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Technical Efficiency in Rice Production at Farm Level in Northern Thailand: A Stochastic Frontier with Maximum Entropy Approach

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Abstract : In this paper, we apply a maximum entropy estimation method that is used as substitute for maximum likelihood method approach. This methodology was applied to sticky rice production data from Northern area, Chiang Mai province, Thailand. The mean square error (MSE) and standard error of parameters confirm that the method of GME in stochastic frontier model is more accurate than the conventional stochastic frontier model. In particular, the standard stochastic frontier model underestimate the technical inefficiency scores for the lower rankings and overestimate for the higher rankings. In this study, 95% of the sticky rice farmers were found to have high inefficiencies and most of them

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had technical inefficiencies between 0.6 and 0.8. These findings suggest that a considerable amount of productivity is lost due to inefficiency.

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1 Introduction

For many decades, rice is a vital to the national economic development of many countries. Especially in Thailand, rice is not only the economic crop of Thailand agriculture but it also plays the important role of Thailand export products. In 2016, Office of Agricultural Economics, reported that 58.427 million rais (93,483.2 million square metres) were planted to rice and produced 25.578 million tonnes and rice products contributed more than $6,600 billion to the internationally exported annually. The reason is that rice is not only a subsistence crop but also a cash crop for Thai farmers. It was estimated that approximately 4 million people are highly dependent on rice farms [1].

The development of the rice farming, relying on resources and technique, serves as an important role of the rice production. As for research regarding technical efficiency on the rice production, efficiency improvement is often concerned as one of a crucial targets behind social and economic policies and reforms (see, [2]). Concerning about rice production in Thailand, many economists have raised the question of the technical efficiency due to the lack of knowledge in production. Thus, stochastic frontier model (SFM) is become one of the most successful method in agricultural production analysis. In general, SFM is a parametric method used to investigate technical efficiency and productivity of the production function, such as Cobb-Douglas, Leontief, translog etc. A previous studied related to a stochastic frontier model on rice, such as [3] discussed the productive efficiency of jasmine rice in Thailand by using a sample selection procedure in stochastic frontier models. [4] applied data envelopment analysis (DEA) and SFM to determine the technical efficiency on certified organic Jasmine rice farms in Thailand. The result indicate that SFM and DEA (CRS) are more consistent than those from SFM and DEA (VRS) for efficiency measurement.[5] measured technical efficiency of the rice production in China. Moreover, the method of SFM is applied in various areas, such as [6, 7, 8, 9, 10], and among others.

Technically speaking, SFM needs the definition of a specific functional form for the production function and for the inefficiency error component. Literally, The method of maximum likelihood was applied to estimates all the parameters in the model (see, [11, 12, 13]). Another choice of estimation procedure is a non-parametric method [14], such as a quantile estimation, which is a semi-parametric estimation using a concept of Least absolute deviations method (LAD), The method minimizes the sum of absolute errors. The LAD estimate also arises as the ML estimate if the errors have a Laplace distribution (see,[10, 15, 16, 17]).

In this paper, we measure and investigate factors affecting technical efficiency of rice production in Northern of Thailand. We applied the stochastic frontier method to farm-level survey data of rice farms in Chiang Mai provinces. Alternatively, we used the method of maximum entropy (ME) to estimate the parameter in a classical stochastic frontier. The principle of maximum entropy is a postulate stating that, subject to known constraints, the probability distribution which best represents the current state of knowledge is the one with maximum entropy. A related works on ME in econometrics, For instance, [18, 19, 20, 21, 22, 23, 24, 25]. Specially, the ME method has the following properties ([26]) first, ME works well when the sample size is small and the covariates are highly correlated. Second, It avoids strong parametric assumptions and works well when the matrix is ill conditioned. Finally, ME method is more efficient.

The remainder of the paper is structured as follows. Section 2 gives the back ground of maximum entropy, In section 3, we describe a methodology and data. Section 4 reports the empirical results, and final section gives conclusions.

2 Theoretical Background

[20] defines Entropy as expected information. Entropy represents what we expect to learn from data on average. Technically, entropy is a measure of uncertainty of a single random variable. Let $\mathbf{A} = \{a_1, a_2, ..., a_M\}$ be a finite set with a corresponding probability distribution function $\mathbf{p}$. In communication theory, [27] developed Hartley's formula for measuring the amount of information needed to fully characterize all of the elements in A. The formula is $I(\mathbf{A}_M) = \log_2 M$. Shannon [28] developed the information criterion from Hartley's formula in the context of communication process called Shannon's information. The Shannon's information is defined as

$$h(a_i) = h(p_i) \equiv \log_2 \frac{1}{p_i} \quad i = 1, ..., M. \tag{2.1}$$

Shannon's entropy representing the expected information of an outcome is then defined as

$$H(\mathbf{p}) \equiv \sum_{i=1}^{M} p_i \log_2 \frac{1}{p_i} = E[\log_2(1/p(X))], \tag{2.2}$$

where X is the random variable with probability distribution $\mathbf{p}$. The entropy (information criterion) measures the uncertainty or informational content of X implied by $\mathbf{p}$.

In the context of Econometrics, we can view $H(p)$ as a measure of the economic system uncertainty. Technically, researchers never knows the true underlying values that characterizes the economic system. Hence, they may incorporate their knowledge of the system in estimating the unknown parameters of the system. This knowledge usually can be represented by some global macro-level quantities such as moments. Researchers could select the parameters of the system such that the entropy of the system is maximized while retaining the moment conditions.

Maximum entropy (ME) principle was proposed by [29] to address the question of how to draw inferences from limited and insufficient data. The ME principle was applied in many fields by a large number of researchers. In econometrics, where researchers are concerned about an estimation of unknown distribution of a random variable by imposing the minimal assumption on the underlying likelihood function, the principle was widely applied to solve such a problem (see, e.g.[30]. The rationale behind the ME principle was provided by [31] through the entropy concentration theorem. The theorem states that among the distributions that satisfy the observed-data moments, a significantly large portion of these distributions are concentrated sufficiently close to the one of maximum entropy. Readers are referred to [30] for more details about Entropy Econometrics.

2.1 GME Stochastic Frontier Model

The stochastic frontier model (SFM) is a widely used model to measure inefficiency of production units. [11], [32], and [33] are independently developed the SFM for a cross-section production data. Technically, SFM is just a linear regression model with composed (of two) error components. It has a two-sided error component as in the linear regression model capturing random variation of the production frontier and an additional one-sided error component that measures inefficiency relative to the production frontier. In conventional estimation, researchers have to specify the distribution assumption for those two random components. Thus, it requires a correct model specification and complete data in order to make a valid estimation and testing.

Another alternative for inefficiency measurement is data envelopment analysis (DEA) [34]. DEA does not require any ad hoc assumptions about the inefficiency distribution and also a functional form relating inputs and outputs. However, it has some limitations such as statistical hypothesis testings are difficult and any measurement errors or other noise can cause significant problems for inefficiency estimation.

In this paper, we use the generalized maximum entropy (GME) approach for stochastic frontier model. GME approach has the advantage over conventional method, since GME does not require an assumption on the inefficiency component of stochastic frontier model. Another advantage of GME estimation is that it also works very well with a small sample size. The advantage of GME estimation over DEA is that GME approach considers stochastic noise in data and is easier to perform the statistical testing of hypotheses. GME approach gives a potential alternative measure of inefficiency that exhibits the strengths of both SFM and DEA.

We consider a stochastic frontier model of the following form:

$$y_i = x_i'\boldsymbol{\beta} + \epsilon_i, \quad \epsilon_i = v_i - u_i, \quad u_i \geq 0, \tag{2.3}$$

where $i = 1, 2, ..., N$ indicates production units, y_i is log of output, x_i is a column vector of function of inputs, $\boldsymbol{\beta}$ is a column vector of parameters, v_i is a two-sided error component, and u_i is a one-sided error component capturing inefficiency of production unit. Some common functional forms for production functions are listed and discussed in [35]. In this paper, we assume the Cobb-Douglas production function [36].

The GME method is based on the maximum entropy principle [37]. This method treats both the parameter vector β, the errors v_i and u_i as discrete random variables with bounded support. In line with tradition, we assume that the set of true (unknown) parameters is bounded: $\beta \in B$ where B is a convex set. The probability distribution corresponding to the discrete random variables are approximated by maximizing the Shannon entropy under data-consistency constraints.

Let each element (k) of the parameter vector β be bounded below by $\underline{z}_k$ and above by $\bar{z}_k$:

$$B = \{\beta \in \mathcal{R}^k | \beta_k \in (\underline{z}_k, \bar{z}_k), k = 1, 2, ..., K\}. \tag{2.4}$$

Let $\boldsymbol{z}_k \equiv \{\underline{z}_k, \cdots, \bar{z}_k\} = \{z_{k1}, \cdots, z_{kM}\}$ denote the support space of component β_k of $\boldsymbol{\beta}$. We denote the probability distribution of each element β_k by $\boldsymbol{p}_k = (p_{k1}, \ldots, p_{kM})'$. Similarly, the two-sided error v_i is assumed to be a discrete random variable with support $\{r_1, \cdots, r_J\}$ and probability distribution $\boldsymbol{w}_i = (w_{i1}, \ldots, w_{iJ})$. Finally, the one-sided error u_i is assumed to be a discrete random variable with support $\{q_1, \cdots, q_J\}$ and probability distribution $\Phi_i = (\phi_{i1}, \ldots, \phi_{iJ})$. In the absence of prior knowledge about the possible values of support spaces, researchers can define the support space of component β_k to be centered on zero with wide range. For the support space of one-sided error, we define the support space with zero lower bound. For the support space of two-sided error component, we may use the three-sigma rule, as suggested by [30], to establish the lower and upper bounds on the error component.

According to the ME principle, the probability distributions $\boldsymbol{p}_k$, $\boldsymbol{w}_i$ and Φ_i can be chosen to maximize the following Shannon entropy function,

$$H(p, w, \phi) = -\sum_{k=1}^{K}\sum_{m=1}^{M} p_{km} \log p_{km} - \sum_{i=1}^{n}\sum_{j=1}^{J} w_{ij} \log w_{ij} - \sum_{i=1}^{n}\sum_{j=1}^{J} \phi_{ij} \log \phi_{ij}, \tag{2.5}$$

under the normalization constraints

$$\sum_{m=1}^{M} p_{km} = 1, \quad k = 1, \ldots, K$$

$$\sum_{j=1}^{J} w_{ij} = 1, \quad i = 1, \ldots, n$$

$$\sum_{j=1}^{J} \phi_{ij} = 1, \quad i = 1, \ldots, n$$

and some data-consistency constraints

$$y_i = \sum_{k=1}^{K} x_{ik} \sum_{m=1}^{M} p_{km} z_{km} + \sum_{j=1}^{J} w_{ij} r_j - \sum_{j=1}^{J} \phi_{ij} q_j, \quad i = 1, \ldots, n. \tag{2.7}$$

The optimal solution $\widetilde{\boldsymbol{p}_k}$, $\widetilde{\Phi_i}$ and $\widetilde{\boldsymbol{w}_i}$ yield the point estimates

$$\widetilde{\beta}_k = \sum_{m=1}^{M} \widetilde{p}_{km} z_{km}, \tag{2.8}$$

$$\widetilde{u}_i = \sum_{j=1}^{J} \widetilde{\phi}_{ij} q_j, \tag{2.9}$$

and

$$\widetilde{v}_i = \sum_{j=1}^{J} \widetilde{w}_{ij} r_j. \tag{2.10}$$

Finally, technical inefficiency (TI) can be measured by $\exp(-\widetilde{u}_i)$ for each production unit i.

3 Data and Formulation

3.1 Data

In this paper, we used a survey data which was collected in the crop year 2014 by interviewing rice farmers in Chiang Mai province, a province located in the north of Thailand where farmers have greater access to irrigation. A total of 125 farmers were interviewed of sticky rice production. The data consists of four variables, where the dependent variable is a quantity of rice in kilogram per rai, the explanatory variables are amount of fertilizer and seed in kilogram per rai and the number of labor per day. This data might suffer from measurement errors due to the lack of precise input and output records for each farmer. Mostly, farmers did not precisely record their inputs used in their rice productions. The interviewers can only ask them to approximate their inputs used in their farms. Therefore, we expect to have the measurement errors in this data set.

3.2 Model Formulation

The rice production frontier is specified as

$$\log(output_i) = \beta_0 + \beta_1 \log(fertilizer_i) + \beta_2 \log(seed_i) + \beta_3 \log(labor_i) + v_i - u_i, \tag{3.1}$$

where $\log(output_i)$ is the rice output, $\log(fertilizer_i)$ is the amount of fertilizer, $\log(seed_i)$ is the amount of seed, and $\log(labor_i)$ is the number of labor per day, for each farmer i. Notice that all variables are in logarithm form.

In order to implement the GME method, we have to define the support spaces for the error components and the parameters. The support space for each coefficient $\beta_k, k = 0, 1, 2, 3$ were selected based on estimates obtained from conventional SFM. We chose the number of elements for supports equal three. A simple strategy to define the support space is to select the support space to be centered on zero with wide range. For example, to define the support space for the parameter of log(*seed*), we knew from the conventional SFM model that the estimated parameter is 0.046 with standard error of 0.050. Therefore, to cover the wide range we select the lower and upper bounds for the support to be ten times of 0.050 and make it centered on zero. Notice that the choice of support space is subject to researchers judgment.

The chosen support were set to $(-10, 0, 10)$, $(-0.5, 0, 0.5)$, $(-0.5, 0, 0.5)$, and $(-0.6, 0, 0.6)$ for intercept, fertilizer, seed, and labor, respectively. The supports for the two-sided errors are chosen to be $(-0.8, 0, 0.8)$ the same for all observations. For the one-sided errors (inefficiency component), the lower bound of supports were set to zero and the upper bound were set to 0.8. According to the upper bound of 0.8, the minimum efficiency can be reached at $\exp(-0.8) = 44.9\%$. Notice that the minimum zero efficiency is only obtained when the upper bound of support approaches infinity. We remark here that the mean efficiency of production unit depends on the choice of one-sided error supports. The optimal choice for the supports has to be investigated in further study.

3.3 Empirical Results and Discussion

In this section, we present the estimated parameters and the technical inefficiency from both conventional SFM and GME-SFM (Generalized Maximum Entropy Stochastic Frontier Model). For the conventional SFM, we made the usual assumptions of half-normal and normal distributions for one-sided and two-sided errors, respectively. Table 1 shows parameter estimates form both models. In the conventional SFM, the variance of technical inefficiency $\exp(-u_i)$ was bigger than the GME-SFM. The mean square errors (MSE) of the production frontiers were also computed to compare the goodness-of-fit between two approaches of estimation. As we expected, the MSE of GME approach was slightly lower than the conventional approach.

In conventional SFM, the coefficients of all explanatory variables were not statistically significant except for the constant coefficient. This might suggest that the conventional SFM suffer from the measurement errors in the data as previously discussed. As we expect, the better results for rice production frontier could be achieved by GME approach. In GME-SFM, the coefficient of variable labor was statistically significant and had positive sign. The coefficient of labor can be interpreted as rice production elasticity for labor. GME approach is more robust to the measurement errors when compares with the conventional method.

Table 1: Parameter estimated in the conventional Stochastic Frontier model (SFM) and GME Stochastic Frontier model

Input variables	SFM		GME-SFM	
	Estimates	StdErr	Estimates	StdErr
Constant	6.929*	0.149	6.728*	0.126
$\log(fertilizer)$	-0.027	0.020	-0.009	0.018
$\log(seed)$	0.046	0.050	0.004	0.040
$\log(labor)$	-0.003	0.074	0.181*	0.051
σ_u	0.468	-	-	-
σ_v	0.142	-	-	-
Mean of inefficiency	0.723	0.159	0.683	0.051
MSE	5.472	-	5.169	-

Note: $*$ indicates a significance level of 0.05.

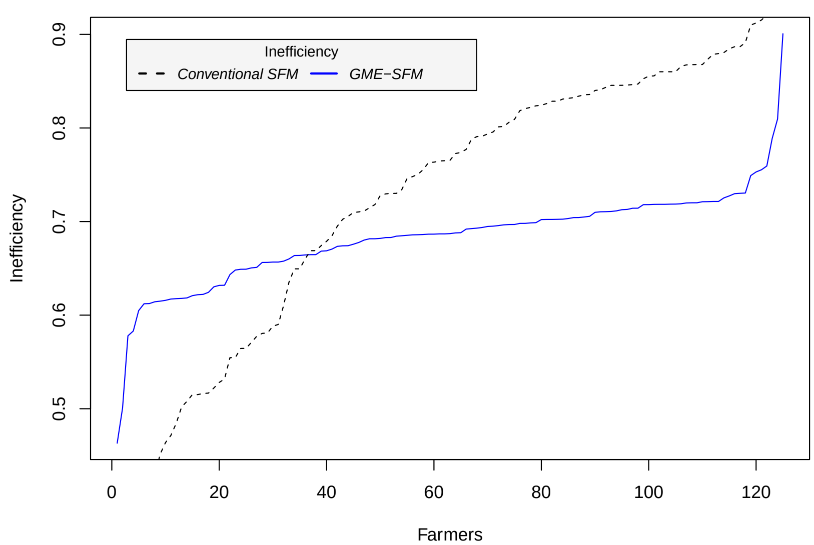

Figure 1: Technical Inefficiency Comparison between the conventional SFM and GME-SFM

Figure 1 illustrates the cumulative technical inefficiency distribution based on SFM and GME-SFM. The two shapes are different where the distribution from GME-SFM shows steeper curve in both upper and lower tails but smoother curve in the middle. The line represents technical inefficiencies GME-SFM model, while the dotted line illustrates technical inefficiencies of the conventional SFM models. Given the specification of GME-SFM model, all farmers have a range of 0.10 to 0.90 inefficiency scores with an average efficiency 0.683, while the technical inefficiency of all farmers are from 0.45 to 0.99 according to conventional stochastic frontier model, and the average technical efficiency is 0.723. Obviously, the traditional SFM of technical inefficiency overestimate the technical inefficiencies.

4 Conclusions

In this paper, we have applied the classical stochastic frontier model using maximum entropy estimation method that is used as substitute for maximum likelihood method thereby possibly generalizing the use of the GME-SFM approach. GME-SFM allows the models of interest to incorporates as much (or as insufficient) information as there is available. The mean square error (MSE) and standard error of parameters confirmed that the method of GME in stochastic frontier model is more appropriate than the conventional stochastic frontier model.

We used the GME-SFM method to investigate the sticky rice production associated with seed, labor and fertilizer. The results showed that GME-SFM approach had better performance than traditional stochastic frontier model which overestimate the technical efficiency in this study. The graph of the SFM and GME-SFM model in term of technical inefficiency suggested that SFM underestimated the technical inefficiency scores for the lower rankings and overestimated for the higher rankings.

This study also found some significant results which showed that 95% the sticky rice farmers had the production inefficiencies are more than 0.5 from the maximum scale of 0.9 inefficiency. This suggested that policy makers should seriously improve the efficiency of the sticky rice production

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