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New Closed Sets and Maps via Ideals

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Abstract : The purpose of this paper is to study a new class of closed sets, called generalized semi-closed sets with respect to an ideal, which is an extension of generalized semi closed sets. Then, by using these sets, we introduce the concept of Igs-compact spaces along with some new classes of maps via ideals and obtain analogues of some known results for compact spaces, continuous maps and closed maps in general topology.

Keywords : ideal; generalized semi-closed set; Igs-closed set; Igs-continuous; Igs-closed map; Igs-compact spaces.
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1 Introduction

The closed sets are very important in topology. The study of generalized closed sets has found considerable interest during the last few years among general topologists. The reason is, these sets are generalizations of closed sets. In 1963, Levine [1] introduced the concept of semi-open sets and semi-closed sets. T.M. Nour [2] further studied the topic. The concept of generalized semi-closed sets was introduced by S.P. Arya and T. Nour [3]. This concept is further studied by H. Maki et al. [4]. In terms of these sets a weaker form of continuous maps called generalized semi-continuous maps was introduced and studied by J.H. Park [5].

M. Navaneethakrishnan and J.P. Joseph [6] introduced the concept of g-closed sets in ideal topological spaces. The concept of ideals was introduced by K. Ku-

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ratowski [7]. Newcomb [8], Hamlet and Jankovic [9, 10] and Vaidyanathaswamy [11] further studied the properties of general topology with respect to ideals. On the other hand, Acikgoz et al. [12] and Ahmad et al. [13] have studied some new classes of maps in ideal topological spaces.

In this paper, we introduce and investigate a new class of closed sets, generalized semi-closed sets with respect to an ideal, called Igs-closed sets. Then, by using these sets, we introduce the concept of Igs-compact spaces along with some new classes of maps via ideals and obtain analogues of results for compact spaces, continuous maps and closed maps in general topology.

2 Preliminaries

Definition 2.1. [9] A non empty collection I of subsets on a topological space (X, τ) is called a *topological ideal* if it satisfies following two conditions:
(i) $A \in I$ and $B \subseteq A$ implies $B \in I$(*heredity*).
(ii) $A \in I$ and $B \in I$ implies $A \cup B \in I$(*finite additivity*).

We denote a topological space (X, τ) with an ideal I defined on X by (X, τ, I). If (X, τ, I) is an ideal space, (Y, σ) is a topological space and $f : (X, \tau, I) \longrightarrow (Y, \sigma)$ is a map, then $f(I) = \{f(I_1) : I_1 \in I\}$ is an ideal of Y [8]. If I is ideal of subsets of X and Y is subset of X, then $I_Y = \{Y \cap I_1 : I_1 \in I\}$ is an ideal of subsets of Y [8]. If $f : (X, \tau) \longrightarrow (Y, \sigma, I)$ is an injection then $f^{-1}(I) = \{f^{-1}(B) : B \in I\}$ is an ideal on X [8].

Definition 2.2. [1] Let (X, τ) be a topological space. A subset A of X is said to be *semi-open set* if $A \subseteq cl(intA)$ and a semi-closed set if $int(clA) \subseteq A$.

Remark 2.3. *Union of semi-open sets is semi-open.*

Remark 2.4. [1] *Intersection of semi-open set and open set is semi-open.*

Definition 2.5. [14] The intersection of all semi-closed sets containing a subset A of a space X is called *semi-closure* of A and is denoted by $scl(A)$. Also $scl(A) = A \cup int(cl(A))$.

Definition 2.6. [14] The union of all semi-open sets which are contained in A is called *semi-interior* of A and is denoted by $sint(A)$. Also $sint(A) = A \cap cl(int(A))$.

Let $A \subset B \subset X$, Then $scl_B(A)$ (resp. $sint_B(A)$) denotes semi-closure of A (resp. semi-interior of A) with respect to B.

Definition 2.7. [15] Let A be a subset of X. A point x in X is a *semi-limit point* of A if every semi-open set containing x intersects A in a point different from x. The set of all semi-limit points of A is called the *semi-derived set* of A and is denoted by $D_S[A]$.

Remark 2.8. [16] *In a topological space* (X, τ), *if* A *is subset of* X *then* $D_S[A] \subseteq D[A]$ *and* $scl(A) = A \cup D_S[A]$.

The following lemma is useful in this sequel.

Lemma 2.9. [3] *For subsets A and B of X, the following assertions are valid*
1. $sint(X - A) = X - scl(A)$
2. $scl(X - A) = X - sint(A)$
3. $scl(A) \subset cl(A)$
4. $sint(A) \cup sint(B) \subset sint(A \cup B)$
5. $scl(scl(A)) = scl(A)$
6. $scl(A \cap B) \subset scl(A) \cap scl(B)$
7. $scl(A) \cup scl(B) \subset scl(A \cup B)$

Definition 2.10. [3] A subset A of a space X is called *generalized semi-closed* (briefly, gs-closed) if $scl(A) \subset U$ whenever $A \subset U$ and U is open in X. A is generalized semi-open (briefly, gs-open) if its complement $X - A$ is generalized semi-closed.

Definition 2.11. [5] A map $f : (X, \tau) \to (Y, \sigma)$ is called *generalized semi-continuous* (briefly gs-continuous) if $f^{-1}(G)$ is gs-closed in X for every closed set G of Y.

Definition 2.12. [5] A map $f : (X, \tau) \to (Y, \sigma)$ is called *gs-irresolute* if $f^{-1}(G)$ is gs-closed in X for every gs-closed set G of Y.

Definition 2.13. [8] An ideal topological space X is said to be I*- compact* or *compact modulo ideal* if for every cover $\{U_\lambda : \lambda \in \Lambda\}$ of X by open sets of X, there exist a finite subset $\Lambda_\circ$ of Λ such that $X - \cup\{U_\lambda : \lambda \in \Lambda_\circ\} \in I$.

By a space, we always mean a topological space (X, τ) with no separation axioms assumed.

3 Igs-Closed Sets and Igs-Compact Spaces

In this section, we introduce and investigate the concept of generalized semi-closed sets with respect to an ideal (briefly Igs-closed sets) which is an extension of generalized semi-closed sets defined by Arya and Nour [3]. Further, we introduce the concept of Igs-compact spaces in ideal topological spaces and obtain analogues of results for compact spaces in general topological spaces.

Definition 3.1. Let (X, τ) be a topological space and I be an ideal on X. A subset A of X is said to be *generalized semi-closed with respect to an ideal* (briefly, Igs-closed) if $scl(A) - B \in I$ whenever $A \subset B$ and B is open in X. A subset $A \subset X$ is said to be *generalized semi-open with respect to an ideal* (briefly Igs-open) if $X - A$ is Igs-closed.

We have the following implication.

Remark 3.2. *Closed $\Rightarrow$ semi $-$ closed $\Rightarrow$ gs $-$ closed $\Rightarrow$Igs$-$closed.*

The following example 3.3 shows that the converse of above implication is not true.

Example 3.3. Let $X = \{a, b, c, d\}$ with topology $\tau = \{\phi, \{a\}, \{b, c\}, \{a, b, c\}, X\}$ and ideal $I = \{\phi, \{d\}\}$. Here for set $A = \{a, b\}$, we have $cl\{a, b\} = X$ and $scl\{a, b\} = \{a, b\} \cup int(cl\{a, b\}) = X$. It can be seen that $scl(A) - B \in I$ for $A \subset B$ where B is open set in X, but $scl(A) \nsubseteq C$ for open set $C = \{a, b, c\}$ containing A. Hence A is Igs-closed but neither gs-closed nor semi-closed nor closed.

Theorem 3.4. *If A and B are Igs-closed subsets of (X, τ, I) such that $D[A] \subset D_S[A]$ and $D[B] \subset D_S[B]$, then $A \cup B$ is also Igs-closed.*

Proof. Let A and B be Igs-closed subsets of (X, τ, I) such that $D[A] \subset D_S[A]$ and $D[B] \subset D_S[B]$. As for any subset A, $D_S[A] \subset D[A]$. Therefore $D[A] = D_S[A]$ and $D[B] = D_S[B]$. That is $cl(A) = scl(A)$ and $cl(B) = scl(B)$. Let $A \cup B \subset U$ and U open, then $A \subset U$ and $B \subset U$. Since A and B are Igs-closed, $scl(A) - U \in I$ and $scl(B) - U \in I$. Now $scl(A \cup B) - U = cl(A \cup B) - U = (cl(A) \cup cl(B)) - U = (scl(A) \cup scl(B)) - U = (scl(A) - U) \cup (scl(B) - U) \in I$. So $scl(A \cup B) - U \in I$, thereby implying that $A \cup B$ is Igs-closed. □

Corollary 3.5. *If A and B are Igs-open subsets of (X, τ, I) such that $D[X - A] \subset D_S[X - A]$ and $D[X - B] \subset D_S[X - B]$, then $A \cap B$ is also Igs-open.*

Arbitrary union of Igs-closed sets may not be Igs-closed, as the following example 3.6 shows.

Example 3.6. Let $X = \{a, b, c, d\}$ with topology $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}, X\}$ and ideal $I = \{\phi, \{a\}\}$. Here for set $A = \{a\}$, we have $cl\{a\} = \{a, d\}$ and $scl\{a\} = \{a\} \cup int(cl\{a\}) = \{a\}$ and for set $B = \{b, c\}$, we have $cl\{b, c\} = \{b, c, d\}$ and $scl\{b, c\} = \{b, c\} \cup int(cl\{b, c\}) = \{b, c\}$. It can be seen that $scl(A) - C \in I$ for $A \subset C$ where C is open set in X and $scl(B) - D \in I$ for $B \subset D$ where D is open set in X, so A and B are Igs-closed, but union $A \cup B = \{a, b, c\}$ is not Igs-closed, since $scl\{a, b, c\} = X$ and $scl\{a, b, c\} - U \notin I$ for open set $U = \{a, b, c\}$ containing $A \cup B$.

Remark 3.7. *Every subset of Igs-closed set is not Igs-closed as the following example 3.8 shows, but this is possible under the condition given in theorem 3.9.*

Example 3.8. Let $X = \{a, b, c, d\}$ with topology $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}, X\}$ and ideal $I = \{\phi, \{a\}\}$. Here for set $A = \{b, c\}$, we have $cl\{b, c\} = \{b, c, d\}$ and $scl\{b, c\} = \{b, c\} \cup int(cl\{b, c\}) = \{b, c\}$. It can be seen that A is Igs-closed, but subset $\{b\}$ of $\{b, c\}$ is not Igs-closed, as $cl\{b\} = \{b, c, d\}$, $scl\{b\} = \{b, c\}$ and $scl\{b\} - U \notin I$ for open set $U = \{a, b\}$.

Theorem 3.9. *Let A be an Igs-closed subset of an ideal topological space (X, τ, I) such that $A \subset B \subset scl(A)$ in X, then B is Igs-closed in (X, τ, I).*

Proof. Let U be an open subset of X containing B. Then $A \subset U$ and A is Igs-closed implies $scl(A) - U \in I$. Since $B \subset scl(A)$, $scl(B) \subset scl(scl(A)) = scl(A)$. Therefore $scl(B) - U \subset scl(A) - U \in I$. Hence $scl(B) - U \in I$, for $B \subset U$ and U open in X, implying thereby that B is Igs-closed. □

Corollary 3.10. *If $sint(A) \subset B \subset A$ and A is Igs-open in X, then B is Igs-open in X.*

Remark 3.11. *Intersection of two Igs-closed sets is not Igs-closed, this may be seen from the following example.*

Example 3.12. Let $X = \{a, b, c, d\}$ with topology $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}, X\}$ and ideal $I = \{\phi, \{a\}\}$. Here for set $A = \{b, c\}$, we have $cl\{b, c\} = \{b, c, d\}$ and $scl\{b, c\} = \{b, c\} \cup int(cl\{b, c\}) = \{b, c\}$ and for set $B = \{a, b, d\}$, we have $cl\{a, b, d\} = X$ and $scl\{a, b, d\} = \{a, b, d\} \cup int(cl\{a, b, d\}) = X$. It can be seen that $scl(A) - C \in I$ for $A \subset C$ where C is open set in X and $scl(B) - D \in I$ for $B \subset D$ where D is open set in X, so A and B are Igs-closed, but intersection $A \cap B = \{b\}$ is not Igs-closed, since $cl\{b\} = \{b, c, d\}$, $scl\{b\} = \{b, c\}$ and $scl\{b\} - U \notin I$ for open set $U = \{a, b\}$ containing $\{b\}$.

Theorem 3.13. *If a subset A is Igs-closed in (X, τ, I), then $F \subset sclA - A$ for some closed set F implies $F \in I$.*

Proof. Let A be an Igs-closed set in (X, τ, I) and let F be any closed set contained in $sclA - A$. Then we have $F \subset X - A$ or $A \subset X - F$ where $X - F$ is an open set. Since A is Igs-closed, $F = F \cap scl(A) = scl(A) - (X - F) \in I$. □

Theorem 3.14. *A subset A of an ideal topological space (X, τ, I) is Igs-closed if $F \subset sclA - A$ for some semi-closed set F implies $F \in I$.*

Proof. Suppose the given condition holds. Let A be a subset of an ideal topological space (X, τ, I) and U be any open set containing A. Then $sclA - U \subset sclA - A$. Take $F = scl(A) - U$. Then F is semi-closed and $F \subset sclA - A$ therefore by hypothesis $F \in I$ that is $sclA - U \in I$. Hence A is Igs-closed. □

Theorem 3.15. *Let A be an Igs-closed subset of an ideal topological space (X, τ, I) and F be any closed subset of X, then $A \cap F$ is an Igs-closed subset of (X, τ, I).*

Proof. Let U be any open subset of X containing $A \cap F$. Then $A \subset U \cup (X - F)$. Since A is Igs-closed, $scl(A) - (U \cup (X - F)) \in I$. Now $scl(A \cap F) \subset scl(A) \cap F$ as $scl(F) \subset cl(F) = F$. Therefore $scl(A \cap F) - U \subset (scl(A) \cap F) - U = scl(A) - (U \cup (X - F)) \in I$. Hence $A \cap F$ is Igs-closed. □

Definition 3.16. An ideal topological space (X, τ, I) is said to be *Igs-compact* if for every Igs-open cover $\{U_\lambda : \lambda \in \Lambda\}$ of X, there exists a finite subset $\Lambda_\circ$ of Λ such that $X - \cup\{U_\lambda : \lambda \in \Lambda_\circ\} \in I$.

The following results from 3.17-3.21 for Igs-compact spaces in ideal topological spaces are analogues of results for compact spaces in general topology.

Theorem 3.17. *Every Igs-closed subset of Igs-compact space is Igs-compact.*

Proof. Let A be Igs-closed subset of (X,τ,I). Let $\{U_\lambda : \lambda \in \Lambda\}$ be an Igs-open cover of A. Since A is Igs-closed, so $X-A$ is Igs-open. Now $\{U_\lambda : \lambda \in \Lambda\} \cup \{X-A\}$ is an Igs-open cover of X, which is Igs-compact, therefore there exists a finite subset $\Lambda_\circ$ of Λ such that either $X - (\cup[U_\lambda : \lambda \in \Lambda_\circ] \cup \{X-A\}) \in I$ or $X - \cup[U_\lambda : \lambda \in \Lambda_\circ] \in I$. In each case $A - \cup[U_\lambda : \lambda \in \Lambda_\circ] \subset \{X - \cup[U_\lambda : \lambda \in \Lambda_\circ]\} \in I$. Hence A is Igs-compact. □

Corollary 3.18. *Every gs-closed subset of Igs-compact space is Igs-compact.*

Theorem 3.19. *If A and B are Igs-compact subsets of ideal topological space (X,τ,I), then $A\cup B$ is Igs-compact subset of X.*

Proof. Let $\{U_\lambda : \lambda \in \Lambda\}$ be an Igs-open cover of $A\cup B$ in X. Then $\{U_\lambda : \lambda \in \Lambda\}$ is Igs-open cover of A and B. Since A and B are Igs-compact, there exists $I_1, I_2 \in I$ and finite subset $\Lambda_\circ$ and Λ_1 of Λ such that
$A - \cup[U_{\lambda_i} : \lambda_i \in \Lambda_\circ] = I_1$ and $B - \cup[U_{\lambda_K} : \lambda_K \in \Lambda_1] = I_2$.
$A = \cup[U_{\lambda_i} : \lambda_i \in \Lambda_\circ] \cup I_1$ and $B = \cup[U_{\lambda_K} : \lambda_K \in \Lambda_1] \cup I_2$.
Now,$A \cup B = (\cup[U_{\lambda_i} : \lambda_i \in \Lambda_\circ]) \cup (\cup[U_{\lambda_K} : \lambda_K \in \Lambda_1]) \cup (I_1 \cup I_2)$.
$A \cup B = \cup[U_{\lambda_i} \cup U_{\lambda_K} : \lambda_i \in \Lambda_\circ, \lambda_K \in \Lambda_1] \cup (I_1 \cup I_2)$
This implies $A \cup B = \cup[U_{\lambda_i} \cup U_{\lambda_K} : \lambda_i \in \Lambda_\circ, \lambda_K \in \Lambda_1] \cup I$ where $I_1 \cup I_2 = I$
$(A \cup B) - \cup[U_{\lambda_i} \cup U_{\lambda_K} : \lambda_i \in \Lambda_\circ, \lambda_K \in \Lambda_1] \in I$ implying thereby that $A \cup B$ is Igs-compact in X. □

Corollary 3.20. *Finite union of Igs-compact subsets of an ideal topological space is Igs-compact.*

Theorem 3.21. *The following are equivalent for an ideal topological space* (X,τ,I)
(a) (X,τ,I) is Igs- compact.
(b) For any family $\{F_\lambda : \lambda \in \Lambda\}$ of Igs-closed subsets of X such that $\cap\{F_\lambda : \lambda \in \Lambda\} = \phi$, there exists a finite subset $\Lambda_\circ$ of Λ such that $\cap\{F_\lambda : \lambda \in \Lambda_\circ\} \in I$.

Proof. $(a) \Rightarrow (b)$ Let $\{F_\lambda : \lambda \in \Lambda\}$ be a family of Igs-closed subsets of X such that $\cap\{F_\lambda : \lambda \in \Lambda\} = \phi$. Then $\{X - F_\lambda : \lambda \in \Lambda\}$ is an Igs-open cover of X. Since (X,τ,I) is Igs-compact, there exists a finite subset $\Lambda_\circ$ of Λ such that $X - \cup\{X - F_\lambda : \lambda \in \Lambda_\circ\} \in I$. This implies that $\cap\{F_\lambda : \lambda \in \Lambda_\circ\} \in I$.
$(b) \Rightarrow (a)$ Let $\{U_\lambda : \lambda \in \Lambda\}$ be an Igs-open cover of X, then $\{X - U_\lambda : \lambda \in \Lambda\}$ is a collection of Igs-closed sets and $\cap\{X - U_\lambda : \lambda \in \Lambda\} = \phi$. Hence there exists a finite subset $\Lambda_\circ$ of Λ such that $\cap\{X - U_\lambda : \lambda \in \Lambda_\circ\} \in I$. This implies that $X - \cup\{U_\lambda : \lambda \in \Lambda_\circ\} \in I$. This shows (X,τ,I) is Igs- compact. □

4 Igs-Continuous Maps and Igs-Closed Maps

Having discussed Igs-closed sets, we now turn to introduce the concepts of Igs-continuous maps, Igs-irresolute maps, Igs-closed maps, Igs-resolute maps and study their properties.

Definition 4.1. A map $f : (X, \tau, I) \to (Y, \sigma)$ is called *generalized semi-continuous with respect to an Ideal* (briefly Igs-continuous) if inverse image of every closed subset of Y is Igs-closed in X.

Definition 4.2. A map $f : (X, \tau, I) \to (Y, \sigma, J)$ is called *generalized semi-closed with respect to an ideal* briefly Igs-closed (*generalized semi-open with respect to an ideal* briefly Igs-open) if the image of every closed set (open set) in X is Jgs-closed (Jgs-open) in Y.

Definition 4.3. A map $f : (X, \tau, I) \to (Y, \sigma, J)$ is called *Igs-irresolute* if inverse image of every Jgs-closed subset of Y is Igs-closed in X.

Definition 4.4. A map $f : (X, \tau, I) \to (Y, \sigma, J)$ is called *Igs-resolute* if the image of every Igs-closed set in X is Jgs-closed in Y.

Definition 4.5. Let $x \in X$. A subset $U \subset X$ is called *Igs-neighborhood* of x in X if there exists Igs-open set A such that $x \in A \subset U$.

Theorem 4.6. *Let $f : (X, \tau, I) \to (Y, \sigma)$ be any map. Then $(i) \Rightarrow (ii) \Rightarrow (iii) \Rightarrow (iv)$.*

- (i) *f is Igs-continuous.*
- (ii) *The inverse image of each open set in (Y, σ) is Igs-open.*
- (iii) *For each $x \in X$ and each $G \in \sigma$ containing $f(x)$, there exists an Igs-open set W containing x such that $f(W) \subseteq G$.*
- (iv) *For each $x \in X$ and open set G in Y with $f(x) \in G$, $f^{-1}(G)$ is an Igs-neighborhood of x.*

Proof. $(i) \Rightarrow (ii)$ Let G be open in Y. Then $B = Y - G$ is closed in Y. Here $f^{-1}(B) = f^{-1}(Y) - f^{-1}(G) = X - f^{-1}(G)$. By (i) $f^{-1}(B)$ is Igs-closed in X. Hence $f^{-1}(G)$ is Igs-open in X.
$(ii) \Rightarrow (iii)$ Obvious.
$(iii) \Rightarrow (iv)$ Let G be open set in Y and let $f(x) \in G$. Then by (iii), there exists an Igs-open set W containing x such that $f(W) \subseteq G$. So $x \in W \subset f^{-1}(G)$. Hence $f^{-1}(G)$ is an Igs-neighborhood of x. □

The following theorem for Igs-irresolute is an analogue of the theorem 4.6.

Theorem 4.7. *Let $f : (X, \tau, I) \to (Y, \sigma, J)$ be any map, then $(i) \Rightarrow (ii) \Rightarrow (iii) \Rightarrow (iv)$.*

- (i) *f is Igs-irresolute.*
- (ii) *The inverse image of each Igs-open set in (Y, σ, J) is Igs-open in (X, τ, I).*
- (iii) *For each $x \in X$ and each Igs-open set G containing $f(x)$, there exists an Igs-open set W containing x such that $f(W) \subseteq G$.*

(*iv*) *For each* $x \in X$ *and* Igs*-open set* G *in* Y *with* $f(x) \in G$, $f^{-1}(G)$ *is an* Igs*-neighborhood of* x.

The following theorem 4.8 for Igs-closed maps and Igs-resolute maps is an analogue of Theorem 11.2 of Dugundji [17] for closed maps.

Theorem 4.8. *If* $f : (X, \tau, I) \to (Y, \sigma, J)$ *is* Igs*-closed (*Igs*-resolute) map then for each* $y \in Y$ *and each open set (*Igs*-open set)* U *containing* $f^{-1}(y)$*, there is a* Jgs*-open set* V *of* Y *such that* $y \in V$ *and* $f^{-1}(V) \subset U$.

Proof. We give the proof of non-parenthesis part. The proof of parenthesis part is similar. Let f be Igs-closed map. If $y \in Y$ and U is any open set in X containing $f^{-1}(y)$, then $X - U$ is closed set in X and $(X - U) \cap f^{-1}(y) = \phi$. Since f is Igs-closed, $f(X - U)$ is Jgs-closed in Y and $y \notin f(X - U)$. Let $V = Y - f(X - U)$, then V is Jgs-open set in Y containing y and $f^{-1}(V) \subset U$. □

Theorem 4.9. *If* $f : (X, \tau, I) \to (Y, \sigma, J)$ *is a bijection, then following are equivalent:*

(*i*) $f^{-1} : (Y, \sigma, J) \to (X, \tau, I)$ *is* Jgs*-continuous.*

(*ii*) f *is* Igs*-open map.*

(*iii*) f *is* Igs*-closed map.*

Proof. (*i*) ⇒ (*ii*) Let U be open set of X. Since f^{-1} is Jgs-continuous, $(f^{-1})^{-1}(U)$ $= f(U)$ is Jgs-open in Y. So f is Igs-open.
(*ii*) ⇒ (*iii*) Let U be closed set of X. Then $(X - U)$ is open set in X. By (ii) $f(X - U)$ is Jgs-open in Y. Therefore $f(X - U) = Y - f(U)$ is Jgs-open in Y. Here $f(U)$ is Jgs-closed in Y. Hence f is Igs-closed.
(*iii*) ⇒ (*i*) Let U be closed set of X. By (iii) $f(U)$ is Jgs-closed in Y. Since $f(U) = (f^{-1})^{-1}(U)$ that implies (f^{-1}) is Jgs-continuous. □

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