Thai Journal of Mathematics : (2017) 243–256
Special Issue on Entropy in Econometrics

http://thaijmath.in.cmu.ac.th
ISSN 1686-0209

An Empirical Examination of Maximum Entropy in Copula-Based Simultaneous Equations Model

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Abstract : In our previous work, copulas were used to connect different marginal distributions in a system of linear equations. And in this study, we aim to further improve the way to join the marginal distributions. We propose entropy copula-based simultaneous equations model, in which the copulas are employed to improve the modeling of joint distribution. Under this model, we do not have to assume a specific distribution for the margins; instead, they are derived from the entropy method. This study will provide a conceptual idea of bivariate entropy copula-based simultaneous equations model and an explanation of how this method works. Then, an empirical experiment will be conducted to explore the performance of our proposed model.

Keywords : maximum entropy; copulas; joint distribution; system of equations; simultaneous equations.
2010 Mathematics Subject Classification : 62P20; 91B84.

This research was supported by Puay Ungpakorn Centre of Excellence in Econometrics, Faculty of Economics, Chiang Mai University.

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1 Introduction

Usually, a conventional system of equations for time series data, particularly the seemingly unrelated regressions (SUR) model and the simultaneous equations (SE) model, has a strong assumption of normally distributed residuals. Consequencely, in our previous works, we relaxed this assumption of normality by using copulas to link the different marginal distributions in the system of equations and introduced the copula-based simultaneous equations model and also the copula-based seemingly unrelated regressions model, and applied in several areas of economic research. (see, [1], and [2]) In this study, we aim to further develop our copula-based SE model with a special concern about more flexibility of dependency between equations. The copula approach offers a flexible way to construct a joint distribution independently from the marginal distributions [3]. This, in turn, makes the copula-based SE model more flexible and possible to capture nonlinear and asymmetric dependence between two or more equations. However, univariate and multivariate statistical analysis in either (pure) economics or applied economics typically involves inference of probability distribution, so the appropriate specification of marginal distributions and copula function appears so difficult to obtain. Uncertainty distributions can be specified either by fitting to data or by prior information (Bedford and Wilson, 2014). The data or prior information provides several specifications of functions of the variables for which the uncertainty distribution must satisfy. These are known as constraints of the parametric copula-based SE model. However, if there is some knowledge or partial information of the data or the outcome, the parametric copula-based SE model may give bias results.

Therefore, instead of using a parametric copula function, this study proposes to construct the joint distribution using the copula method with the marginal distribution derived by the entropy method. The principle of maximum entropy allows for the inclusion of the available information in assignment of probabilities to different data or outcome of a model [4]. The concept of maximum entropy for probability density inference has been applied extensively in a variety of areas, including copulas. Accordingly, this study considers the entropy copula method as a promising technique for modelling the dependency, particularly for joining the error terms in the SE model. Subsequently, the maximum primal discrete entropy estimator is used to estimate the unknown parameters of the copula-based SE model. The rest of this paper is structured as follows. In Section 2, we will explain the theoretical foundation of the entropy copula-based SE model, including the conceptual idea of the SE model, copula theory, entropy theory, and the GME estimator for the bivariate entropy copula-based SE model. Then, Section 3 is mainly for application in which the proposed model is extended to the analysis of demand and supply of Thai rice and the results are presented. And finally Section 5 contains conclusions.

2 Methodology

2.1 Review of Simultaneous Equations Model

In this section, we review the formulas to estimate simultaneous equations (SE) model. We then provide some background of entropy and Copula theories. Finally, the estimation steps are presented to show how to estimate the coefficients using entropy copula-based SE model.

Economic systems are usually described in terms of the behavior of various economic agents, and the equilibrium is reached when these behaviors are reconciled [5]. For example, with the demand and supply behaviors; the market clearing process will respond to the behavioural equations for demand and supply, creating simultaneous or joint determination of the equilibrium quantities. This causes the correlation between explanatory variables and errors in the estimation of this system equations. This system equations are called "simultaneous equations (SE) model". The system of equations of SE model is written as

$$y_i = [\Gamma'_i, X'_i]\beta_i + \varepsilon_i \qquad i = 1, ..., M, \tag{2.1}$$

where y_i is a $T \times 1$ vector of dependent variables, Γ_i is $T \times K$ matrix of M endogenous y_i, X_i is $T \times K$ a matrix of K exogenous variables, and β_i is a $(M + K) \times 1$ vector of unknown parameters to be estimated. ε_i is a $T \times 1$ vector of error terms in equation i^{th}. Hence, we can write the extension form of SE as

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_M \end{bmatrix} = \begin{bmatrix} \beta_{21}y_2 + \beta_{31}y_3 + \cdots + \beta_{M1}y_M + \beta_{11}X_1 + \cdots \beta_{K1}X_K \\ \beta_{12}y_1 + \beta_{32}y_3 + \cdots + \beta_{M2}y_M + \beta_{12}X_1 + \cdots \beta_{K2}X_K \\ \vdots \\ \beta_{1M}y_1 + \cdots + \beta_{M-1,M}y_{M-1} + \beta_{1M}X_1 + \cdots + \beta_{KM}X_K \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_M \end{bmatrix} \tag{2.2}$$

In the real application study, we can restrict some coefficients of endogenous variables to be zero. In addition, the vital assumption of the SE model is that it assumes that the errors have no correlation across observations but there are correlations across equations., so that

$$E[\,\varepsilon_{it}\,\varepsilon_{js}\,|X\,] \;=\; 0 \qquad ;\; t \neq s,\, i \neq j \tag{2.3}$$

where i and j indicate the number of each equation and t and s denote the observations at each time. However, we explicitly allow for contemporaneous correlation, i.e.,

$$E[\varepsilon_{it}\,\varepsilon_{jt}\,|X] \;=\; \sigma_{ij} \tag{2.4}$$

That indicates that the SE model allows non-zero covariance between the error terms of different equations in the model. In the conventional approach, the errors are assumed to have a normal distribution, that is $\varepsilon_t = (\varepsilon_{1t}, ..., \varepsilon_{Mt})\tilde{}N(0, \Sigma)$ where Σ, is a variance-covariance matrix for M equations, thus

$$E(\varepsilon_t \varepsilon\prime_t) = \Omega = \Sigma \otimes I_T, \tag{2.5}$$

where I is an T-dimensional identity matrix and $\otimes$ denotes the matrix Kronecker product, and this system equation can be estimated by least squares (LS), Maximum likelihood (ML), Bayesian, and entropy estimators.

2.2 Copula Theory

Nowadays, one of the most popular dependence theories is copula. The copula theory has been recently employed to construct the joint distribution of multiple variables. It provides a flexible way for constructing the joint distribution to measure the dependence of multivariate random variables, says marginal distributions. For example, in the bivariate case, the marginal distributions for the continuous random variables X and Y are denoted as $F(X)$and $G(Y)$, respectively. The joint cumulative distribution functions (CDF) of $C(u,v)$ or $F(x,y)$ can be constructed with the copula C as

$$P(X \leq x, Y \leq y) = C(F(x), G(y); \theta) = C(u, v; \theta), \tag{2.6}$$

where C is copula distribution function of a two-dimensional realization of random variables, u, v. θ is the copula parameter that relates to the dependence structure or family. The copula C can link two marginal distributions by mapping them into a joint distribution. If the marginal distribution is continuous, the copula function is unique. According to Nelsen [6], the copula function satisfies the properties on $[0,1]^2$.

(1) Boundary condition:

$$C(u, 0) = 0 = C(0, v), \tag{2.7}$$

$$\begin{aligned} C(u,1) &= u \\ C(1,v) &= v. \end{aligned} \tag{2.8}$$

(2) Monotonicity:

For every u_1, u_2, v_1 and v_2 in $[0,1]$ such that $u_1 \leq u_2$ and $v_1 \leq v_2$:

$$C(u_2, v_2) - C(v_1, v_2) - C(u_1, v_2) + C(u_1, v_1) \geq 0. \tag{2.9}$$

There is a variety of copula families such as elliptical copulas (Gaussian and t-copula), Archimedean copulas (Frank, Clayton, Gumbel copulas), extreme copulas and vine copulas, and mixed copulas. The best fit copulas can be assessed using AIC, BIC and goodness of fit [7].

2.3 Entropy Theory

The basic concept of maximum entropy consists of making inferences on the probability distribution that maximizes information entropy subject to a set of constraints, which are the assignment of probabilities that best represent the current state of knowledge [8]. In this study, we use a maximum entropy estimator, in discrete form, to estimate our unknown parameters in copula-based SE model. In the discrete case, for a random variable X with probabilities $\mathbf{p} = p_1, ..., p_T$ on $X = x_1, ..., x_T$, the Shannon entropy ([9], [10]) is defined as

$$H(\mathbf{p}) = -\sum_{i=1}^{T} p_i \log p_i \ , \tag{2.10}$$

where $0 \log 0 = 0$ and $\sum_{i=1}^{T} p_i = 1$. The probability distribution $\mathbf{p}$ is derived by the principle of maximum entropy based on discrete form given a set of constraints. The entropy measures the uncertainty of a distribution and reaches a maximum when p_i has uniform distribution. For example, suppose the expected value of the variable is known, we can specify another constraint as:

$$H(\mathbf{p}) = -\sum_{i=1}^{T} p_i x_i = \bar{x}. \tag{2.11}$$

The optimization of this problem can be solved using the Lagrangian method which takes the form as

$$\mathbf{L} = \sum_{i=1}^{T} p_i \log p_i - (\lambda_0 - 1) \left(\sum_{i=1}^{T} p_i - 1 \right) - \lambda_1 \left(\sum_{i=1}^{T} p_i x_i - \bar{x} \right), \tag{2.12}$$

where λ_0 and λ_1 are the Lagrangian multiplier. Thus, the resulting first-order conditions become

$$p_i = \exp(-\lambda_0 - \lambda_1 x_i) = \frac{\exp(-\lambda_1 x_i)}{\sum_{i=1}^{T} \exp(-\lambda_1 x_i)}. \tag{2.13}$$

2.4 Concept of Bivariate Entropy Copula-Based SE Model

To apply the entropy concept to an estimator of bivariate copula-based SE model, we use generalized maximum entropy to inverse a problem to the regression framework (see, [11]). First of all, the estimated parameter can be classified into two groups, namely SE parameter and error group and copula parameter group. These two parameter groups can be estimated jointly using the discrete entropy approach. Before we derive the entropy copula-based SE model, we need to understand the basic concept of the estimated parameters in each group. For the SE parameters, the point estimates β_k is computed as expectations of random variables p_{kd} with $z_k = [\underline{z}_{k1}, ..., 0, ..., \bar{z}_{kd}]$ support value, whereas $\underline{z}_{k1}$ and $\bar{z}_{km}$ denote the lower bound and upper bound, respectively. Thus we can compute parameter β_k as

$$\beta_k = \sum_{d=1}^{D} p_{kd} z_{kd} \tag{2.14}$$

where p_{kd} are the D dimensional support values or estimated probability distribution defined on the set z_{kd}. Similarly, the error ε_t is also constructed as the expected value of some random variable w_{td} with $v_t = [v_{t1}, ..., 0, ..., v_{td}]$ whereas w_t is a D dimensional proper probability weights defined on the set v_t such that

$$\varepsilon_t = \sum_{d=1}^{D} w_{td} v_{td} \tag{2.15}$$

For the copula parameter group, we need to find the appropriate constraint to model the dependence structure $h(\varepsilon_{1i}, \varepsilon_{2i})$ where ε_{1i} and ε_{2i} are the error term in the first and and the second equations respectively of SE model. Note that the study uses the discrete entropy copula as a dependence structure to join the expected errors in bivariate SE model. Therefore, in this section, we show in the following the derivation of maximum entropy distributions in discrete form of the bivariate entropy copula-based SE model.

2.5 Generalized Maximum Entropy Estimator to Bivariate Entropy Copula-Based SE Model

The maximum entropy can be approximated by discrete form of constraints to derive a copula-based SE model with maximum entropy. This estimation is classified as the discrete density maximum entropy copula-based model. We employ the maximum entropy checkerboard copula (MECBC) of Piantadosi et al. ([12],[13]) to join the residuals of entropy SE model In addition, the grade correlation is imposed as a constraint to preserve the dependence structure in the original data. In this section, the bivariate entropy checkerboard copula based SE model is proposed and constructed.

Suppose that the input space has been discretized into the points $(\varepsilon_i^1, \varepsilon_j^2)$ for$i, j = 1, ..., T$, we construct the discrete copula entropy as proposed by Piantadosi1, Howlett, and Borwein, [12]; Bedford and Wilson [14]; and Hao and Singh [8], to join the expected error term of two equations in bivariate SE model. Those studies suggested that the entropy copula density function$c(u, v)$ can be approximated by the discrete form. Suppose the probability $B(i, j) = b_{ij}$, $0 \le i$, $j \le T$ is the discrete copula probability partitioned within the interval $[0, 1] \times [0, 1]$ on the point $(\varepsilon_i^1, \varepsilon_j^2)$, p_{kd}^1, p_{kd}^2, w_{id}^1 and w_{jd}^2 are the discrete coefficients and error probabilities of equations 1 and 2 in SE model, therefore the discrete form of the entropy copula-based SE model can be expressed by

$$\begin{aligned} H(\mathbf{b}, \mathbf{p}, \mathbf{w}) = &- \sum_{i=1}^{T} \sum_{j=1}^{T} b_{ij} \log b_{ij} - \sum_{k=1}^{K1} \sum_{d=1}^{D} p_{kd}^1 \log p_{kd}^1 - \sum_{k=1}^{K2} \sum_{d=1}^{D} p_{kd}^2 \log p_{kd}^2 \\ &- \sum_{i=1}^{T} \sum_{d=1}^{D} w_{id}^1 \log w_{id}^1 - \sum_{j=1}^{T} \sum_{d=1}^{D} w_{jd}^2 \log w_{jd}^2 \,, \end{aligned} \tag{2.16}$$

subject to the following constraints.

1) Marginal probabilities constraints

$$\sum_{i=1}^{T} b_{ij} = \frac{1}{T} \tag{2.17}$$

$$\sum_{j=1}^{T} b_{ij} = \frac{1}{T} \tag{2.18}$$

Bedford and Wilson [14] mentioned that these constraints on the functions of the discretized variables, $E(h_l) = \alpha_l,\ l = 1, ..., L$ can be defined from the Kronecker deltas:

$$\begin{aligned} \delta_q^{(r)}(i,j) &= \begin{cases} 1, & if \ \ i = q, \\ 0, & if \ \ i \neq q, \end{cases} \\ \delta_q^{(c)}(i,j) &= \begin{cases} 1, & if \ \ j = q, \\ 0, & if \ \ j \neq q, \end{cases} \end{aligned} \tag{2.19}$$

which indicate whether we are in the q^{th} row and q^{th} column, respectively. The marginal constraints Eqs.(2.17-2.18) are then considered to be the expectations of these Kronecker deltas.That is,

$$\begin{aligned} E\left[\delta_q^{(r)}(i,j)\right] &= \sum_{i=1}^{T}\sum_{j=1}^{T} \delta_q^{(r)}(i,j) b_{ij} = \frac{1}{T} \qquad ; q = 1, ..., T, \\ E\left[\delta_q^{(c)}(i,j)\right] &= \sum_{i=1}^{T}\sum_{j=1}^{T} \delta_q^{(c)}(i,j) b_{ij} = \frac{1}{T}. \end{aligned} \tag{2.20}$$

2) Dependence structure constraint

$$\sum_{i=1}^{T}\sum_{j=1}^{T} h_l \left(\sum_{d=1}^{D} w_{id}^1 v_{id}^1, \sum_{d=1}^{D} w_{jd}^2 v_{jd}^2 \right) b_{ij} = \alpha_l, \tag{2.21}$$

where α_l is the sample mean of the dependence function h_l, which is encompassed by a variety of dependence measures.

3) The constraints of SE equations can be expressed as

$$\begin{aligned} y_{1t} &= \sum_{k=1}^{K1} x'_{1k} \sum_{d=1}^{D} p_{kd}^1 z_{kd}^1 + \sum_{t=1}^{T}\sum_{d=1}^{D} w_{id}^1 v_{id}^1 \\ y_{2t} &= \sum_{k=1}^{K2} x'_{2k} \sum_{d=1}^{D} p_{kd}^2 z_{kd}^2 + \sum_{t=1}^{T}\sum_{d=1}^{D} w_{jd}^2 v_{jd}^2 \end{aligned} , \tag{2.22}$$

where $K1$ and $K2$ are the number of coefficient in the first and the second equation, respectively. For a simple derivation, x'_{1k} is denoted as either endogenous variable or exogenous variable.

4) Additional constraints

$$\sum_{d=1}^{D} p_{kd}^{1} = 1, \sum_{d=1}^{D} p_{kd}^{2} = 1, \sum_{d=1}^{D} w_{id}^{1} = 1, \sum_{d=1}^{D} w_{jd}^{2} = 1 \tag{2.23}$$

By using Lagrangian method, solution of the optimization problem to derive the maximum entropy copula-based SE model subject to all the above constraints can be expressed as:

$$\begin{aligned} L = & -H\left(\mathbf{b}, \mathbf{p}, \mathbf{w}\right) + \lambda'_1 \left(y_{1t} - \sum_{k=1}^{K1} x'_{1k} \sum_{d=1}^{D} p_{kd}^{1} z_{kd}^{1} - \sum_{t=1}^{T} \sum_{d=1}^{D} w_{id}^{1} v_{id}^{1} \right) \\ & + \lambda'_2 \left(y_{2t} - \sum_{k=1}^{K2} x'_{2k} \sum_{d=1}^{D} p_{kd}^{2} z_{kd}^{2} - \sum_{t=1}^{T} \sum_{d=1}^{D} w_{jd}^{2} v_{jd}^{2} \right) \\ & + \lambda'_3 \left(\sum_{d=1}^{D} p_{kd} - 1 \right) + \lambda'_4 \left(\sum_{d=1}^{D} w_{td} - 1 \right) \\ & + \theta'_1 \left(\sum_{i=1}^{T} \sum_{j=1}^{T} \delta_q^{(r)}(i,j) b_{ij} - \frac{1}{T} \right) + \theta'_2 \left(\sum_{i=1}^{T} \sum_{j=1}^{T} \delta_q^{(c)}(i,j) b_{ij} - \frac{1}{T} \right) \\ & + \theta'_3 \left(\sum_{i=1}^{T} \sum_{j=1}^{T} h_l \left(\sum_{d=1}^{D} w_{id}^{1} v_{id}^{1}, \sum_{d=1}^{D} w_{jd}^{2} v_{jd}^{2} \right) b_{ij} - \alpha_l \right), \end{aligned} \tag{2.24}$$

where θ'_i, λ'_i, $i = 1, 2, 3$ are the vectors of Lagrangian multiplier. Thus, the resulting first-order conditions this optimization yields are as follows ([11], [14]).

$$\hat{p}_{kd}^{1} = \frac{\exp(-z_{kd}^{1} \lambda_1 x'_{1k})}{\sum\limits_{d=1}^{D} \exp(-z_{kd}^{1} \lambda_1 x'_{1k})}, \tag{2.25}$$

$$\hat{p}_{kd}^{2} = \frac{\exp(-z_{kd}^{1} \lambda_1 x'_{1k})}{\sum\limits_{d=1}^{D} \exp(-z_{kd}^{2} \lambda_2 x'_{2k})}, \tag{2.26}$$

$$\hat{w}_{id}^{1} = \frac{\exp(-\hat{\lambda}_{3t} v_{id})}{\sum\limits_{d=1}^{D} \exp(-\hat{\lambda}_{3t} v_{id})}, \tag{2.27}$$

$$\hat{w}_{jd}^{1} = \frac{\exp(-\hat{\lambda}_{3t} v_{jd})}{\sum\limits_{d=1}^{D} \exp(-\hat{\lambda}_{3t} v_{jd})}, \tag{2.28}$$

$$b_{ij} = \frac{\exp\left[-\sum\limits_{q=1}^{T-1} \left(\theta_q^{(r)} \delta_q^{(r)}(i,j) + \theta_q^{(c)} \delta_q^{(c)}(i,j) \right) - \sum\limits_{l=1}^{L} \theta_l h_l \left(\sum\limits_{d=1}^{D} w_{id}^{1} v_{id}^{1}, w_{jd}^{2} v_{jd}^{2} \right) \right]}{\sum\limits_{i=1}^{T} \sum\limits_{j=1}^{T} \exp\left[-\sum\limits_{q=1}^{T-1} \left(\theta_q^{(r)} \delta_q^{(r)}(i,j) + (\theta_q^{(c)} \delta_q^{(c)}(i,j) \right) - \sum\limits_{l=1}^{L} \theta_l h_l \left(\sum\limits_{d=1}^{D} w_{id}^{1} v_{id}^{1}, w_{jd}^{2} v_{jd}^{2} \right) \right]}. \tag{2.29}$$

Summing up the above equations, we maximize the joint-entropy objective, Eq. (2.16), subject to the regression Eqs. (2.17-2.23). The solution to this maximization problem is unique by forming the Lagrangian and solving for the first-order conditions to obtain the optimal solution for each probabilities. Then these estimated probabilities are used to derive the point estimates for the regression coefficients and error term, as well as the joint copula dependence structure $h_l(\cdot)$. For the dependence structure between two error terms in SE model, the Spearman correlation [13] is used as a dependence measure here to illustrate the flexibility of entropy copula for dependence modelling. Thus, the copula dependence structure $h_l(\cdot)$ in Eq. (2.29) can be measured by the Spearman correlation ρ which is expressed as:

$$\rho = 12\left[\frac{1}{n^3}\cdot\sum_{i=1}^{T}\sum_{j=1}^{T} b_{ij}(i-1/2)(j-1/2) - 1/4\right] \tag{2.30}$$

For more discussion, formulation and solution of the discrete copula entropy in the primal problem can be found in Piantadosi, Howlett, Borwein [12]. In this method, the probability density b_{ij} is only available for the points defining the partition on $[0,1]^2$. For example, given $T = 20$, $M = 2$, and $\rho = 0.6$, Figure 1 illustrates the density function that is consistent with continous entropy copula density function. (see, [15]).

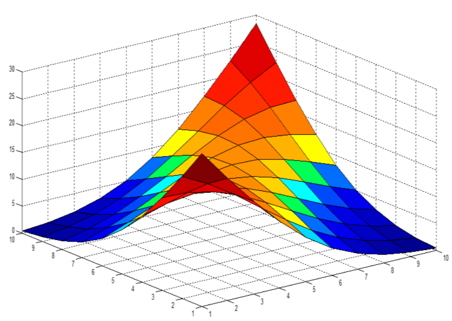

Figure 1: Example of copula density function from discrete entropycopula methods.

3 An Application

So far we have explained the theoretical foundation of the entropy copula-based SE model, starting from conceptual idea of the SE model, copula theory, and entropy theory, followed by the GME estimator for the bivariate entropy copula-based SE model. Right now, this part is conducted as an empirical study to roughly evaluate the performance of this approach and explore how it can work through the real data set.

We consider the analysis of demand and supply of Thai rice as a case study due to two reasons. First, rice is considered one of the most important agricultural crops in Thailand. It creates a large economic value for the Thai economy through exportation and provides income to a large number of Thai farming families. Second, the analysis of demand and supply is an obvious example of simultaneous equations model. For example, the analysis of Thai rice consists of two equations, demand and supply, describing the behavior of consumer and producer, respectively, and also the equilibrium price and quantity of rice that result when the demand and supply behaviors are reconciled. In essence, the market clearing process feeds back equilibrium price into the demand and supply equations, creating simultaneous determination of the equilibrium quantity.

The demand and supply analysis of Thai rice can be specified as:

$$Q_t^d = \beta_1 + \beta_2 P_t + \beta_3 P_t^{VN} + \beta_4 P_t^{PK} + \beta_5 P_t^{Ind} + \varepsilon_t, \tag{3.1}$$

$$Q_t^s = \alpha_1 + \alpha_2 P_t + \alpha_3 WS_t + \alpha_4 R_t + \alpha_5 P_t^{fm} + \varepsilon_t. \tag{3.2}$$

Eqs.(3.1) and (3.2) show demand and supply for Thai rice, respectively. The quantity demanded depends on the export price of Thai rice(P) as well as the prices of other rice-exporting countries namely Vietnam (P^{VN}), Pakistan (P^{PK}), and India (P^{Ind}) -which are main competitors of Thailand's rice exports. The quantity supplied typically depends on the export price, but also reacts to essential factors for rice cultivation like water storage (WS), rain (R), and producer price for rice or so-called farm price (P^{fm}). All the data is collected monthly from January 2006 to December 2015, from different sources. We retrieved the data regarding rice cultivation, and Thailand's export prices and quantities from the government institutions. But other countries' rice prices were obtained from the International Rice Research Institute (IRRI), with some help from the Bank of Thailand staff. Prior to the model estimation, we checked the stationarity of the data set by using the Augmented Dickey-Fuller (ADF) unit root test. We found that all the series were non-stationary. Then, we transformed the data into stationary form through taking growth rates, before estimating the model. Table 1 is descriptive statistics a simple quantitative summary of the variables.

Table 1: Descriptive statistics

	Q	P	P^{VN}	P^{PK}	P^{Ind}	W^S	R	P^{fm}
Mean	0.0002	0.004	0.007	0.001	−0.001	0.001	0.680	0.003
Median	0.003	−0.007	0	0	0	−0.021	0	0.002
Maximum	0.373	0.413	0.428	0.135	0.059	0.216	11.506	0.362
Minimum	−1	−0.118	−0.213	−0.138	−0.089	−0.126	−0.958	−0.113
Range	1.373	0.531	0.641	0.274	0.149	0.342	12.465	0.475
Std.Dev.	0.147	0.071	0.094	0.045	0.020	0.084	2.004	0.059
Skewness	−2.580	2.032	1.231	0.193	−0.404	0.764	2.836	2.762
Kurtosis	20.730	11.450	7.572	4.747	7.195	2.523	12.319	17.726

Source: Calculation.

In this subsection, the entropy copula-based SE model is compared to the relevant benchmarking models in terms of in-of-sample predictive ability. Firstly, it is compared to the conventional SE model estimated by least squares (LS) estimator using the R package systemfit, contributed by Henningsen and Hamann [16]. Secondly, we compare our proposed approach to the copula-based SE model, which is estimated using the maximum likelihood estimator (MLE). The distribution of the innovations for each equation and the copula are chosen according to the AIC from a range of different innovation distributions, which consist of normal, student-t, and various copulas capturing different types of dependence, e.g., tail independent Gaussian copula and Frank copula, symmetric-tail-dependent student-t, Student-t copula with symmetric tail dependence, lower-tail-dependent Clayton, upper-tail dependent Gumbel, Joe, and Ali-Mikhail-Haq. This parametric copula-based model was described in our previous work, Pastpipatkul et al. [1]. Predictions are evaluated in terms of two different loss functions, so the classical root mean squared error is used to evaluate the performance of the proposed method in this work.

$$RMSE = \sqrt{\frac{1}{T}\sum_{t=1}^{T}\left(\hat{y}_t - y_t\right)^2} \tag{3.3}$$

Table 2 presents the RMSE values for the conventional SE model (denoted by LS-SE), the copula-based SE model (denoted by parametric Cop-SE), and the entropy copula-based SE model (denote by Entropy-Cop-SE).

Table 2: Model comparison

Model	LS-SE	Parametric Cop-SE	Entropy Cop-SE
RMSE	0.1368	0.1709	0.1381

Source: Calculation.
Note: Gaussian copula with student-t and normal margins shows the lowest AIC in parametric Cop-SE model.

The results show that the conventional SE model is the best model for this data set as it holds the minimum value of RMSE compared to the others. However, apart from the LS-SE model, if we compare the Entropy-Cop-SE model to the parametric Cop-SE, we will find that our newly proposed model can perform strongly better than the parametric Cop-SE model due to the smaller RMSE value. In fact, this value is pretty close to the one of LS-SE model. We expect that the error bound or support in the entropy estimator is possibly too wide which, in turn, leads a higher variance of the GME estimator. Golan, Judge, and Miller [11] noted that the bias increases as we widen the error bounds. Thus, there exists the higher value of RMSE.

We have to note that this experiment is conducted under one specific data set. So, the results do not indicate that this model is certainly better than the other models. In this specific case, using the conventional SE approach is preferable, but the entropy copula-based SE model is still acceptable and can be interesting for the analysis of demand and supply of Thai rice as an alternative model. In addition, for the case that the distribution is unknown, the entropy copula-based

SE model is better. This is because we do not need to spend time selecting the best-fit model as happened in either the conventional SE or the parametric Cop-SE models.

Table 3: Estimated results

	Conventional SE		Entropy Copula based SE	
	Coefficient	Std. Error	Coefficient	Std. Error
Demand equation				
Intercept	0.002	0.0124	−0.0034	0.0125
P	−0.9697 * **	0.1996	−0.8872 * **	0.2020
PVN	0.4534**	0.1527	0.3829*	0.1544
PPK	−0.223	0.2949	0.0069	0.2984
PInd	1.2819*	0.6539	0.4959	0.6615
Supply equation				
Intercept	−0.0026	0.0152	−0.0192	0.0133
P	0.1082	0.2137	0.0377	0.1866
WS	0.0421	0.1796	0.0336	0.1568
R	0.0041	0.0074	0.0066	0.0065
Pfm	−0.1369	0.2502	0.0290	0.2185
Spearman correlationa	0.8577		0.8912	

Source: Calculation.
Note: *, **, and *** denote rejections of the null hypothesis at the 10%, 5% and 1% significance levels, respectively. "a" is correlation of the residuals.

The estimated parameters for the demand and supply equations (Eqs.3.1-3.2) are shown in Table 3. According to results shown in Table 3, we estimated the parameters under the form of the conventional SE model and also the entropy copula-based SE model. As we can see, the results from both models are slightly different. In the case of the conventional model, we find that the quantity demanded for Thai rice depends on the export price and the prices of other rice-exporting countries namely Vietnam and India. On the other hand, the entropy copula-based SE model discovers that only Thailand's export price and Vietnam's export price can significantly create the impact on the demand for Thai rice. But surprisingly, we failed to find statistically significant impacts for the rest of the variables.The results show that an increase in export price creates the negative effect on the demand for Thai rice with the predicted coefficients -0.9697 (for the conventional SE) and -0.8872 (for entropy copula-based SE). Moreover, the process of other rice-exporting countries is also relevant to the quantity demanded, but in the opposite direction to Thai price. An increase in export prices of Vietnam and India cause the increasing demand for Thai rice as they are substitute goods. Rice consumers think that rice from those countries can satisfy the same necessity, when the price of one country rises, the consumers tend to import rice from other countries instead, creating the positive relationship between Vietnamese and Indian export prices and the demand for Thai rice.

Additionally, high value of Spearman correlation coefficients, and 0.8912, just support our efforts to estimate the SE model using a correlated joint distribution.The demand and supply residuals based on the maximum entropy copula are shown in Figure 1, which provide multivariate return period information based on both demand and supply residuals. The level curves of the entropy copula are also plotted to compare with the Gaussian copula.As we expected, contour plots give evidence that demand and supply tend to have positive dependence and symmetry.

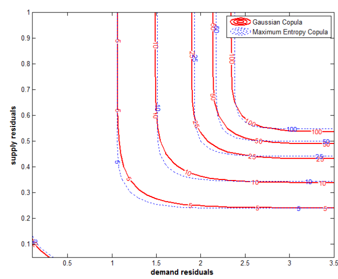

Figure 2: Bivariate return period of the demand and supply residuals based on entropy copula.

4 Conclusion and Future Work

The copula approach has been used so far to relax the strong assumption of normally distributed residuals in a conventional system of equations. In this study, we aimed to develop this further by applying the entropy approach to the copula-based simultaneous equations (SE) model. In essence, we generate the joint distribution by using copulas, whereas the marginal distributions are derived by the entropy method. As a result, the entropy copula-based SE model is introduced in this paper.Then, we conducted an empirical experiment to explore the performance of this proposed model through real analysis of demand and supply for Thai rice. According to this experiment, we found that the conventional SE model was best-fit to this data set. However, this experiment can also lead to a general suggestion that when the distribution is unknown, the entropy copula-based SE model is more advantageous and it can be a promising technique for modeling the dependency, particularly for joining the error terms in the SE model.

Acknowledgements : We are grateful for financial support from Puay Ungpakorn Centre of Excellence in Econometrics, Faculty of Economics, Chiang Mai University. Also, we thanks Prof. Nguyen, H. T. for valuable comments to improve this paper.

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(Received 31 August 2017)
(Accepted 30 October 2017)