric space $(X, M, *)$:

(vi) $\lim_{t\to\infty} M(x, y, t) = 1$, for all $x, y \in X$.

It is important to note that every metric space (X, d) induces a fuzzy metric space $(X, M, *)$ where $a * b =$ a b (or $a * b =$ min $\{a, b\}$) and for all $x, y \in X$, we have $M(x, y, t) = \frac{t}{t+d(x,y)}$, for all $t > 0$, and $M(x, y, 0) = 0$, so-called the fuzzy metric space induced by the metric d and it is often referred to as the standard fuzzy metric.

Definition 1.5 ([3]). A sequence $\{x_n\}$ in a fuzzy metric space $(X, M, *)$ is called *Cauchy sequence* if, $\lim_{n\to\infty} M(x_{n+p}, x_n, t) = 1$ for every $t > 0$ and for each $p > 0$.

A fuzzy metric space $(X, M, *)$ is complete if, every Cauchy sequence in X converges in X.

Definition 1.6 ([3]). A sequence $\{x_n\}$ in a fuzzy metric space $(X, M, *)$ is said to be *convergent to* x in X if, $\lim_{n\to\infty} M(x_n, x, t) = 1$ for each $t > 0$.

It is noted that since $*$ is continuous, it follows from the condition (iv) of Definition 1.4 that the limit of a sequence in a fuzzy metric space is unique.

Definition 1.7 ([7]). Two self mappings A and S of a fuzzy metric space $(X, M, *)$ are said to be *compatible* or *asymptotically commuting* if, for all $t > 0$, $\lim_{n\to\infty} M(ASx_n, SAx_n, t) = 1$ whenever $\{x_n\}$ is a sequence such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = p$ for some p in X.

It is noted that mappings A and S are noncompatible maps, if there exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Ax_n = p = \lim_{n\to\infty} Sx_n$, but either $\lim_{n\to\infty} M(ASx_n, SAx_n, t) \neq 1$ or the limit does not exist for all p in X.

Definition 1.8 ([23])**.** Two self mappings A and S of a fuzzy metric space $(X, M, *)$ are said to be *weakly compatible* if, they commute at coincidence points. That is, $Ax = Sx$ implies that $ASx = SAx$ for all x in X.

It is important to note that a compatible mappings in a fuzzy metric space are weakly compatible but weakly compatible mappings need not be compatible (see [23, Example 2, page 160]).

Definition 1.9 ([18])**.** Two self mappings A and S of a fuzzy metric space $(X, M, *)$ are said to *satisfy (E.A.) property* if,there exist a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = x$ for some x in X.

Definition 1.10 ([20])**.** Mappings A, B, S, and T of a fuzzy metric space $(X, M, *)$ are said to *satisfy (E.A.) property* if,there exists sequences $\{x_n\}$ and $\{y_n\}$ in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = x$ for some x in X.

It is noted that compatible and non-compatible maps satisfy (E.A.) property but the converse is not in general true. Also, the weakly compatible and property (E.A.) are independent to each other (see [24, Example 2.2]).

Lemma 1.11 ([7])**.** *Let $(X, M, *)$ be a fuzzy metric space. If there exists $h \in (0, 1)$ such that $M(x, y, ht) \geq M(x, y, t)$, then $x = y$.*

Let Φ a class of implicit relations be the set of all continuous functions $\phi : [0, 1]^5 \to [0, 1]$ which are increasing in each coordinate and $\phi(t, t, t, t, t) > t$ for all $t \in [0, 1)$.

If $\{A_i\}, i = 1, 2, 3, \ldots, S$ and T are self mappings of fuzzy metric space $(X, M, *)$ in the sequel, we shall denote

$$M_{1i}(x, y, t) = \{M(A_1x, Sx, t), M(A_iy, Ty, t), M(Sx, Ty, t), M(A_1x, Ty, \alpha t)\\ M(Sx, A_iy, (2-\alpha)t)\},$$

for all $x, y \in X$, $\alpha \in (0, 2)$, $t > 0$ and $\phi \in \Phi$.

2 Main Results

Theorem 2.1. *Let $(X, M, *)$ be a fuzzy metric space. Let $\{A_i\}, i = 1, 2, 3, \ldots, S$ and T be mappings of a fuzzy metric space from X into itself such that*

(i) $A_1X \subseteq TX$, $A_iX \subseteq SX$, for $i > 1$, and

(ii) there exists a constant $r \in (0, 1/2)$ such that

$$M(A_1x, A_iy, rt) \geq \phi(M_{1i}(x, y, t)),$$

for all $x, y \in X$, $\alpha \in (0, 2)$, $t > 0$ and $\phi \in \Phi$. If one of A_iX, SX and TX is a closed subset of X; for some $k > 1$ if the pair (A_1, S) and (A_k, T) are weakly compatible, and the pair $\{A_1, S\}$ or $\{A_k, T\}$ satisfies (E.A.) property, then all the mappings A_i, S and T have a unique common fixed point in X.

Proof. Suppose that a pair $\{A_k, T\}$ satisfy the property (E.A.), then by definition, there exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} A_k x_n = \lim_{n\to\infty} T x_n = z$ for some z in X. Since $A_k X \subseteq SX$, so there exists a sequence $\{y_n\}$ in X such that $A_k x_n = S y_n$. So that, for $\alpha = 1$, setting $x = y_n$ and $y = x_n$ in condition (ii), we get

$$M(A_1 y_n, A_k x_n, rt) \geq \phi(M(A_1 y_n, S y_n, t), M(A_k x_n, T x_n, t), M(S y_n, T x_n, t), t),$$
$$M(A_1 y_n, T x_n, t), M(S y_n, A_k x_n, t)).$$

Taking limit as $n \to \infty$, we get

$$M\left(\lim_{n\to\infty} A_1 y_n, A_k x_n, rt\right) \geq \phi\left(M\left(\lim_{n\to\infty} A_1 y_n, z, t\right), M(z,z,t), M(z,z,t),\right.$$
$$\left. M\left(\lim_{n\to\infty} A_1 y_n, z, t\right), M(z,z,t)\right).$$

Since ϕ is increasing in each of its coordinate and $\phi(t,t,t,t,t) > t$ for all $t \in [0,1)$, so, we get $M(\lim_{n\to\infty} A_1 y_n, z, rt) > M(\lim_{n\to\infty} A_1 y_n, z, t)$. Using Lemma 1.11, we get $\lim_{n\to\infty} A_1 y_n = z$.

Also, suppose that SX is closed subspace of X. Then, $z = Su$, for some u in X. Therefore, setting x by u and y by x_{2n+1} in condition (ii) with $\alpha = 1$, we get

$$M(A_1 u, A_k x_{2n+1}, rt) \geq \phi(M(A_1 u, Su, t), M(A_k x_{2n+1}, T x_{2n+1}, t), M(Su, T x_{2n+1}, t),$$
$$M(A_1 u, T x_{2n+1}, t), M(Su, A_k x_{2n+1}, t)).$$

Taking limit as $n \to \infty$, we get

$$M(A_1 u, z, rt) \geq \phi(M(A_1 u, z, t), M(z,z,t), M(z,z,t), M(A_1 u, z, t), M(z,z,t)).$$

This implies that $M(A_1 u, z, rt) > M(A_1 u, z, t))$, and hence, we get $z = A_1 u$. Therefore, we have $z = A_1 u = Su$.

Again, since $A_1 X \subseteq TX$, so there exists v in X such that $z = Tv$. So, setting $x = u$ and $y = v$ in condition (ii) with $\alpha = 1$, we get

$$M(A_1 u, A_k v, rt) \geq \phi(M(A_1 u, Su, t), M(A_k v, Tv, t), M(Su, Tv, t), M(A_1 u, Tv, t),$$
$$M(Su, A_k v, t)).$$

This implies that $M(z, A_k v, rt) > M(z, A_k v, t)$, and hence, we get $z = A_k v$. Therefore, we have $z = A_k v = Tv$. Thus, we have $z = A_1 u = Su = A_k v = Tv$. Now, since $z = A_1 u = Su$, so by the weak compatibility of (A_1, S), it follows that $SA_1 u = A_1 Su$ and so, we get $A_1 z = A_1 Su = SA_1 u = Sz$.

Now, we claim that $z = A_k z$. For this, setting $x = u$ and $y = z$ in condition (ii) with $\alpha = 1$, we get

$$M(A_1 u, A_k z, rt) \geq \phi(M(A_1 u, Su, t), M(A_k z, Tz, t), M(Su, Tz, t), M(A_1 u, Tz, t),$$
$$M(Su, A_k z, t)).$$

This implies that $M(z, A_k z, rt) > M(z, A_k z, t)$, and hence, we get $z = A_k z$.

Similarly, using condition (ii) with $\alpha = 1$, one can show that $z = A_1 z$. Therefore, we have $z = A_1 z = Sz = A_k z = Tz$, for $k > 1$. Hence, the point z is a common fixed point of all mappings A_i, S and T.

Uniqueness: The uniqueness of a common fixed point of the mappings A_i, S and T be easily verified by using (ii). In fact, if u' be another fixed point for mappings A_1, A_k, S and T, for some $k > 1$. Then, for $\alpha = 1$, we have

$$\begin{aligned} M(u,u',rt) = M(A_1u, A_ku', rt) &\geq \phi(M(A_1u, Su, t), M(A_ku', Tu', t), M(Su, Tu', t), \\ &\quad M(A_1u, Tu', t), M(Su, A_ku', t)) \\ &> M(u, u', t), \end{aligned}$$

and hence, we get $u = u'$. This completely establishes the theorem. □

Now, we have the following theorem in fuzzy metric space.

Theorem 2.2. *Let $(X, M, *)$ be a fuzzy metric space. Let $\{A_i\}, i = 1, 2, 3, \ldots, S$ and T be mappings of a fuzzy metric space from X into itself such that*

(i) $A_1X \subseteq TX$, $A_iX \subseteq SX$, for some $i > 1$, and

(ii) there exists a constant $r \in (0, 1/2)$ such that

$$M(A_1x, A_iy, rt) \geq \phi(M_{1i}(x, y, t)),$$

for all $x, y \in X$, $\alpha \in (0, 2)$, $t > 0$ and $\phi \in \Phi$. If TX and SX are closed subset of X; for $k > 1$ if the pair (A_1, S) and (A_k, T) are weakly compatible, and the pair $\{A_1, S\}$ and $\{A_k, T\}$ satisfy common property (E.A.), then all the mappings A_i, S and T have a unique common fixed point in X.

Proof. Suppose that a pair $\{A_1, S\}$ and $\{A_k, T\}$, for some $k > 1$, satisfy a common property (E.A.), then by definition, there exists two sequences $\{x_n\}$ and $\{y_n\}$ in X such that $\lim_{n\to\infty} A_1x_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} A_kx_n = \lim_{n\to\infty} Tx_n = z$, for some z in X. Also, since TX and SX are closed subspace of X, therefore, we have $z = Su = Tv$ for some $u, v \in X$. Now, we claim that $z = A_1u$. For this, setting $x = u$ and $y = y_n$ in condition (ii) with $\alpha = 1$, we get

$$\begin{aligned} M(A_1u, A_ky_n, rt) \geq \phi(&M(A_1u, Su, t), M(A_ky_n, Ty_n, t), M(Su, Ty_n, t), \\ &M(A_1u, Ty_n, t), M(Su, A_ky_n, t)). \end{aligned}$$

Taking limit as $n \to \infty$, we get$M(A_1u, z, rt) > M(A_1u, , z, t)$. This implies that $z = A_1u = Su$, and hence, we have $z = A_1u = Su = Tv$.

Again, setting $x = u$ and $y = v$ in condition (ii) with $\alpha = 1$, we get

$$\begin{aligned} M(Tv, A_kv, rt) = M(A_1u, A_kv, rt) &\geq \phi(M(A_1u, Su, t), M(A_kv, Tv, t), M(Su, Tv, t), \\ &\quad M(A_1u, Tv, t), M(Su, A_kv, t)). \end{aligned}$$

From this, we get $M(z, A_k v, rt) > M(z, A_k v, t)$. This implies that $z = A_k v$, and hence, we have $z = A_1 u = Su = A_k v = Tv$. Finally, using the similar proof as that in Theorem 2.1, we can show that all the mappings A_i, S and T have a unique common fixed point as z in X. □

We now give an example to illustrate the above theorems.

Example 2.3. Let $X = [2, 20]$ and M be the usual fuzzy metric space on $(X, M, *)$ with minimum t-norm. Define A_i, S and $T : X \to X$ as follows:

$$A_1 x = 2 \ \text{ for each } x.$$

$$Sx = \begin{cases} x, & x \leq 8, \\ 8, & 8 < x < 14, \\ (x+10)/3, & 14 \leq x \leq 17, \\ (x+7)/3, & x > 17. \end{cases} \qquad Tx = \begin{cases} 2, & x = 2 \text{ or } x > 6, \\ x + 12, & 2 < x < 4, \\ (x+9)/3, & 4 \leq x < 5, \\ 8, & 5 \leq x \leq 6. \end{cases}$$

$$A_2 x = \begin{cases} 2, & x < 4 \text{ or } x > 6, \\ x + 3, & 4 \leq x < 5, \\ x + 2, & 5 \leq x \leq 6, \end{cases} \qquad A_i x = \begin{cases} 2, & x = 2 \text{ or } x \geq 4, \\ (x+30)/4, & 2 < x < 4. \end{cases}$$

for each $i > 2$. Also, we define $M(A_1 x, A_k y, t) = \frac{t}{t + d(x,y)}$, for some $k > 1$, for all x, y in X and for all $t > 0$. Then, for $\alpha = 1$, the pairs (A_1, S) and (A_k, T), for $k > 1$, are weakly compatible mappings. Also, we define self maps f and g on X as $fx = A_i x = 2$ for $x = 10$; $fx = (x+3)/5$, otherwise, and $gx = Sx = Tx = 20$ for $0 \leq x \leq 10$, $gx = x/2$ for $10 \leq x \leq 20$, then there exists a sequence x_n in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = z$, then by definition, we have $z \in \{10\}$ and so (f, g) satisfy E.A. property. Therefore, these mappings satisfy all the conditions of the above theorem and have a unique common fixed point $x = 2$. However, for $\alpha = 1$, the mappings A_1, A_2, S and T do not satisfy the contractive condition $M(A_1 x, A_2 y, rt) \geq \phi(M_{12}(x, y, t))$, where $r \in (0, 1)$ and $\phi : [0, 1]^5 \to [0, 1]$ is such that $\phi(t, t, t, t, t) > t$ for all $t > 0$.

Remark 2.4. *As the earlier fixed point theorems have been established using stronger contractive conditions, so our results generalize the results of Abbas et al [20], Cho et al. [25], Jha [26], Kutukcu et al. [12], Mihet [27], Pant and Pant [18], Pant [28, 29], Sedghi et al. [30], Singh and Chauhan [31] and that of Sharma [32], Sharma et al. [33]. Consequently, our theorems improve and unify the results of Altun and Torkoglu [19], Balasubramaniam et al. [10], Chugh and Kumar [34], Jha et al. [35], Kumar and Fisher [21], Pant and Jha [11], Singh and Jain [8] and other similar results for fixed points.*

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