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Totally Somewhat Fuzzy Continuous and Totally Somewhat Fuzzy Semicontinuous Mappings

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Abstract : The aim of this paper is to introduce some new class of mappings called totally somewhat fuzzy continuous, totally somewhat fuzzy semicontinuous, slightly somewhat fuzzy semicontinuous, totally somewhat generalized fuzzy semicontinuous, almost somewhat fuzzy continuous and almost somewhat fuzzy semicontinuous. Their compositions, their relationships with other mappings, preservation of some fuzzy spaces under these mappings and some examples are studied and investigated.

Keywords : totally somewhat fuzzy continuous functions; totally somewhat fuzzy semicontinuous functions; slightly somewhat fuzzy semicontinuous functions; totally somewhat generalized fuzzy semicontinuous functions.
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1 Introduction

The fundamental concept of fuzzy sets introduced by Zadeh [1] provided a natural foundation for building new branches. In 1968 Chang [2] introduced the concept of fuzzy topological spaces as a generalization of topological spaces. The concept of totally continuous functions was introduced in [3] and as a consequence of this totally semicontinuous functions was defined and studied in [4]. The class

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of somewhat continuous mappings was first introduced by Gentry and others [5]. Later, the concept of "somewhat" in classical topology has been extented to fuzzy topological spaces. In fact, somewhat fuzzy continuous mappings and somewhat fuzzy semicontinuous mappings on fuzzy topological spaces were introduced and studied by G. Thangaraj and G. Balasubramanian in [6] and [7] respectively. In this paper we have introduced and studied a concept of totally somewhat fuzzy continuous and fuzzy semicontinuous function as a generalizations of the concept of totally fuzzy semicontinuous function defined and studied in [8]. Also a new class of mapping called almost somewhat fuzzy semicontinuous function is defined as a generalization of the concept introduced by B.M. Munshi and D.S. Bassan [9] in fuzzy setting and some of its properties are also studied. In this paper I will denote the unit interval $[0,1]$ of the real line R. X, Y, Z will be nonempty sets. The symbols λ, μ, γ, η,... are used to denote fuzzy sets and all other symbols have their usual meaning unless explicitely stated.

2 Preliminaries

Throughout this paper, we denote μ^c with the complement of fuzzy set μ on a nonempty set X, which is defined by $\mu^c(x) = (1-\mu)(x) = 1-\mu(x)$ for all $x \in X$. If μ is a fuzzy set on a nonempty set X and if ν is a fuzzy set on a nonempty set Y, then $\mu \times \nu$ is a fuzzy set on $X \times Y$, defined by $(\mu \times \nu)(x,y) = \min(\mu(x), \nu(y))$ for every $(x,y) \in X \times Y$. Let $f : X \to Y$ be a fuzzy mapping and let μ be a fuzzy set on X. Then $f(\mu)$ is a fuzzy set on Y defined by

$$f(\mu)(y) = \begin{cases} \sup\limits_{x \in f^{-1}(y)} \mu(x), & \text{if } f^{-1}(y) \neq \phi, y \in Y, \\ 0, & \text{otherwise.} \end{cases}$$

Let ν be a fuzzy set on Y. Then $f^{-1}(\nu)$ is a fuzzy set on X, defined by $f^{-1}(\nu)(x) = \nu(f(x))$ for each $x \in X$. The graph $g : X \to X \times Y$ of f is defined by $g(x) = (x, f(x))$ for each $x \in X$. Then $g^{-1}(\mu \times \nu) = \mu \wedge f^{-1}(\nu)$. The product $f_1 \times f_2 : X_1 \times X_2 \to Y_1 \times Y_2$ of mappings $f_1 : X_1 \to Y_1$ and $f_2 : X_2 \to Y_2$ is defined by $(f_1 \times f_2)(x_1, x_2) = (f_1(x_1), f_2(x_2))$ for each $(x_1, x_2) \in X_1 \times X_2$ [10].

Now let X and Y be fuzzy topological spaces. We denote $Int(\mu)$ and $Cl(\mu)$ with the interior and with the closure of the fuzzy set μ on a fuzzy topological space X respectively. Then (i) $1 - Cl(\mu) = Int(1-\mu)$ and (ii) $Cl(1-\mu) = 1 - Int(\mu)$.

Definition 2.1. Let λ be a fuzzy subset of a fts (X, T), then

(i) λ is called *fuzzy regular closed* [10] if $\lambda = ClInt(\lambda)$,

(ii) λ is called *fuzzy regular open* [10] if $\lambda = IntCl(\lambda)$,

(iii) λ is called *fuzzy semi-clopen* [11] (resp. *fuzzy semi-open* [10]) if and only if there exists a fuzzy regular open (resp. fuzzy open) subset η of X such that $\eta \leq \lambda \leq Cl(\eta)$,

(iv) λ is called *fuzzy semi-closed* [10] if $IntCl(\lambda) \leq \lambda$,

(v) λ is called *fuzzy semi-open* [10] if $\lambda \leq ClInt(\lambda)$.

Definition 2.2. A fts (X, T) is called *fuzzy almost compact* [12] if and only if every fuzzy open cover of X has a finite subcover whose closure covers X.

Definition 2.3. Let $f : (X, T) \to (Y, S)$ be a function from fts (X, T) to another fts (Y, S), then

(i) f is called *fuzzy continuous* [10] if the inverse image of every fuzzy open subset of Y is fuzzy open subset of X,

(ii) f is called *fuzzy semicontinuous* [10] if the inverse image of every fuzzy open subset of Y is fuzzy semi-open subset of X,

(iii) f is called *totally fuzzy semicontinuous* [13] if the inverse image of every fuzzy open subset of Y is fuzzy semi-clopen subset of X,

(iv) f is called *fuzzy completely continuous* [14] if the inverse image of every fuzzy open subset of Y is fuzzy regular open subset of X,

(v) f is called *fuzzy semi-irresolute* [15] if the inverse image of every fuzzy semi-clopen subset of Y is fuzzy semi-clopen subset of X.

Definition 2.4 ([6])**.** Let (X, T) and (Y, S) be any two fuzzy topological spaces. A function $f : (X, T) \to (Y, S)$ is called

(i) *somewhat fuzzy continuous* if $\lambda \in S$ and $f^{-1}(\lambda) \neq 0$ then there exists a fuzzy open set $\mu \neq 0$ in X such that $\mu \leq f^{-1}(\lambda)$,

(ii) *somewhat fuzzy completely continuous* if $\lambda \in S$ and $f^{-1}(\lambda) \neq 0$ then there exists a fuzzy regular open set $\mu \neq 0$ in X such that $\mu \leq f^{-1}(\lambda)$.

Remark 2.5 ([6])**.** *Every fuzzy continuous mapping is a somewhat fuzzy continuous mapping but not conversely.*

3 Totally Somewhat Fuzzy Continuous and Totally Somewhat Fuzzy Semicontinuous Mappings

In this section, the two new classes of functions are introduced. Their characterization examples, compositions and the relationships with other functions are established.

Definition 3.1. Let (X, T) and (Y, S) be any two fuzzy topological spaces. A function $f : (X, T) \to (Y, S)$ is called *totally somewhat fuzzy continuous* if $\lambda \in S$ and $f^{-1}(\lambda) \neq 0$ then there exists a fuzzy clopen set $\mu \neq 0$ in X such that $\mu \leq f^{-1}(\lambda)$.

Example 3.2. Let $X = Y = I = [0, 1]$, λ_1 and λ_2 be fuzzy sets on I as follows. $\lambda_1(x) = x$ and $\lambda_2(x) = 1 - x$. Clearly $T = \{0, 1, \lambda_1, \lambda_2, \lambda_1 \vee \lambda_2, \lambda_1 \wedge \lambda_2\}$ is fuzzy topology on I. Let $f : (I, T) \to (I, T)$ be defined as $f(x) = x$, for each x in I. It is clear that the fuzzy sets $\lambda_1, \lambda_2, \lambda_1 \vee \lambda_2, \lambda_1 \wedge \lambda_2$ are both fuzzy open and fuzzy closed and hence fuzzy clopen in (I, T). Since the inverse image of every fuzzy open subset is fuzzy clopen in (I, T), f is totally somewhat fuzzy continuous.

Definition 3.3. Let (X, T) and (Y, S) be any two fuzzy topological spaces. A function $f : (X, T) \to (Y, S)$ is called *totally somewhat fuzzy semicontinuous* if $\lambda \in S$ and $f^{-1}(\lambda) \neq 0$ then there exists a fuzzy semiclopen set $\mu \neq 0$ in X such that $\mu \leq f^{-1}(\lambda)$.

Remark 3.4. *It is clear that every totally somewhat fuzzy continuous function is totally somewhat fuzzy semicontinuous but the converse is not true as the following example shows:*

Example 3.5. Let $X = \{a, b, c\}$ and $Y = \{p, q\}$. The fuzzy sets $\lambda_i : X \to [0, 1]$, $i = 1, 2, 3$ and $\mu : Y \to [0, 1]$ be defined as follows: $\lambda_1 = \frac{1}{a} + \frac{0}{b} + \frac{0}{c}$, $\lambda_2 = \frac{0}{a} + \frac{1}{b} + \frac{0}{c}$, $\lambda_3 = \frac{1}{a} + \frac{1}{b} + \frac{0}{c}$ and $\mu = \frac{1}{p} + \frac{0}{q}$. Consider $\tau = \{0,\ 1,\ \lambda_1,\ \lambda_2,\ \lambda_3\}$ and $\sigma = \{0,\ 1,\ \mu\}$. Then $(X,\ \tau)$ and (Y, σ) are fts. Define $f : X \to Y$ by $f(a) = p$ and $f(b) = f(c) = q$. Then f is totally somewhat fuzzy semicontinuous but not totally somewhat fuzzy continuous as the fuzzy set μ is fuzzy open in Y and $f^{-1}(\mu) = \lambda_1$ is fuzzy open but not fuzzy closed in X.

Remark 3.6. *It is clear that every totally fuzzy semicontinuous function is totally somewhat fuzzy semicontinuous but the converse is not true as shown in the following example.*

Example 3.7. Let $X = Y = \{a,\ b,\ c,\ d\}$. The fuzzy sets $\lambda_i : X \to [0, 1]$, $i = 1, 2, 3, 4$ be defined as follows: $\lambda_1 = \frac{1}{a} + \frac{0}{b} + \frac{1}{c} + \frac{0}{d}$, $\lambda_2 = \frac{0}{a} + \frac{1}{b} + \frac{0}{c} + \frac{0}{d}$, $\lambda_3 = \frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{0}{d}$ and $\lambda_4 = \frac{0}{a} + \frac{0}{b} + \frac{0}{c} + \frac{1}{d}$. Consider $\tau = \{0,\ 1,\ \lambda_1,\ \lambda_2,\ \lambda_3\}$ and $\sigma = \{0,\ 1,\ \lambda_4\}$. The identity mapping $f : (X, \tau) \to (Y, \sigma)$ is totally somewhat fuzzy semicontinuous but not totally fuzzy semicontinuous as the fuzzy set λ_4 is fuzzy open in Y and $f^{-1}(\lambda_4) = \lambda_4$ is not fuzzy semi-clopen in X.

Remark 3.8. *Every totally somewhat fuzzy semicontinuous function is fuzzy semicontinuous but the converse is not true which can be seen from the following example.*

Example 3.9. Let $X = Y = I = [0, 1]$. Let λ, μ, γ and δ be fuzzy sets on I defined as follows:

$$\lambda(x) = \begin{cases} 0, & \text{if } 0 \leq x \leq \frac{1}{2} \\ 2x - 1, & \text{if } \frac{1}{2} \leq x \leq 1, \end{cases} \quad \mu(x) = \begin{cases} 1, & \text{if } 0 \leq x \leq \frac{1}{2} \\ 2 - 2x, & \text{if } \frac{1}{2} \leq x \leq \frac{3}{4} \\ 2x - 1, & \text{if } \frac{3}{4} \leq x \leq 1, \end{cases}$$

$$\gamma(x)=\begin{cases}0, & \text{if } 0\le x\le \frac{1}{2}\\ 2x-1, & \text{if } \frac{1}{2}\le x\le \frac{3}{4}\\ 2-2x, & \text{if } \frac{3}{4}\le x\le 1,\end{cases} \quad \delta(x)=\begin{cases}1, & \text{if } 0\le x\le \frac{1}{4}\\ -4x+2, & \text{if } \frac{1}{4}\le x\le \frac{1}{2}\\ 0, & \text{if } \frac{1}{2}\le x\le 1.\end{cases}$$

Clearly, $S=\{0,\lambda,\lambda^{'},\mu,\gamma,1\}$ and $T=\{0,\lambda,\delta,\lambda\vee\delta,1\}$ two fuzzy topologies on I. Let $g:(I,T)\to(I,S)$ be a function defined by $g(x)=\min(2x,1)$ is fuzzy semicontinuous. Here $g^{-1}(1)=1$, $g^{-1}(\lambda)=\delta^{'}$, $g^{-1}(\lambda^{'})=\delta$. Here $g^{-1}(\lambda^{'})=\delta\in T$. It is not fuzzy semiclopen in (I,T) shows that g is not totally somewhat fuzzy semicontinuous.

Definition 3.10 ([16]). A function $f:(X,T)\to(Y,S)$ is said to be *strongly fuzzy semicontinuous* $\Leftrightarrow$ $f^{-1}(\lambda)$ is fuzzy semiclopen whenever $\lambda\in I^Y$.

Remark 3.11. *Strongly fuzzy semicontinuity [16] $\Rightarrow$ totally fuzzy semicontinuity $\Rightarrow$ totally somewhat fuzzy semicontinuity $\Rightarrow$ fuzzy semicontinuity [10](Azad 1981) $\Leftarrow$ fuzzy continuity [2](Chang 1968).*

The following is an example of a function which is totally somewhat fuzzy semicontinuous but not strongly fuzzy semicontinuous.

Example 3.12. Let $X=Y=\{a,\ b,\ c\}$. Define $\mu_i: X\to[0,1]$, $i=1,2$ as follows: $\mu_1=\frac{1}{a}+\frac{0}{b}+\frac{0}{c}$ and $\mu_2=\frac{0}{a}+\frac{1}{b}+\frac{1}{c}$. Consider $\tau=\{0,\ 1,\ \mu_1,\ \mu_2\}$ and $\sigma=\{0,\ 1,\ \mu_1\}$. The identity mapping $f:(X,\tau)\to(Y,\sigma)$ is totally somewhat fuzzy semicontinuous but not strongly fuzzy semicontinuous as the fuzzy set μ_1 in Y, $f^{-1}(\mu_1^c)=\mu_2$ is fuzzy semiopen but it is not fuzzy semiclosed in X.

Definition 3.13 ([2]). A fuzzy topological space (X,T) is called a *discrete fuzzy topological space* if $T=I^X$. The following proposition is easy to establish.

Remark 3.14. *Every totally somewhat fuzzy semicontinuous function onto a discrete fuzzy topological space need not be strongly fuzzy semicontinuous as shown in the following counter example.*

Example 3.15. Let $X=\{a,\ b,\ c\}$ and consider the fuzzy topological spaces (X,τ) and (X,σ): $\tau=\{0,\ 1,\ \lambda\}$ where $\lambda=\frac{0.3}{a}+\frac{0.4}{b}+\frac{0.2}{c}$ and $\sigma=1^X$. Let f be the identity mapping from (X,τ) onto (X,σ): Let $\mu=\frac{0.5}{a}+\frac{0.7}{b}+\frac{0.6}{c}$. Here, λ is fuzzy semiclopen and $f^{-1}(\mu)=\mu\ge\lambda$, but μ is not fuzzy semiopen in (X,τ). Thus, f is totally somewhat fuzzy semicontinuous but not strongly fuzzy semicontinuous.

Definition 3.16. Let (X,T) and (Y,S) be any two fuzzy topological spaces. A function $f:(X,T)\to(Y,S)$ is called *slightly somewhat fuzzy semicontinuous* if λ is fuzzy clopen and $f^{-1}(\lambda)\ne 0$ then there exists a fuzzy semiclopen set $\mu\ne 0$ in X such that $\mu\le f^{-1}(\lambda)$.

Remark 3.17. *In the Example 3.15, every slightly somewhat fuzzy semicontinuous function into a discrete fuzzy topological space need not be strongly fuzzy semicontinuous.*

A fts (X, T) is said to be *fuzzy extremally disconnected* (FED) if and only if the closures of every fuzzy open subset is fuzzy open in X. By using Lemma 3.3 of [11] if (X, T) is FED space then $sCl(\lambda) = Cl(\lambda)$ for every fuzzy semiopen λ in X.

Remark 3.18. *A somewhat fuzzy continuous function from a fuzzy extremally disconnected space to another fuzzy topological space need not be totally somewhat fuzzy semicontinuous. The converse is not true as shown in the following example.*

Example 3.19. Let $X = I$ and λ be a subset of X be defined as follows

$$\lambda(x) = \begin{cases} \frac{1}{2}, & \text{if } 0 \leq x \leq \frac{1}{2} \\ x, & \text{if } \frac{1}{2} \leq x \leq 1. \end{cases}$$

Clearly, $T = \{0, \lambda, 1\}$ is a fuzzy topology on X and (X, T) is a fuzzy extremally disconnected space. Consider the identity mapping $f : (X, T) \to (X, T)$. Then f is fuzzy continuous. Clearly $f^{-1}(\lambda) = \lambda$ is not semiclosed. Thus, f is not totally fuzzy semicontinuous.

Theorem 2.6 in [13], if the somewhat fuzzy continuous function is somewhat fuzzy completely continuous, then it is valid. See the following Theorem 3.21. First we use the following lemma.

Lemma 3.20 ([13])**.** *Let X be a fuzzy extremally disconnected space. If λ is fuzzy regular open in X, then λ is also fuzzy closed in X.*

Theorem 3.21. *Let (X, T) be a fuzzy extremally disconnected space. If $f : (X, T) \to (Y, S)$ is somewhat fuzzy completely continuous, then it is also totally somewhat fuzzy continuous.*

Proof. Let $\lambda \neq 0 \in T$, then $f^{-1}(\lambda)$ is fuzzy regular open in (X, T). Since (X, T) is fuzzy extremally disconnected, $f^{-1}(\lambda)$ is fuzzy closed in (X, T) by Lemma 3.20 Clearly, $f^{-1}(\lambda) \in T$. Hence, f is also totally somewhat fuzzy continuous. A fuzzy completely continuous function need not be totally somewhat continuous as the following. $\square$

Example 3.22. Let $X = I$ and λ be a subset of X be defined as follows

$$\lambda(x) = \begin{cases} \frac{1}{2}, & \text{if } 0 \leq x \leq \frac{1}{2} \\ 1 - x, & \text{if } \frac{1}{2} \leq x \leq 1. \end{cases}$$

Then, $T = \{0, \lambda, 1\}$ is a fuzzy topology on X. Consider the identity mapping $f : (X, T) \to (X, T)$. Since $f^{-1}(\lambda) = \lambda$ and $Int(Cl\lambda) = \lambda$, f is fuzzy completely continuous. Clearly, λ is not closed. Hence, f is not totally somewhat fuzzy semicontinuous.

Here the relations between totally somewhat semicontinuous function and somewhat fuzzy completely continuous function, somewhat fuzzy continuous function have been discussed. See the following statement.

Theorem 3.23. *A somewhat fuzzy completely continuous function is totally somewhat fuzzy semicontinuous.*

Proof. It is clear that a fuzzy regular open set is both fuzzy semiopen and fuzzy semiclosed. Thus, the theorem holds.

The converse of the above theorem need not be true. Even a totally somewhat fuzzy semicontinuous function need not be somewhat fuzzy completely continuous. □

Example 3.24. Considering Example 3.22, and the subset μ of X defined as follows:

$$\mu(x) = \begin{cases} \frac{1}{4}, & \text{if } 0 \leq x \leq \frac{1}{2} \\ \frac{1-x}{2}, & \text{if } \frac{1}{2} \leq x \leq 1. \end{cases}$$

Then, $S = \{0, \mu, 1\}$ is a fuzzy topology on X. Consider the identity mapping $f : (X, S) \to (X, T)$. Then $f^{-1}(\lambda) = \lambda$. By easy computations it follows that
$\mu \leq f^{-1}(\lambda) = \lambda \leq Cl(\mu) = \mu^{'}, \mu = Int(\mu^{'}) \leq f^{-1}(\lambda) = \lambda \leq \mu^{'}$,
i.e., $f^{-1}(\lambda) = \lambda$ is fuzzy semiclopen. Hence, f is totally somewhat fuzzy semicontinuous. Obviously f is not somewhat fuzzy continuous.

Definition 3.25. A fuzzy topological space (X, T) is said to be *fuzzy semiconnected* [16] if there does not exist fuzzy semiopen sets λ and μ such that $\lambda + \mu = 1, \lambda \neq 0, \mu \neq 0$.

Proposition 3.26. *If f is totally somewhat fuzzy semicontinuous function from a fuzzy semiconnected space X into any fuzzy topological space Y, then Y is indiscrete fuzzy topological space.*

Proof. If possible suppose Y is not indiscrete. Then Y has a proper ($\neq 0$ and $\neq 1$) fuzzy open set λ (say). Then by hypothesis on f, $f^{-1}(\lambda)$ is a proper fuzzy semiclopen set of X, which is a contradiction to the assumption that X is fuzzy semiconnected. Hence the proposition. □

Definition 3.27. Let (X, T) be any fuzzy topological space. X is called *fuzzy T_0* [17] if and only if for any pair of distinct fuzzy points x_t and y_s, there is a fuzzy open set λ such that $x_t \in \lambda,\ y_s \notin \lambda$ or $x_t \notin \lambda$, $y_s \in \lambda$.

Definition 3.28 ([18])**.** Let (X, T) be any fuzzy topological space. (X, T) is called *fuzzy semi T_2* $\Leftrightarrow$ For any pair of distinct fuzzy points x_t and y_s there exist fuzzy semiopen sets λ and μ such that $x_t \in \lambda,\ y_s \in \mu$ and $sCl\lambda \leq 1 - sCl\mu$.

Proposition 3.29. *Let $f : (X, T) \to (Y, S)$ be an injective totally somewhat fuzzy semicontinuous function. If Y is fuzzy T_0, then X is fuzzy semi T_2.*

Proof. Let x_t and y_s be any two distinct fuzzy points of X. Then $f(x_t) \neq f(y_s)$. That is $(f(x))_t \neq (f(y))_s$. Since Y is fuzzy T_0, there exists a fuzzy open set say $\lambda \neq 0$ in Y such that $f(x_t) \in \lambda$ and $f(y_s) \notin \lambda$. This means $x_t \in f^{-1}(\lambda)$ and $y_s \notin f^{-1}(\lambda)$. Since f is totally somewhat fuzzy semicontinuous, there exists fuzzy semi-clopen set $\mu \neq 0$ in X such that $\mu \leq f^{-1}(\lambda)$ is fuzzy semi-clopen set of X.

Also $x_t \in f^{-1}(\lambda)$ and $y_s \in 1 - f^{-1}(\lambda)$. Now put $\mu = 1 - f^{-1}(\lambda)$. Then $f^{-1}(\lambda) = sCl(\lambda)$ and $sCl(1 - f^{-1}(\lambda)) = sCl\mu = 1 - f^{-1}(\lambda)$ and $sCl(\lambda) = f^{-1}(\lambda) = 1 - sCl(\mu) \leq 1 - sCl(\mu)$. Hence the Proposition. □

Proposition 3.30. *Let (X, T) be any fuzzy semiconnected space. Then every totally somewhat fuzzy semicontinuous function from a space X onto any fuzzy T_0-space Y is constant.*

Proof. Given that (X, T) is fuzzy semiconnected. Suppose $f : X \to Y$ be any totally somewhat fuzzy semicontinuous function and we assume that Y is a fuzzy T_0- space. Then by Proposition 3.26, Y should be an indiscrete space. But an indiscrete fuzzy topological space containing two or more points cannot be fuzzy T_0. Therefore, Y must be singleton and this proves that f must be a constant function. □

Proposition 3.31. *If $f : X \to Y$ is a slightly somewhat fuzzy semicontinuous and $g : Y \to Z$ is totally somewhat fuzzy continuous, then $g \circ f$ is totally fuzzy semicontinuous.*

Proof. Let λ be any fuzzy open set of Z. Then by hypothesis on $g, g^{-1}(\lambda)$ is fuzzy clopen in Y. Now $(g \circ f)^{-1}(\lambda) = f^{-1}[g^{-1}(\lambda)]$ and therefore by hypothesis on $f, (g \circ f)^{-1}(\lambda)$ is fuzzy semiclopen. This proves that $g \circ f$ is totally fuzzy semicontinuous. □

Proposition 3.32. *Let $f : (X, T) \to (Y, S)$ be a totally somewhat fuzzy semicontinuous function and (Y, S) is a fuzzy T_1-space. If $(A, T/A)$ is fuzzy semiconnected, then $f(A)$ is single point.*

Definition 3.33. Let (X, T) be a fuzzy topological space. A fuzzy set λ in X is called *generalized fuzzy closed* (in short gfc) [8](Balasubramanian 1997) $\iff$ $Cl\lambda \leq \mu$, whenever $\lambda \leq \mu$ and μ is fuzzy open. μ is called generalized fuzzy open (in short gfo) if $(1 - \mu)$ is gfc.

Definition 3.34 ([16])**.** A fuzzy set λ in (X, T) is called *generalized fuzzy semi-closed* $\iff$ $sCl\lambda \leq \mu$, whenever $\lambda \leq \mu$ and μ is fuzzy semiopen.

Definition 3.35. Let (X, T) and (Y, S) be any two fuzzy topological spaces. A function $f : (X, T) \to (Y, S)$ is called *totally somewhat generalized fuzzy semicontinuous* if $\lambda \in S$ is gf-open set in Y and $f^{-1}(\lambda) \neq 0$ then there exists a fuzzy semiclopen set $\mu \neq 0$ in X such that $\mu \leq f^{-1}(\lambda)$.

Definition 3.36 ([16])**.** A function $f : X \to Y$ is called *strongly gf-semicontinuous* if the inverse image of every gf-open set in Y is fuzzy semiopen in X.

Definition 3.37 ([16])**.** A fuzzy topological space X is called *fuzzy semi* $T_{\frac{1}{2}}$*-space*, if every generalized fuzzy semiclosed set in X is fuzzy semiclosed in X.

Remark 3.38. *Totally somewhat generalized fuzzy semicontinuity coincides with totally somewhat fuzzy semicontinuity when* Y *is fuzzy semi* $T_{\frac{1}{2}}$*-space.*

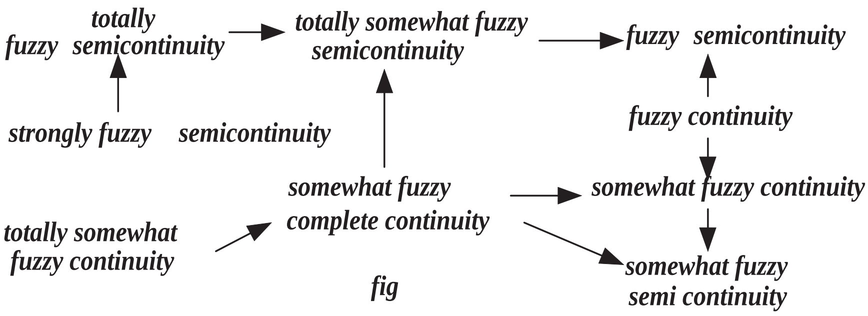
Proposition 3.39. *If* $f : X \to Y$ *is totally somewhat* gf*-semicontinuous, then* f *is strongly* gf*- semicontinuous.*

The converse of the above proposition is not true as the following example shows:

Example 3.40. Let $X = \{a,\ b,\ c\}$ and $Y = \{p,\ q,\ r\}$. Define $\lambda_i : X \to [0,1]$, $i = 1,2,3$, $\mu, \gamma : Y \to [0,1]$ be defined as follows: $\lambda_1 = \frac{1}{a} + \frac{0}{b} + \frac{0}{c}$, $\lambda_2 = \frac{0}{a} + \frac{1}{b} + \frac{0}{c}$, $\lambda_3 = \frac{1}{a} + \frac{1}{b} + \frac{0}{c}$, $\mu = \frac{1}{p} + \frac{1}{q} + \frac{0}{r}$ and $\gamma = \frac{1}{p} + \frac{0}{q} + \frac{0}{r}$. Consider $\tau = \{0,\ 1,\ \lambda_1,\ \lambda_2,\ \lambda_3\}$ and $\sigma = \{0,\ 1,\ \mu\}$. Then (X, τ) and (Y, σ) are fts. Define $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = f(b) = p$ and $f(c) = q$. Then f is strongly gf-semicontinuous but f is not totally somewhat gf-semicontinuous as the fuzzy set γ is gf-semiopen in Y, $f^{-1}(\gamma) = \lambda_3$ is fuzzy semiopen in X but not fuzzy semiclosed in X.

Proposition 3.41. $F : (X, T) \to (Y, S)$ *is totally somewhat* gf*-semicontinuous* $\Leftrightarrow$ *the inverse image of every* gf*- semiclosed set in* Y *is both fuzzy semiopen and fuzzy semiclosed.*

The following diagram shows the different implications established among various types of functions and counter examples in this paper.



fig

4 On Almost Fuzzy Semicontinuous Mappings

Definition 4.1 ([16])**.** Let λ be a fuzzy set in the fuzzy topological space (X, T). λ is called Δ *- fuzzy open* if $\lambda = \vee_{k\in\Gamma}\lambda_k$ where λ_k's are regular fuzzy open sets. λ is called Δ- fuzzy closed if and only if $1 - \lambda$ is Δ- fuzzy open.

$[\lambda]_\Delta$ stands for the fuzzy Δ - closure of λ, defined as $[\lambda]_\Delta = \wedge_{k\in\Gamma}\lambda_k$ where λ_k's are Δ - fuzzy closed and $\lambda_k \geq \lambda$. Similarly $[\lambda]_S$ stands for fuzzy semiclosure of λ.

Definition 4.2. A map $f : (X,T) \to (Y,S)$ is called *almost somewhat fuzzy semi-continuous* if for every regular fuzzy open (closed set) λ in (Y,S), $f^{-1}(\lambda) \neq 0$ is fuzzy semiopen (closed) in X.

Proposition 4.3. *$f : (X,T) \to (Y,S)$ is almost somewhat fuzzy semi-continuous $\Leftrightarrow$ for every fuzzy setλ in X*

$$f([\lambda]_S) \leq [f(\lambda)]_\Delta.$$

Proof. Suppose f is almost somewhat fuzzy semicontinuous and λ be any fuzzy set in X. Then $[f(\lambda)]_\Delta$ is Δ- fuzzy closed set in Y. Then $f^{-1}\{[f(\lambda)]_\Delta\}$ is fuzzy semiclosed in X. Now,

$$\lambda \leq f^{-1}(f(\lambda)) \leq f^{-1}\{Cl(f(\lambda))\} \leq f^{-1}\{[f(\lambda)]_\Delta\}$$

i.e., $\lambda \leq f^{-1}\{[f(\lambda)]_\Delta\}$; but $\lambda \leq [\lambda]_s$ and so

$$([\lambda]_s) \leq f^{-1}\{[f(\lambda)]_\Delta\}$$

and

$$f([\lambda]_s) \leq ff^{-1}\{[f(\lambda)]_\Delta\} = [f(\lambda)]_\Delta.$$

$$(i.e.,)f([\lambda]_s) \leq [f(\lambda)]_\Delta.$$

Conversely, suppose μ is Δ - fuzzy closed set in Y. Then $[\mu]_\Delta = \mu$. Let $\lambda = f^{-1}(\mu)$. Then by hypothesis

$$f([f^{-1}(\mu)]_s) \leq [f[f^{-1}(\mu)]]_\Delta \leq [\mu]_\Delta = \mu,$$

$$\text{i.e.,} \qquad [f^{-1}(\mu)]_s \leq f^{-1}(\mu);\ \text{but}\ [f^{-1}(\mu)]_s \geq f^{-1}(\mu)$$

and so

$$[f^{-1}(\mu)]_s = f^{-1}(\mu).$$

That is $f^{-1}(\mu)$ is fuzzy semiclosed and so f is almost somewhat fuzzy semicontinuous. □

The following proposition is easy to establish.

Proposition 4.4. *$f : (X,T) \to (Y,S)$ is almost somewhat fuzzy semicontinuous $\Leftrightarrow$ for any fuzzy setλ of Y*

$$[f^{-1}(\lambda)]_s \leq [f^{-1}(\lambda)_\Delta].$$

5 Mappings on Product Spaces

Definition 5.1. $f : (X, T) \to (Y, S)$ is called *almost somewhat fuzzy continuous* if $f^{-1}(\lambda) \neq 0$ is fuzzy semiopen whenever $\lambda \neq 0$ in S.

The following Proposition 5.2 is easy to establish.

Proposition 5.2. *If $f : X \to Y$ is somewhat fuzzy semi continuous and $g : Y \to Z$ is somewhat fuzzy continuous, then $g \circ f : X \to Z$ is almost somewhat fuzzy continuous.*

Proposition 5.3 ([16])**.** *Let $T_R^{X_1}$ and $T_R^{X_2}$ denote the fuzzy topologies generated by regular open sets of X_1 and X_2 respectively. If $T^{X_1 \times X_2}$ denotes the product fuzzy topology of $X_1 \times X_2$ and $T_R^{X_1 \times X_2}$ denotes the fuzzy topology generated by the regular fuzzy open sets of $X_1 \times X_2$, then*

$$T_R^{X_1} \times T_R^{X_2} = T_R^{X_1 \times X_2}.$$

Proposition 5.4. *Suppose $f_i : X_i \to Y_i$ be almost somewhat fuzzy semicontinuous functions for $i = 1$ to 2. Let $f : X_1 \times X_2 \to Y_1 \times Y_2$ be defined as $f(x_1, x_2) = (f_1(x_1), f_2(x_2))$. Then f is almost somewhat fuzzy semicontinuous.*

Proof. Let λ be any regular fuzzy open set in $Y_1 \times Y_2$. Then $IntCl(\lambda) = \lambda$. And by the Proposition 5.3, we have $\lambda = \bigvee_{k \in \Gamma} \left(\lambda_k^1 \times \lambda_k^2\right)$ where λ_k^1 and λ_k^2 are regular fuzzy open sets in X_1 and X_2 respectively. Further,

$$f^{-1}(\lambda) = \vee_{k \in \Gamma} f^{-1}(\lambda_k^1 \times \lambda_k^2) \quad = \vee_{k \in \Gamma} f^{-1}(\lambda_k^1) \times f^{-1}(\lambda_k^2).$$

This implies that $f^{-1}(\lambda)$ is a fuzzy semi-open set in $X_1 \times X_2$. Hence the Proposition. □

6 Preservation of Some Fuzzy Spaces Under Totally Fuzzy Semicontinuous Functions

It was proved in Lemma 3.9 of [19] that if λ is fuzzy semi-clopen and A is a fuzzy open crisp subset of a fts (X, T), then $\lambda \cap A$ is fuzzy semi-clopen in (X, T). We have the following Theorem.

Theorem 6.1. *If $f : X \to Y$ is totally somewhat fuzzy semicontinuous function and A is fuzzy open crisp subset of X, then $f_A : A \to Y$ is also totally somewhat fuzzy semicontinuous.*

Proof. Let λ be a fuzzy open subset of Y, then $f^{-1}(\lambda)$ is fuzzy semiclopen in X. Now, by Lemma 3.9 of [19], $f^{-1}(\lambda) \cap A = f_A^{-1}(\lambda)$ is fuzzy semi-clopen in X. Hence, the theorem. □

Theorem 6.2. *If $f : X \to Y$ is a totally somewhat fuzzy semicontinuous onto function and X is fuzzy s-closed, then Y is fuzzy compact.*

Proof. Let $\{\lambda_a : a \in \wedge\}$; $\wedge$ being the index set be a fuzzy open cover of Y. Then $\{f^{-1}(\lambda_a) : a \in \wedge\}$ is a fuzzy semi-clopen cover of X. By fuzzy s-closedness of X, there exists a finite subfamily of $\{f^{-1}(\lambda_a)\}$ such that $\bigcup_{i=1}^{n} f^{-1}(\lambda_{a_i}) = 1_X$. Now $1_Y = f(1_X) = f(\bigcup_{i=1}^{n} f^{-1}(\lambda_{a_i})) \subseteq \bigcup_{i=1}^{n} \lambda_{a_i}$ which implies that Y is fuzzy compact. □

Theorem 6.3. *If $f : X \to Y$ is a somewhat fuzzy semicontinuous onto function and X is fuzzy s-closed, then Y is fuzzy almost compact.*

Proof. Let $\{\eta_a : a \in \wedge\}$ be a fuzzy open cover of Y, then $\{f^{-1}(\eta_a) : a \in \wedge\}$ is a fuzzy semiopen cover of X. By fuzzy s-closedness of X, there is a finite subfamily of $f^{-1}(\eta_\alpha)$ such that $\bigcup_{i=1}^{n} sClf^{-1}(\eta_{a_i}) = 1_X$. Now $1_Y = f(1_X) = f(\bigcup_{i=1}^{n} sClf^{-1}(\eta_{a_i})) \subseteq f(\bigcup_{i=1}^{n} Clf^{-1}(\eta_{a_i})) \subseteq \bigcup_{i=1}^{n} Cl\eta_{a_i}$ which implies Y is fuzzy almost compact. □

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