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Portfolio Optimization of Energy Commodity Futures Returns with Minimum Information Copula

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Abstract : Energy commodity futures returns are modeled using GARCH and EGARCH processes. Their dependence structures are constructed from vine copula approach. Minimum information methods were applied to approximate bivariate copulas for all pairs in vine structure. A copula that satisfies a set of data constraints and that has minimum relative entropy (with respect to the independence copula) among the class of all copulas satisfying those constraints is called minimum information copula. The vine copula built from minimum information copulas is used to quantify the risks of energy commodity portfolio. Optimal portfolio that minimizes the risks with a given expected return are obtained for 4 energy commodity products (crude oil, natural gas, gasoline, and heating oil) using data from the New York Mercantile Exchange (NYMEX).

Keywords : energy commodity futures return; EGARCH; minimum information copula.
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1 Introduction

Nowadays, energy is key to almost all human activities such as lighting, transportation, industrial production et al. Hence, population growth entails an increasing energy consumption. In 2014, the world total final energy consumption is 9,425 Mtoe increased from 1973 (4,661 Mtoe) about 49.45%. While, the world total final energy supply is 13,699 Mtoe increased from 1973 (6,101 Mtoe) about 44.54%(see [1]). The energy prices are invariably very volatile because of many factors such as politics, wars, climate change including speculation. Many researchers have tried to study about the changes in energy prices such as [2] and [3]. Because uncertain situation of energy prices can pose a lot of risk for investors, financial institutions, and portfolio managers. Thus, risk management is important. While there are many methods for management of risks, value at risk (VaR) and conditional value at risk (CVaR) are widely used to measure risk. According to these risk measures, we can construct optimal portfolio by minimizing the risks for any given expected returns.

The goal of portfolio optimization is to find the proportions of various assets to be held in a portfolio with the highest returns. In 1952, [4] introduced modern portfolio theory in which dependence between financial returns is explained by correlation coefficient. However, it may be not appropriate for the financial analysis. For example, [5], [6], [7], and [8] found that the performance of portfolio based on dependence structure is better than portfolio based on normal distribution model. While, we found that copula is widely used in dependence structure model (see [9] and [10]). A copula is a multivariate probability distribution with the uniformly distributed marginal distributions. However, in higher dimensions, the selection of parametric copula is still rather limited. This has led [11] to develop a pair-copula construction (PCC) which further explored and discussed by [12]. Finally, [13] put the PCC in an inferential context. After that PCC has been applied to many fields such as finance, insurance, genetic, marketing, and hydrology.

Pair-copula construction (PCC) or vine copula demonstrates high flexibility and advantages in constructing multivariate distributions. PCC have been used to construct dependence structure and build the joint distribution of portfolio returns (see [14], [15], and [16]). Meanwhile, [13] and [17] show that the optimal portfolio via vine copula is better than Student t copula model.

In 2013, [18] proposed the minimum information copula which can be considered the most independent copula satisfying the data constraints. The concept of minimum information was first introduced by [19] and considered by [20].

Minimum information copula that was proposed by [18] had led [21] to develop minimum information copula for risk analysis. The results show that the minimum information copula can model from weak to strong upper tail dependence in all of the parametric copulas chosen and the minimum information copula is better to capture the conditional relationship between variables than gaussian copula. Similarly, [22] showed that the log-likelihood of the minimum information copula is larger than gaussian copula and t-copula. However, this method has the source of error that the algorithm works more slowly and this becomes more important

when the distribution being modeled is far from uniform. Therefore, [23] addressed the problem of approximating a conditional copula and developed a two-stage procedure. This showed that the two-stage procedure is both feasible and competent; and it can be applied to relatively higher-dimensional vine copulas than [21].

This paper aims to investigate the dependence among energy commodities future prices by using minimum information copula in order to calculate VaR and CVaR, and to construct the optimal portfolio.

The remainder of this paper is organized as follows. Section 2 provides the theoretical background of vine copula, relative information, and minimum information copula. Section 3 gives the discussion on vine copula approximation from minimum information methods. Application to energy commodity is given in Section 4. Finally, we conclude our paper in Section 5.

2 Vine Copula

The dependence among stock returns is a crucial information for portfolio optimization. In this paper, we model the dependence structure of stock returns using copula. An n-dimensional copula is a joint distribution function (restricted to $[0,1]^n$) with uniform marginal distributions. When any two different joint distributions are constructed using the same copula function, they will exhibit the same type of dependence regardless of their marginal distributions.

Theorem 2.1 (Sklar's Theorem [24])**.** *Let $X_1, ..., X_n$ be a given random variables having continuous distribution functions, $F_1(x_1), ..., F_n(x_n)$ and a joint distribution function, $H(x_1, ..., x_n)$. There exists a unique copula $C(.)$ such that*

$$H(x_1, ..., x_n) = C\left(F_1(x_1), ..., F_n(x_n)\right), \forall (x_1, ..., x_n) \in \mathbb{R}^n. \tag{2.1}$$

Conversely, for a given continuous distribution function $F_1(x_1), ..., F_n(x_n)$ and a copula function $C(.)$, $H(x_1, ..., x_n)$ defined in (2.1) is a joint distribution function.

Let $\mathcal{V} = \{T_1, ..., T_{n-1}\}$ be a vine on n variables with the set of edges $\mathcal{E}(\mathcal{V}) = \mathcal{E}_1 \cup ... \cup \mathcal{E}_{n-1}$; where T_1 is a tree with nodes $\mathcal{N}_1 = \{1, ..., n\}$ and a set of $(n-1)$ edges denoted by $\mathcal{E}_1$; for $i = 2, ..., n-1$, T_i is a tree with nodes $\mathcal{N}_i = \mathcal{E}_{i-1}$. If for every $e = \{e_1, e_2\} \in \mathcal{E}_i, i = 2, .., n-1$, the cardinality of the symmetric difference (union without intersection) between e_1 and e_2 is 2, then $\mathcal{V}$ is called a regular vine on n variables [25].

Let U_e^* denote the constraint set related to the edge $e = \{e_1, e_2\} \in \mathcal{E}_i, i = 2, .., n-1$. The constraint set is the subset of $\{1, .., n\}$ reachable from e. Let $D_e = U_{e_1}^* \cap U_{e_2}^*$ denote the conditioning set of e and the conditioned set of e can be defined as $\{U_{e_1}^* \setminus D_e, U_{e_2}^* \setminus D_e\}$, where $\{U_{e_1}^* \setminus D_e$ represent the relative complement of D_e in $U_{e_1}^*$. To simplify the notation, we would write $\dot{e}_1$ and $\dot{e}_2$ for $U_{e_1}^* \setminus D_e$ and $U_{e_2}^* \setminus D_e$, respectively.

Figure 1 illustrates a regular vine on 4 variables. T_1 is a tree connecting nodes $\mathcal{N}_1 = \{1, ..., 4\}$ with corresponding edges $\mathcal{E}_1 = \{\{1, 2\}, \{2, 3\}, \{3, 4\}\}$. T_2 is a tree

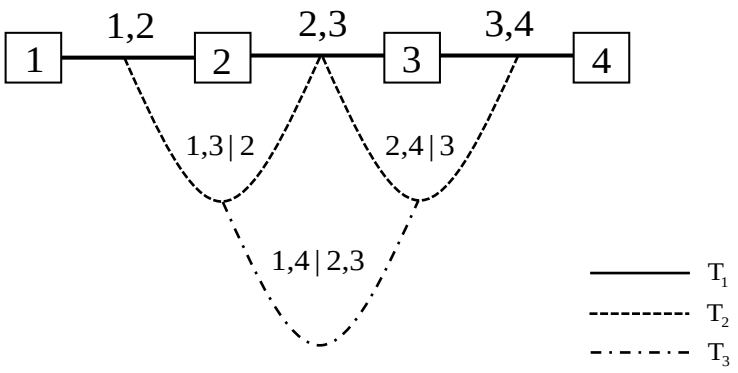


Figure 1: An example of regular vine or called D-vine on 4 variables

connecting nodes $\mathcal{N}_2 = \mathcal{E}_1$ with corresponding edges $\mathcal{E}_2 = \{\{1,3|2\},\{2,4|3\}\}$. For T_3 with nodes $\mathcal{N}_2 = \mathcal{E}_2$ and edge $\mathcal{E}_3 = \{\{1,4|2,3\}\}$, the constraint set of $\mathcal{E}_3$ is $\{1,...,4\}$. Thus, the conditioned set is $\{1,4\}$ and the conditioning set is $\{2,3\}$. Notice that; for the tree T_1, $e = \{e_1, e_2\} \in \mathcal{E}_1$, we have $U_e^* = \{e_1, e_2\}$ and D_e is empty.

Suppose that we have a regular vine $\mathcal{V}$. For any edge $e \in \mathcal{E}(\mathcal{V})$ with the conditioned set $\{\dot{e}_1\dot{e}_2\}$ and corresponding conditioning set D_e, we define $\mathbf{X}_e = (X_v | v \in D_e)$ as the vector of random variables associated with the conditioning set D_e. Note that a bold letter denotes a vector. Let $C_{\dot{e}_1\dot{e}_2|D_e}(.)$ be a bivariate copula for the edge $e \in \mathcal{E}(\mathcal{V})$ and $c_{\dot{e}_1\dot{e}_2|D_e}(.)$ be a copula density associated with $C_{\dot{e}_1\dot{e}_2|D_e}(.)$. We denote the values of $X_{\dot{e}_1}, X_{\dot{e}_1}$, and $\mathbf{X}_e$ by $x_{\dot{e}_1}, x_{\dot{e}_1}$, and $\mathbf{x}_e$, respectively. We have the following theorem from [25].

Theorem 2.2. *Let $\mathcal{V} = \{T_1, ..., T_{n-1}\}$ be a regular vine for the random variables $\{X_1, ..., X_n\}$. For each edge $e \in \mathcal{E}_i, i = 2, .., n-1$ with the conditioning set D_e and the conditioned set $\{\dot{e}_1\dot{e}_2\}$, let the conditional copula and copula density be and $C_{\dot{e}_1\dot{e}_2|D_e}$ and $c_{\dot{e}_1\dot{e}_2|D_e}$, respectively. Let the marginal distribution functions $F_i(x_i)$ with density functions $f_i(x_i), i = 1, 2, .., n$ be given. Then the vine-dependent distribution for the random variables $\{X_1, ..., X_n\}$ is uniquely determined with density function*

$$f(x_1, ..., x_n) = \prod_{i=1}^{n} f_i(x_i) \times \prod_{e \in \mathcal{E}} c_{\dot{e}_1\dot{e}_2|D_e}\left(u_{\boldsymbol{x}_e}, w_{\boldsymbol{x}_e} | \boldsymbol{X}_e = \boldsymbol{x}_e\right), \tag{2.2}$$

where $u_{\boldsymbol{x}_e} = F_{\dot{e}_1|D_e}(x_{\dot{e}_1}|\boldsymbol{X}_e = \boldsymbol{x}_e)$ and $w_{\boldsymbol{x}_e} = F_{\dot{e}_2|D_e}(x_{\dot{e}_2}|\boldsymbol{X}_e = \boldsymbol{x}_e)$ are conditional marginal distributions (conditional on $\boldsymbol{X}_e$).

A regular vine copula for the regular vine structure shown in Figure (1) can be written as

$$f(x_1,...,x_4) = \prod_{i=1}^{4} f_i(x_i) \times c_{12}\left(F_1(x_1), F_1(x_1)\right) \times c_{23}\left(F_2(x_2), F_3(x_3)\right) \times c_{34}\left(F_3(x_3), F_4(x_4)\right) \times c_{13|2}\left(F_1(x_1|x_2), F_3(x_3|x_2)\right) \times c_{24|3}\left(F_2(x_2|x_3), F_4(x_4|x_3)\right) \times c_{14|23}\left(F_1(x_1|x_2,x_3), F_4(x_4|x_2,x_3)\right).$$

3 Vine Copula Approximation

3.1 Relative Information

For any given bivariate densities f and g, we define the relative information of f with respect to g as

$$I(f|g) = \int\int \ln\left(\frac{f(x_1,x_2)}{g(x_1,x_2)}\right) f(x_1,x_2)dx_1dx_2. \tag{3.1}$$

The relative information is a non-symmetric measure of the degree of deviation of f from g. Its value equates zero (the minimum value of relative information) when $f = g$. [21] used this information as a criterion for selecting a copula by. The reason that this information is a good criterion for copula selection is its monotonic transformation-invariant property. According to this property, $I(f|g)$ is the same as $I(c_f|c_g)$, where c_f and c_g are two copula densities of f and g, respectively. Equation (3.2) can be written with copula density function as

$$I(f|g) = \int\int \ln\left(\frac{c_f(F_1(x_1), F_2(x_2))}{c_g(F_1(x_1), F_2(x_2))}\right) dF_1(x_1)dF_2(x_2). \tag{3.2}$$

To measure the degree of dependency in a copula, it is natural to choose the copula of g as an independent bivariate with the same marginal distributions as f [21].

Using information and entropy framework [19], one can use data moments as the dependency constraints for copula approximation. [21] use minimum information methods to operationalize the approximation in the class of copulas used. Notice that one can either specify on the underlying joint density or on the copula since we can transform from one to another if we know the margins. Minimum (relative) information copula is a copula that its relative information from the independence copula is minimal. We can view the minimum information copula as the most independent copula among the class of qualified copulas satisfying the data moment constraints.

Suppose that we have a multivariate data set and a regular vine structure for the involved random variables, we can find the minimum information copula for every edge in the regular vine structure. From the minimum information (bivariate) copulas we can build up the regular vine copula to approximate the multivariate distribution representing the random variables. [23] showed that a

regular vine copula is minimally informative if and only if all its bivariate copulas are minimally informative. This means we can get a minimum information regular vine copula from the minimum information copula for every edge in the regular vine.

3.2 Bivariate Minimum Information Copula

Let $X_1, ...X_n$ be n uniformly distributed random variables. In a practical use of copula modeling, researchers can transform any samples from random variables to uniformly distributed samples by taking the probability integral transformation. Let $h_1, ..., h_k : [0,1]^2 \to \mathbb{R}$ be k basis functions for which we specify the mean values $\alpha_1, ..., \alpha_k$ corresponding to these functions. Furthermore, we assume that h_i, h_j are linearly independent for $i \neq j$. [21] suggested to use the expectation constraints in the following form:

$$\alpha_l = \int_0^1 \int_0^1 h_l(x_i, x_j) c_e(x_i, x_j) dx_i dx_j, l = 1, .., k, \tag{3.3}$$

as the copula dependency constraints discussed in the previous section, where X_i and X_j, for $1 \leq i < j \leq n$, are the two uniform random variables joined by the edge e and c_e is the bivariate copula density for the edge e. This expectation form of constraint is flexible for capturing the shape of dependence in the data. For example, when $h_k(X_i, X_j) = X_i X_j$, it means that the rank correlation of a qualified copula should be α_k. Therefore, this expectation constraints can capture a wider range of correlations within the data.

[26] showed that there exists a unique minimum information copula satisfying the constraints in Equation (3.3) with the following copula density function:

$$\hat{c}_e(x_i, x_j) = d_1(x_i) d_2(x_j) \exp\left(\lambda_1 h_1(x_i, x_j) + ... + \lambda_k h_k(x_i, x_j)\right), \tag{3.4}$$

for some functions $d_1(.)$ and $d_2(.)$, and $\lambda_1, ...\lambda_k$ are the unknown Lagrange multipliers. These Lagrange multipliers also depend nonlinearly on $\alpha_1, ...\alpha_k$. Notice that $d_1(.)$ and $d_2(.)$ have no closed-form expression. Thus, $\hat{c}_e(x_i, x_j)$ has to be determined numerically. [23] gives the algorithm for determining $d_1(.)$, $d_2(.)$, and $\lambda_1, ...\lambda_k$.

How well does $\hat{c}_e(x_i, x_j)$ can approximate the true copula density $c_e(x_i, x_j)$ [21] has given the proof that by selecting sufficiently many basis functions one can approximate $\log(c_e(x_i, x_j))$ to any required level of precision by a linear combination of the basis functions $h_1, ..., h_k$.

We end this section by discussing about how to evaluate α_l, for $l = 1, .., .k$ from data. For the practical implementation, we calculate the sample mean of the random variable $h_l(X_i, X_j)$ as $\hat{\alpha}_l$ to approximate the α_l, for $l = 1, .., .k$. For an edge e in tree T_1, the conditioning set D_e is empty. We can use all observations in the data to approximate the sample mean of $h_l(X_i, X_j)$. However, for tree $T_i, i = 1, ..., n-1$ the conditioning set is no longer empty. Thus, we have to take into account the value of $\mathbf{X}_e$ when calculating the sample mean.

Using the previous notations in Section 2, see Equation (2.2) , the constraints for an edge e in tree $T_i, i = 1, ..., n-1$ can be written as

$$\alpha_l(\mathbf{x}_e) = \int_0^1 \int_0^1 h_l(u_{\mathbf{x}_e}, w_{\mathbf{x}_e}) c_{\dot{e}_1\dot{e}_2|D_e}\left(u_{\mathbf{x}_e}, w_{\mathbf{x}_e}|\mathbf{X}_e = \mathbf{x}_e\right) du_{\mathbf{x}_e} dw_{\mathbf{x}_e}, \tag{3.5}$$

for $l = 1, ..., k$.

Ideally, if we have a sizable sample of $U_{\mathbf{x}_e}$ and $W_{\mathbf{x}_e}$, we can calculate the sample means and use them as the approximation of $\alpha_l(x_e)$. However, in reality, we have only one realization of $U_{\mathbf{x}_e}$ and $W_{\mathbf{x}_e}$ for any point $\mathbf{X}_e = \mathbf{x}_e$. In this paper, we assume that the conditional copula $c_{\dot{e}_1\dot{e}_2|D_e}\left(u_{\mathbf{x}_e}, w_{\mathbf{x}_e}|\mathbf{X}_e = \mathbf{x}_e\right)$ does not depend on the value of $\mathbf{X}_e$. Thus, we treat an approximation of $\alpha_l(\mathbf{x}_e)$ as a constant:

$$\alpha_l = \int_0^1 \int_0^1 h_l(u_{\mathbf{x}_e}, w_{\mathbf{x}_e}) c_{\dot{e}_1\dot{e}_2|D_e}\left(u_{\mathbf{x}_e}, w_{\mathbf{x}_e}\right) du_{\mathbf{x}_e} dw_{\mathbf{x}_e}. \tag{3.6}$$

3.3 The Lagrange Multipliers Determination

Given random variables $h_l(X_i, X_j)$ and their expected values α_l, for $l = 1, ..., k$, the Lagrange multipliers $\lambda_1, ..., \lambda_k$ can be numerically determined by solving the equality constraints in Equation (3.5). [23] has suggested to numerically solve the discretized version of Equation (3.5) as follows. For a large enough integer m, we can discretize (X_i, X_j) into

$$(x_i^{(r)}, x_j^{(v)}) = \left(\frac{2r-1}{2m}, \frac{2v-1}{2m}\right), \text{for} \quad 1 \leq r, v, \leq m.$$

The integral in Equation (3.5) can be approximated by

$$\frac{1}{m^2} \sum_{r=1}^{m} \sum_{v=1}^{m} d_1(x_i^{(r)}) d_2(x_j^{(v)}) A(x_i^{(r)}, x_j^{(v)}) h_l(x_i^{(r)}, x_j^{(v)}), \tag{3.7}$$

where $A(x_i^{(r)}, x_j^{(v)}) = \exp(\lambda_1 h_1(x_i^{(r)}, x_j^{(v)}) + ... + \lambda_k h_k(x_i^{(r)}, x_j^{(v)}))$. The Lagrange multipliers is the root of the equation [23]:

$$\sum_{l=1}^{k} L_l^2(\lambda_1, ..., \lambda_k) = 0, \tag{3.8}$$

where

$$L_l(\lambda_1, ..., \lambda_k) = \frac{1}{m^2} \sum_{r=1}^{m} \sum_{v=1}^{m} d_1(x_i^{(r)}) d_2(x_j^{(v)}) A(x_i^{(r)}, x_j^{(v)}) h_l(x_i^{(r)}, x_j^{(v)}) - \alpha_l.$$

The elements in the two vectors $\left(d_1(x_i^{(r)}), r = 1, .., m\right)$ and $\left(d_2(x_j^{(v)}), v = 1, .., m\right)$ can be numerically determined by a simple algorithm suggested by [27]. Starting with any initial values of $d_1(x_i^{(r)})$ and $d_2(x_j^{(r)})$, we can iteratively evaluate

$\left(d_1(x_i^{(r)}), r = 1, .., m\right)$ and $\left(d_2(x_j^{(v)}), v = 1, .., m\right)$ until convergence by using the following two mappings:

$$d_1(x_i^{(r)}) \mapsto \frac{m}{\sum_{v=1}^{m} d_2(x_j^{(v)}) A(x_i^{(r)}, x_j^{(v)})},$$
$$d_2(x_j^{(v)}) \mapsto \frac{m}{\sum_{r=1}^{m} d_1(x_i^{(r)}) A(x_i^{(r)}, x_j^{(v)})}.$$

Finally, finding the root of Equation (3.8) can be accomplished by the Nelder-Mead simplex method [28].

4 Application to Energy Commodity Returns

4.1 Data

Table 1: Descriptive statistics and unit root test

Statistics	Crude oil	Natural gas	Gasoline	Heating oil
Mean	-0.000032	-0.000137	-0.000015	-0.000003
Median	0.000000	0.000000	0.000000	0.000000
Maximum	0.071267	0.116266	0.094046	0.049049
Minimum	-0.056742	-0.064681	-0.070189	-0.045013
Std. Dev.	0.010441	0.013675	0.010548	0.008847
Skewness	0.171172	0.710985	0.014087	-0.059824
Kurtosis	7.717443	7.993057	8.953872	5.813096
Jarque-Bera	2664.069*	3209.607*	4221.431*	944.0719*
ADF-Test				
None	-56.16297*	-57.27366*	-54.02022*	-55.60386*
Constant	-56.15364*	-57.26913*	-54.01083*	-55.59412*
Constant and trend	-56.15087*	-57.26685*	-54.00738*	-55.59616*
PP-Test				
None	-56.19851*	-57.35277*	-54.02605*	-55.57505*
Constant	-56.18915*	-57.36142*	-54.01659*	-55.56547*
Constant and trend	-56.18711*	-57.36161*	-54.01255*	55.56845*

We used daily data of four energy commodity futures prices including crude oil(C), natural gas(N), gasoline(G), and heating oil(H). These data were obtained from Thomson Reuters. The 2,859 energy commodity futures price observations were collected from 17 January 2006 to 30 December 2016, traded in the New York Mercantile Exchange (NYMEX). Then, we transformed energy commodity futures prices to log return and we checked all log return data are stationary by using Augmented Dickey-Fuller (ADF) and Phillip-Perron (PP) tests. The descriptive statistics and the unit root tests are shown in Table (1).

4.2 GARCH Models

[29] proposed the generalized autoregressive conditional heteroscedasticity (GARCH) model that relaxed the constant variance assumption of the innovations (ϵ_t) in conventional time series model. After that there are many variations of GARCH models that can be used to capture the volatility of price movement in the market. In this paper, we use the GARCH and exponential GARCH (EGARCH) model of [30]. We chose best fitted GARCH from three types of GARCH include GARCH, GJR-GARCH, and EGARCH based on lowest akaike information criterion(AIC). For a log return series (r_t), a GARCH model can generally be specified as

$$\begin{aligned} r_t &= \mu + \sum_{i=1}^{p} \phi_i r_{t-i} + \sum_{j=1}^{q} \psi_j \epsilon_{t-j} + \epsilon_t \\ \epsilon_t &= \sigma_t z_t \\ \sigma_t^2 &= \omega + \alpha_1 \epsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \end{aligned}$$

An EGARCH model can generally be specified as

$$\begin{aligned} r_t &= \mu + \sum_{i=1}^{p} \phi_i r_{t-i} + \sum_{j=1}^{q} \psi_j \epsilon_{t-j} + \epsilon_t \\ \epsilon_t &= \sigma_t z_t \\ \log(\sigma^2) &= \omega + \sum_{i=1}^{k} a_i \epsilon_{t-i}^2 + \sum_{j=1}^{l} b_j \log(\sigma_{t-j}^2) + \sum_{i=1}^{k} \gamma_i \left(\frac{|\epsilon_{t-i}|}{\sigma_{t-i}} - \sqrt{\frac{2}{\pi}} \right), \end{aligned}$$

where $\epsilon_t, t = 1, ..., T$ is the innovation process and z_t is assumed to be the skew student's t and student's t distributions. GARCH and EGARCH models for the log returns of energy commodity futures prices were estimated and the results are shown in Table (2). We obtained the standardized residuals for four log return series from GARCH and EGARCH models. These standardized residuals will be used as marginal distributions in the regular vine copula models for four energy commodity futures prices. However, EGARCH has some drawback such as incorrect conditions, no clear explanation which lead to misleading interpretation(see [31], [32]).

In order to guarantee that the marginals are uniformly distributed, we use the Kolmogorov-Smirnov test(KS-test) and we use Ljung-Box test to guarantee that the residuals are independent and identically distributed random variable(i.i.d) as shown in Table (3).

Table 2: ARMA-GARCH models

	Crude oil	**Natural Gas**	**Gasoline**	**Heating Oil**
Mean equation				
μ	-0.000276**	-0.00014	0.00013	-0.00002
ϕ_1	0.951031***	-1.27407***	0.61796***	-0.23643***
ϕ_2	0.016525***	0.38937***	-0.99968***	-0.44929***
ϕ_3	0.020726***	0.70122***	-	-0.86503***
ψ_1	-0.980883***	1.25744***	-0.61891***	0.21179***
ψ_2	-	-0.42430***	0.99885***	0.45908***
ψ_3	-	-0.71923***	-	0.85306***
Variance equation				
ω	-0.043347***	0.00000	-0.04220***	-0.02470***
a_1	-0.070283***	0.05461***	-0.04286***	-0.03473***
b_1	0.995345***	0.92986***	0.99559***	0.99745***
γ_1	0.087843***	-	0.07654**	0.07469***
Skewness	0.936700***	1.07324***	-	-
Shape	9.535924***	7.14716***	5.26904***	7.90749***
LL	9547.568	8486.483	9382.053	9851.077
AIC	-6.6736	-5.9304	-6.5585	-6.8853

Table 3: KS-test and Ljung-box test

	Crude oil	**Natural gas**	**Gasoline**	**Heating oil**
KS-test	0.4891***	0.4912***	0.4884***	0.4897***
Q(5)	17.215***	22.673***	9.5582***	9.5782***
Q(10)	19.624***	25.560***	11.557***	12.138
Q(15)	23.068***	30.844***	29.154***	14.953
Q(20)	28.282	34.816***	31.47***	18.413

4.3 Minimum Information Copula Approximation

Given the standardized residuals obtained from the previous section, we let C, N, G, and H denote the four variables for the standardized residuals of crude oil, natural gas, gasoline and heating oil, respectively. The technique of [33] is used to construct the Vine copula structure and we found that C-vine structure is appropriate for this data. Figure 2 illustrates the C-vine structure on 4 variables.

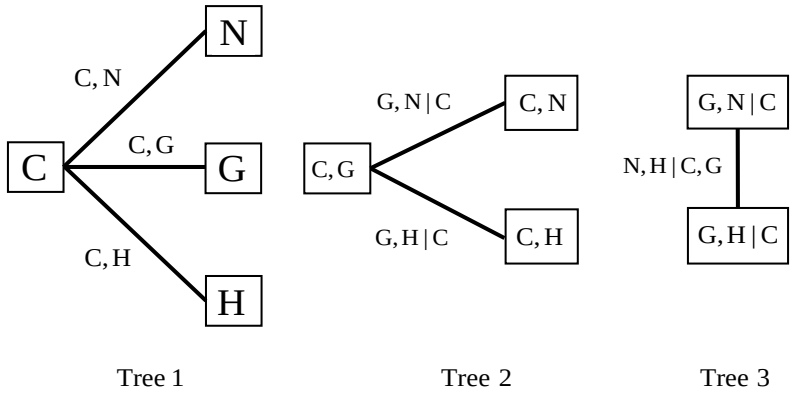


Figure 2: C-vine structure on 4 variables

To construct minimally informative copulas between each set of two adjacent variables in the C-vine structure. We must decide on which bases to take and how many discretization points to use in each case. The basis function could be chosen, starting with simple polynomials and moving to more complex ones, and include them until we are satisfied with our approximation. [23] did the intensive simulation study and found that the polynomial of degree six is optimal in the sense that increasing the power more than six will improve the approximation a little but will impose a lot of additional computational load.

Table 4: Constraints and parameter values for C_{CN}, C_{CG}, and C_{CH}

$h_i(C,N)$	α_l	λ_l	$h_i(C,G)$	α_l	λ_l	$h_i(C,H)$	α_l	λ_l
C^4N^2	0.07989	-0.6651	C^2G^2	0.1743	8.0669	C^2H^2	0.1475	-4.3102
CN^2	0.1834	2.2768	CG^2	0.2276	2.1432	CH^2	0.2024	5.5931
C^2N	0.1826	0.6543	C^2G	0.2274	4.5078	C^2H	0.2022	4.7538

In this paper, we select only three basis functions out of all possible polynomial basis functions $\{u^p w^q : 0 \leq p, q \leq 6\}$ that give the largest increase in the log-likelihood to reduce the burden of computational time. A grid size of 200×200

is chosen for all bivariate copula densities. Notice that, for three basis functions, the level of precision is about 10^{-6}. The optimal basis functions for all edges in tree 1 of C-vine structure are given in Table (4). For tree 2 and tree 3 of C-vine structure, the optimal basis functions, constraints and parameter values are given in Table (5). The surface plots for all bivariate copula densities of the C-vine copula are illustrated in Figure 3.

The log-likelihood of minimum information copula, obtained by summing the log-likelihoods of each of the component minimum information copulas, is 2,634.89. This is larger than that using the vine construction of bivariate t-copulas which the log-likelihood is 1,676.88. Same as, the results of [21] and [22] showed that the log-likelihood of minimum information copula is larger than the log-likelihood of t-copulas. The results are shown in Table (6).

Table 5: Constraints and parameter values for $C_{GN|C}, C_{GH|C}$, and $C_{NH|CG}$

$h_i(G,N)$	α_l	λ_l	$h_i(G,H)$	α_l	λ_l	$h_i(N,H)$	α_l	λ_l
G^4N^2	0.0774	-1.0253	GH	0.2852	2.7361	N^2H^4	0.0721	-0.2326
G^2N	0.1802	2.2834	G^2H	0.2005	-1.5334	NH^2	0.1732	1.1654
GN^2	0.1798	0.5094	GH^2	0.2011	4.5406	N^2H^3	0.0893	-0.0763

Table 6: Comparison between Vine copula and Minimum information copula

Types of copula	Variables	Log-likelihood
Vine Copula	$(C,N)-(C,G)-(C,H)$ $(G,N\|C)-(G,H\|C)$ $(N,H\|C,G)$	1,676.88
MI Copula	$(C,N)-(C,G)-(C,H)$ $(G,N\|C)-(G,H\|C)$ $(N,H\|C,G)$	2,634.89

4.4 Portfolio Optimization

To construct energy commodities portfolio , we use the Monte Carlo simulation to estimate VaR and CVaR. After that, the optimal portfolio is constructed under minimizing CVaR with respect to maximizing returns. First, we use minimum information copula to generate N sample size by inputting the joint minimum information copula densities to Algorithm 4 (see [21]). Second, we use simulates in first step for inverse functions of the probability distributions and use the mean and variance equations of the ARMA-GARCH model to get the N values of each variable at period $t+1$. Finally, optimal portfolio is given by:

$$\text{MIN CVaR} = E[r|r \leq r_\alpha],$$

subject to

$$r_i = w_1 r_{(1,t+1)} + w_2 r_{(2,t+1)} + ... + w_n r_{(n,t+1)},$$
$$w_1 + w_2 + ... + w_n = 1,$$
$$0 \leq w_i \leq 1,\ i = 1, 2, ..., n$$

where r_α is the lower α-quantile and $r_{i,t+1}$ is the return on individual asset at time $t+1$.

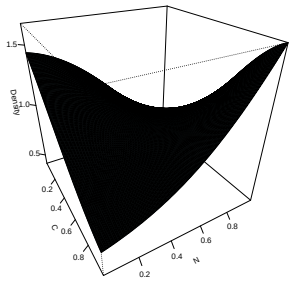
The optimal portfolio shown in Table (7). The results shows that most of the investment proportion are crude oil, while natural gas, gasoline, and heating oil have little of the investment proportion. In case of natural gas, it's prices have been declining for several years and are currently 50 percent off their 2008 peak and the costs to roll the futures contracts are very high. Even though the natural gas prices have been declining for several years, the supply of natural gas continues to increase. There are reasons for the oversupply situation. In case of gasoline, gasoline is seasonal so the prices are lowest in December and their highest price in April-May in preparation for the summer driving season. In case of heating oil, heating oil is seasonal so heating oil prices are increasing in winter according to heating oil demand and heating oil is an important alternative energy source for homes that lack access to natural gas.

Table 7: Optimal investment proportion of energy portfolio with minimum risk (CVaR 5)

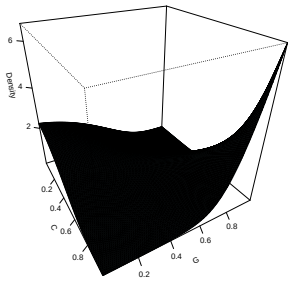
Portfolios	**Investment proportion**				**Returns**	**Risk**
	Crude oil	Natural gas	Gasoline	Heating oil		
1	0.84500	0.06634	0.03396	0.05470	0.74154	-0.26929
2	0.86184	0.05542	0.03271	0.05003	0.74665	-0.26909
3	0.87868	0.04515	0.02749	0.04869	0.75177	-0.26864
4	0.89586	0.03702	0.02656	0.04056	0.75689	-0.26794
5	0.91308	0.02933	0.02452	0.03307	0.76201	-0.26703
6	0.93004	0.02007	0.01935	0.03054	0.76713	-0.26569
7	0.94716	0.01197	0.01463	0.02624	0.77225	-0.26400
8	0.96423	0.00325	0.01132	0.02120	0.77737	-0.26180
9	0.98193	0.00000	0.00671	0.01136	0.78249	-0.25878
10	1.00000	0.00000	0.00000	0.00000	0.78761	-0.25466

5 Conclusion

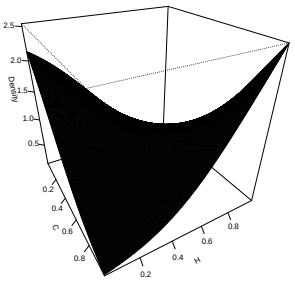
In this paper, we investigated the optimal portfolio of energy market with minimally informative copula. Empirical results showed that ARMA-GARCH and ARMA-EGARCH with student-t distribution and skew student-t distribution are appropriate to estimate parameter. Then, we approximate copula parameter by using minimally informative copula. After we obtained minimally informative



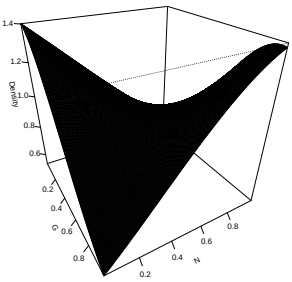
(a) The minimally informative copula between C and N



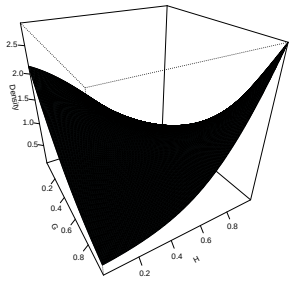
(b) The minimally informative copula between C and G



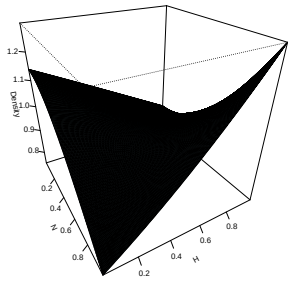
(c) The minimally informative copula between C and H



(d) The minimally informative copula between G and N given C



(e) The minimally informative copula between G and H given C



(f) The minimally informative copula between N and H given C and G

Figure 3: Surface plots of the minimally informative copulas for C-vine structure

copula parameters, we estimated VaR and CVaR based on 10%, 5%, 1% levels, respectively. Finally, the optimal portfolios suggest that crude oil has the large proportion, while gasoline, natural gas, and heating oil should be in small proportion in portfolio.

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