



## Maximum Entropy Quantile Regression with Unknown Quantile

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**Abstract :** Selecting quantile level in quantile regression model has been problematic for some researchers. Thus, this paper extends the analysis of quantile regression model by regarding its quantile level as an unknown parameter, as it can improve the prediction accuracy by estimating an appropriate quantile parameter for regression predictors. We develop a primal generalized entropy estimation to obtain the estimates of coefficients and quantile parameter. Monte Carlo simulations for quantile regression models with unknown quantile show that the primal GME estimator outperforms other alternatives like least squares and maximum likelihood estimators when the true quantile parameter is assumed to deviate from median. Finally, our model is applied to study the effect of oil price on stock index to examine the performance of the model in real data analysis.

**Keywords :** quantile regression; generalized maximum entropy; unknown quantile parameter.

**2010 Mathematics Subject Classification :** 62P20; 91B84.

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## 1 Introduction

Quantile regression is one of the famous regression techniques used in statistics and econometrics. In contrast to linear regression model which estimates the approximate conditional mean of the response variable given certain values of the independent variables, quantile regression aims at estimating either the conditional median or other quantiles of the response variable. One advantage of quantile regression, relative to the linear regression, is that its estimates are more robust against outliers in the response measurements. The main feature of quantile regression goes beyond that. However, we suspect that among all the quantile levels, which one is fit to the data? The traditional quantile regression model will be useful if we want to focus on some or particular quantile level. But if we estimate the quantile model at a given range of quantile, say  $0.1, 0.2, \dots, 0.9$ , we will obtain 9 different regression results, one for each quantile level. The problem that follows is how to interpret each result and how to find the best fit solution. In addition, it is sometimes difficult to answer the question which quantile would be the most likely one to extract the most information from the data. This question can be easily solved by considering the quantile level as a parameter would be an appropriate way to solve this problem. Recently, this problem has received increasing attention in the literature. For instance, Tu, Wang, and Sun [1] also considered the quantile level as an unknown parameter and estimated it with other parameters in the model. In other words, they estimated the quantile parameter from the data and allowed the data to tell their own story. Nevertheless, their purpose is to estimate the quantile parameter by using Bayesian adaptive Lasso on the maximum entropy and the flat prior,  $\pi(\tau) = 1$ , was placed for quantile parameter  $\tau$ . The idea behind this method is that it is defined from the information entropy of the distribution of probabilities  $p$  as an Asymmetric Laplace distribution (ALD) density,  $f(y|\beta)$ , and maximizing entropy measure subject to two moment constraints, therefore

$$f_{ME}(y|\beta) = \max_f - \int f(y|\beta) \log f(y|\beta) dy \quad (1.1)$$

subject to

$$\begin{aligned} E|y - x\beta^\tau| &= c_1, \\ E(y - x\beta^\tau) &= c_2, \end{aligned}$$

where  $\int f(y)dy = 1$ ;  $c_1$  and  $c_2$  are known constants. Although this entropy estimation in quantile regression has already been proposed and it has been extended by Tu, Wang, and Sun [1] to estimate quantile regression regarding the quantile as an unknown parameter, it still adheres to the strong ALD assumption on the entropy measures. Thus, it is greatly desirable to expand the flexibility of entropy estimation by relaxing the ALD in the objective function. Moreover, the appropriate prior for quantile parameter is what we are also concerned and it might be difficult to specify. If we specify an inappropriate prior for this parameter, it would affect the rest of parameters in the model. Thus, this motivates us to develop an

entropy estimation for quantile regression model without assuming the ALD. Our purpose is to estimate the quantile parameter by taking expectations of random variables with  $M$  support value  $h_m$ . Thus, quantile parameter can be expressed as entropy by

$$\tau = \sum_m g_m h_m, \quad (1.2)$$

where  $g_m$  is the  $M$ -dimensional unknown probability of quantile parameter and  $h_m$  is the  $M$ -dimensional support for  $\tau$ .

As we mentioned above, the Generalized Maximum Entropy (GME) estimator of Golan et al. [2] is employed as an alternative estimator for quantile regression with unknown quantile level. In this study, we will show the performance of our method by comparing with Least Square (LS) and Maximum Likelihood (ML) estimators. Finally, our method is applied to real data to analyse the effect of oil price on the stock markets of Thailand, Indonesia, and the Philippines which are the emerging stock markets in Asia.

The rest of this paper is organized as follows. The quantile regression with unknown quantile model is presented in Section 2. In Section 3, the parameter estimation is explained. Section 4 is on simulation studies to compare the performance of the various methods under consideration. In Section 5 we use real data for empirical estimation of quantile regression with unknown quantile model. Final remarks are provided in Section 6.

## 2 Quantile Regression with Unknown Quantile Model

To explain the basic concept of quantile regression, consider the following model:

$$y_t = x'_{i,t} \beta_i^\tau + \varepsilon_t ; i = 1, \dots, k \text{ and } j = 1, \dots, n \quad (2.1)$$

where  $x'_{i,t}$  is  $n \times k$  independent variables,  $\beta_i^\tau$  is  $1 \times k$  vector of coefficients at given  $\tau$ . Note that unlike the traditional quantile regression, we consider the quantile  $\tau$  as an unknown parameter and estimate it jointly with other parameters.  $\varepsilon_t$  is the error which does not assume any distributions. Thus,  $\tau^{th}$  ( $0 < \tau < 1$ ) conditional quantile of  $y_t$  given  $x'_{i,t}$  is simply

$$Q_y(\tau | x) = x'_{i,t} \beta_i^\tau \quad (2.2)$$

In this study, we aim to relax the assumption in the conventional quantile regression by regarding  $\tau$  as the unknown parameter. In the traditional estimation of quantile regression model, the focus is on using ordinary least squares (OLS) with a general technique for estimating families of conditional quantile functions (see,

[3]). The  $\tau$  specific coefficient vector  $\beta^\tau$  can be estimated by minimizing the loss function:

$$\beta^\tau = \arg \min_{\beta(\tau)} \sum_{j=1}^n \rho^\tau(y_t - x_{i,t}\beta_i^\tau), \quad (2.3)$$

where  $\rho^\tau(L) = L(\tau - I(L < 0))$  is called the check function.  $L$  is a loss function which is corresponding to  $y_t - x_{i,t}\beta_i^\tau$  and  $I(\cdot)$  is the indicator function. In addition, the quantile regression model can be estimated by maximizing the likelihood based on the asymmetric Laplace density (ALD):

$$L(\beta^\tau, \sigma | y) = \frac{\tau(1-\tau)}{\sigma} \exp\left(-\sum_{t=1}^T \rho^\tau\left(\frac{y_t - x_{i,t}\beta_i^\tau}{\sigma}\right)\right), \quad (2.4)$$

where  $\sigma$  is a nuisance parameter. Note that, the maximization of the likelihood in Eq.(2.4) with respect to the parameter  $\beta^\tau$  is equivalent to the minimization of the objective function in Eq.(2.3). (see, [4]).

### 3 Parameter Estimation

In this section, we discuss an estimation method for quantile regression with unknown quantile model based on the primal generalized entropy estimation (GME). This estimator transforms the estimated parameter of the model to be described by a discrete probability distribution defined on a certain interval or support bound. We, then maximize these entropy probabilities of the unknown parameters and error term subject to the constraints imposed by the data and subsequently recover estimates of these unknown parameters. Mittelhammer et al. [5] suggested that GME estimation is the alternative way to avoid making any parametric assumptions.

Let us discuss the concept about the entropy approach. The maximum entropy concept consists of inferring the probability distribution that maximizes information entropy given a set of various constraints. Let  $p_k$  be a proper probabilities of the random variable with possible  $k$  outcomes. Shannon [6] developed his information criteria and proposed a classical entropy, that is

$$H(p) = -\sum_{k=1}^K p_k \log p_k, \quad (3.1)$$

where  $\sum_{k=1}^K p_k = 1$ . The entropy measures the uncertainty of a distribution and

reaches a maximum when  $p_k = \frac{1}{K}$ .

This entropy concept is applied in the present model by generalizing the maximum entropy as the inverse problem in the quantile regression framework. Following

Jaynes [7] on the maximum entropy principle; out of all those distributions consistent with the data-evidence, we choose the one that maximizes Eq.(3.1) and thus maximizes the missing information.

In this maximization problem, we extend the estimation steps which were presented in Pipitpojanakarn. et al. [8]. For the point estimates  $\beta^\tau$ , one can view these unknown parameters as expectations of random variables with  $M$  support value for each estimated parameter value ( $k$ ),  $Z = [z_1, \dots, z_K]$  where  $z_k = [\underline{z}_{k1}, \dots, \bar{z}_{km}]$  for all  $k = 1, \dots, K$ . Note that  $\underline{z}$  and  $\bar{z}$  denote the lower bound and upper bound, respectively, of each support  $z_k$ . Thus parameter  $\beta_i$  can be expressed as

$$\beta_i^\tau = \begin{bmatrix} \underline{z}_{11} & 0 & \bar{z}_{1m} \\ \underline{z}_{21} & 0 & \bar{z}_{2m} \\ \vdots & \vdots & \vdots \\ \underline{z}_{k1} & 0 & \bar{z}_{km} \end{bmatrix} \cdot \begin{bmatrix} \underline{p}_{11} & 0 & \bar{p}_{1m} \\ \underline{p}_{21} & 0 & \bar{p}_{2m} \\ \vdots & \vdots & \vdots \\ \underline{p}_{k1} & 0 & \bar{p}_{km} \end{bmatrix} \quad (3.2)$$

$$\beta_k^\tau = \sum_m p_{km} z_{km}, \quad (3.3)$$

where  $p_{km}$  are the  $M$  dimensional estimated probability distribution defined on the set  $z_{km}$ . Next, similar to the above expression,  $\varepsilon_t$  is also constructed as the mean value of some random variable  $v$ . Each  $\varepsilon_t$  is assumed to be a random vector with finite and discrete random variable with  $M$  support value,  $v_t = [v_{t1}, \dots, v_{tM}]$ . Let  $w_t$  be an  $M$  dimension proper probability weights defined on the set  $v_t$  such that

$$\beta_i^\tau = \begin{bmatrix} \underline{v}_{11} & 0 & \bar{v}_{1M} \\ \underline{v}_{21} & 0 & \bar{v}_{2M} \\ \vdots & \vdots & \vdots \\ \underline{v}_{T1} & 0 & \bar{v}_{TM} \end{bmatrix} \cdot \begin{bmatrix} \underline{w}_{11} & 0 & \bar{w}_{1M} \\ \underline{w}_{21} & 0 & \bar{w}_{2M} \\ \vdots & \vdots & \vdots \\ \underline{w}_{T1} & 0 & \bar{w}_{TM} \end{bmatrix} \quad (3.4)$$

$$\varepsilon_t = \rho_\tau \sum_m v_{tm} w_{tm}. \quad (3.5)$$

Note that the study considers the quantile level as an unknown parameter, therefore,

$$\tau = \sum_m g_m h_m, \quad (3.6)$$

thereby obtaining

$$p_\tau(\varepsilon_t) = \varepsilon_t \left( \sum_m g_m h_m - I(\varepsilon_t < 0) \right) \quad (3.7)$$

Using the reparameterized unknowns  $\beta_k^T$ ,  $\tau$ , and  $\varepsilon_t$ , one can rewrite equation as

$$\begin{aligned}
 Y_t &= \sum_m p_{1m} z_{1m}(x'_{1,t}) + \cdots + \sum_m p_{Km} z_{Km}(x'_{K,t}) \\
 &+ \sum_m v_{tm} w_{tm} \left( \sum_m g_m h_m - I(\sum_m v_{tm} w_{tm} < 0) \right) \tag{3.8}
 \end{aligned}$$

where the vector support  $z_{km}$ ,  $v_{tm}$  and  $h_m$  are convex set that is symmetric around zero with  $2 \leq M < \infty$ . Then, the Generalized Maximum Entropy (GME) estimator for this model can be constructed as

$$\begin{aligned}
 H(p, w, g) &= \arg \max_{p, w, g} \{H(p) + H(w) + H(g)\} \\
 &\equiv - \sum_k \sum_m p_{km} \log p_{km} - \sum_t \sum_m w_{tm} \log w_{tm} - \sum_m g_m \log g_m \tag{3.9}
 \end{aligned}$$

subject to

$$\begin{aligned}
 Y_t &= \sum_m p_{1m} z_{1m}(x'_{1,t}) + \cdots + \sum_m p_{Km} z_{Km}(x'_{K,t}) \\
 &+ \sum_m v_{tm} w_{tm} \left( \sum_m g_m h_m - I(\sum_m v_{tm} w_{tm} < 0) \right) \tag{3.10}
 \end{aligned}$$

$$\sum_m p_{km} = 1, \quad \sum_m w_{tm} = 1, \quad \sum_m g_m = 1, \tag{3.11}$$

where  $p_{km}$ ,  $w_{tm}$  and  $g_m$  are on the interval  $[0, 1]$ . For simple explanation, suppose we have one independent variable ( $k = 1$ ), this optimization problem can be solved using the Lagrangian method which takes the form of

$$\begin{aligned}
 L &= H(p, w, g) + \lambda'_1 \left( Y_t - \sum_m p_{1m} z_{1m}(x'_{1,t}) - \sum_m v_{tm} w_{tm} \left( \sum_m g_m h_m - \right. \right. \\
 &\left. \left. I(\sum_m w_{tm} w_{tm} < 0) \right) \right) + a'(1 - \sum_m p_{1m}) + b'(1 - \sum_m w_{tm}) + c'(1 - \sum_m g_m) \tag{3.12}
 \end{aligned}$$

where  $\lambda'_1$ ,  $a'$ ,  $b'$  and  $c'$  are the vectors of Lagrangian multipliers corresponding to the number of constraints. Thus, the resulting first-order conditions are

$$\frac{\partial L}{\partial p_{1m}} = -1 - \log(p_{1m}) - \sum_m \lambda_{1m} z_{1m}(x'_{1,t}) - a_t = 0, \tag{3.13}$$

$$\begin{aligned}
 \frac{\partial L}{\partial w_{tm}} &= -1 - \log(w_{tm}) - \sum_m \lambda_{1m} v_{tm} \left( \sum_m g_m h_m - I(\sum_m v_{tm} w_{tm} < 0) \right) \times \\
 &I(\sum_m v_{tm} < 0) - b_t = 0, \tag{3.14}
 \end{aligned}$$

$$\frac{\partial L}{\partial g_m} = -1 - \log(g_m) - \sum_m \lambda_{1m} w_{tm} v_{tm} \left( \sum_m h_m \right) - c_m = 0, \quad (3.15)$$

$$\frac{\partial L}{\partial \lambda_1} = Y_t - \sum_m p_{1m} z_{1m}(x'_{1,t}) - \sum_m v_{tm} w_{tm} \left( \sum_m g_m h_m - I(\sum_m w_{tm} w_{tm} < 0) \right) = 0, \quad (3.16)$$

$$\frac{\partial L}{\partial a_i} = 1 - \sum_m p_{1m} = 0, \quad (3.17)$$

$$\frac{\partial L}{\partial b_i} = 1 - \sum_m w_{1m} = 0, \quad (3.18)$$

$$\frac{\partial L}{\partial c_i} = 1 - \sum_m h_m = 0. \quad (3.19)$$

Thus, we have

$$p_{1m} = \exp \left\{ -a_t - 1 - \sum_m \lambda_{1m} z_{1m}(x'_{1,t}) \right\}, \quad (3.20)$$

$$w_t = \exp \left\{ -b_t - 1 - \sum_m \lambda_{1m} v_{tm} \left( \sum_m g_m h_m - I(\sum_m v_{tm} w_{tm} < 0) \right) I(\sum_m v_{tm} < 0) \right\} \quad (3.21)$$

and

$$g_m = \exp \left\{ -c_m - 1 - \sum_m \lambda_{1m} w_{tm} v_{tm} \left( \sum_m h_m \right) \right\}. \quad (3.22)$$

Due to  $\sum_m p_{km} = 1$ ,  $\sum_m w_{tm} = 1$ ,  $\sum_m g_m = 1$ , and  $\exp\{-a_t - 1\}$ ,  $\exp\{-b_t - 1\}$ ,  $\exp\{-c_m - 1\}$  are constant, thus by solving the first order conditions, we yield

$$p_{1m} = \frac{\exp \left\{ -z_{1m} \sum_m \lambda_{1m} x'_{1,t} \right\}}{\sum_m \exp \left\{ -z_{1m} \sum_m \lambda_{1m} (x'_{1,t}) \right\}}, \quad (3.23)$$

$$w_{tm} = \frac{\exp \left\{ \sum_m \lambda_{1m} v_{tm} \left( \sum_m g_m h_m - I(\sum_m v_{tm} w_{tm} < 0) \right) I(\sum_m v_{tm} < 0) \right\}}{\sum_m \exp \left\{ \sum_m \lambda_{1m} v_{tm} \left( \sum_m g_m h_m - I(\sum_m v_{tm} w_{tm} < 0) \right) I(\sum_m v_{tm} < 0) \right\}} \quad (3.24)$$

$$g_m = \frac{\exp \left\{ \sum_m \lambda_{1m} w_{tm} v_{tm} \left( \sum_m h_m \right) \right\}}{\sum_m \exp \left\{ \sum_m \lambda_{1m} w_{tm} v_{tm} \left( \sum_m h_m \right) \right\}} \quad (3.25)$$

### 4 Simulation Study

In this section, a simulation study was conducted to evaluate performance and accuracy of GME estimation in quantile regression with unknown quantile and compare it with the classical methods, namely Least Squares (LS), Maximum Likelihood estimation (MLE). We simulated the data from the quantile regression model where the error term is assumed to have asymmetric Laplace distribution (ALD),  $\varepsilon_{i,t}^\tau \sim ALD(0, 1)$ . To this end, we consider the quantile regression equation as follows:

$$y_{1,t} = \beta_0^\tau + \beta_1^\tau x_{1,t} + \varepsilon_{1,t}^\tau, \tag{4.1}$$

where  $\beta_0^\tau = 2$  and  $\beta_1^\tau = 3$ . We simulated the independent variables  $x_{1,t}$  from  $N(0, 1)$ .

We consider two scenarios as follows: 1. Case 1:  $T = 20, 50, 100$ .

2. Case 2:  $\tau = 0.5, 0.25, 0.75$

Our interest is on the bias of the parameters which are obtained from three estimation techniques. We carry out all the experiments with 100 replications.

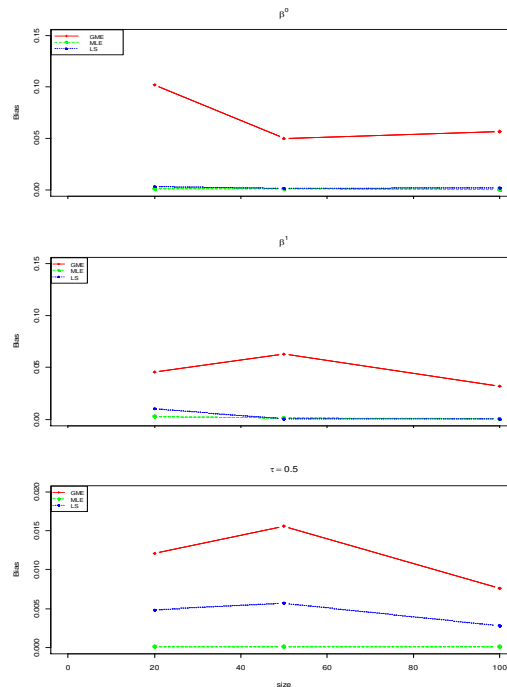


Figure 1: Comparison of the Bias of parameters at quantile 0.5



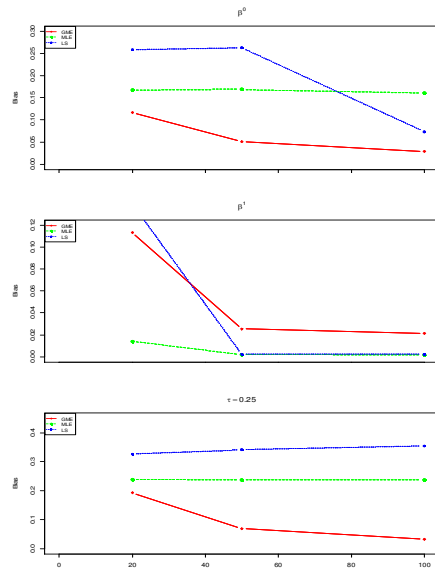


Figure 2: Comparison of the Bias of parameters at quantile 0.25

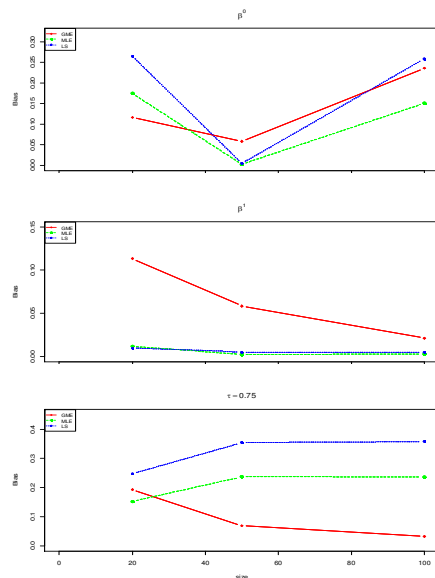


Figure 3: Comparison of the Bias of parameters at quantile 0.75

Figures 1-3 report the results of the Monte Carlo simulation study performed with samples of different sizes, and quantile levels. In all cases we compute the bias

with respect to  $\beta_0, \beta_1$  and  $\tau$ . We observe that our proposed model can perform well through this simulation study. From these three figures, we observe that the overall bias of GME estimation of parameters is likely to be low and tends to approach zero for large observation. Thus, this indicates that GME performs well with accuracy and asymptotically unbiased in this simulation study.

By comparing the GME and two other estimations at all quantile levels, we note that when the quantile level is assumed to be located at the median  $\tau = 0.5$ , the bias of the GME is higher than those of MLE and LS. However, the bias of the GME generally gets lower than those of MLE and LS when the true quantile parameter is assumed to deviate from median, like when  $\tau = 0.25$  and  $\tau = 0.75$ . This result suggests that when the asymmetric assumption of the error exists as true, the GME estimation has a smaller risk and is more precise than those conventional methods.

In summary, the entropy approach to quantile regression modeling is effective and it generally outperforms MLE and LS when there are considerably biased estimates at the extreme quantile levels.

## 5 Real Data Application

### 5.1 Data Description

The time series data used in this paper include stock indexes of Thailand (SET), Indonesia (IDX), and Philippines (PSE) and Brent oil price (OIL). These data are collected from Thomson Reuters datastream.

Table 1: Descriptive statistics

	SET	IDX	PHE	OIL
<b>Mean</b>	0.0001	0.0001	0.0002	-0.0001
<b>Median</b>	0.0003	0.0005	0.0001	0
<b>Maximum</b>	0.0328	0.0331	0.0306	0.0483
<b>Minimum</b>	-0.0482	-0.0476	-0.0568	-0.0475
<b>Std. Dev.</b>	0.0055	0.0059	0.0061	0.0094
<b>Skewness</b>	-0.6962	-0.649	-0.84	-0.0245
<b>Kurtosis</b>	11.0753	11.2608	9.6702	6.7412
<b>Jarque-Bera</b>	6488.317	6756.549	4571.655	1352.652
<b>Probability</b>	0	0	0	0
<b>ADF-test</b>	-45.6570***	-43.2178***	-42.7283***	-51.1885***

Source: Calculation

Note :\*\*\* is significant at 1% level

Our time series quantile regression model takes the following form:

$$\text{SET}_t = \beta_0 + \beta_1 \text{OIL}_t + \varepsilon_t,$$

$$\text{IDX}_t = \beta_0 + \beta_1 \text{OIL}_t + \varepsilon_t,$$

$$\text{PHP}_t = \beta_0 + \beta_1 \text{OIL}_t + \varepsilon_t,$$

The daily data is collected from January, 2, 2008 to June, 30, 2017. The data is transformed to be log-return. Table 1 gives the descriptive statistics for the series.

## 5.2 Results

In this section, we illustrate applicability of our proposed model and methods to the oil price and stock indexes data described in previous section. In this application, the relationship is between oil price and Thailand, Indonesia, and Philippines stock indexes.

Table 2: Estimates of the parameters from quantile regression with unknown quantile model.

Parameter	SET	IDX	PSE
$\beta_0$	0.0011***	0.0005*	-0.0011***
	-0.0002	-0.0002	-0.0001
$\beta_1$	0.0466***	-0.0041	-0.0992***
	-0.0177	-0.0178	-0.0271
$\tau$	0.5722***	0.5046***	0.3631***
	-0.0078	-0.0078	-0.0095
SSE of Reg	0.0692	0.082	0.0872
SSE of QReg	0.0625	0.0819	0.0871

Source : Calculation

Note “\*” , “\*\*” , and “\*\*\*” are significant at 1%, 5%, and 10% levels and their standard errors (in parentheses)

Table 2, reports the estimated coefficients and standard errors, as well as the sum of squared errors obtained through our proposed model (QReg) and conventional regression model (Reg). The results reveal that the appropriate values for explaining the effect of oil price on SET, IDX, and PSE are 0.5722, 5046, and 0.3631, respectively. Then, in making a comparison of model fit, we consider the Sum of Squared errors (SSE) to compare the performance of our model with the conventional linear regression model. The results in Table 2 reveal that the SSE of QReg is relatively low compared to linear regression. Therefore, from this application study, we can conclude that quantile regression with unknown quantile could be the appropriate alternative model to linear regression.

In the economic point of view, the results show that oil price has a positive and significant effect on SET index but it has a negative and significant effect on PSE index. For IDX index, we cannot obtain the significant effect of oil on IDX index. Additionally, Figure 1 is constructed to plot the fitted regression lines obtained from our proposed model and linear regression. We can observe that if the linear regression is employed to study the effect of oil price on stock markets, it will not

provide the accurate result and the effect would be underestimated since the slope of fitted lines of linear regression ( red line) is less steep than the slope of fitted lines of quantile linear regression with unknown quantile ( blue dotted line).

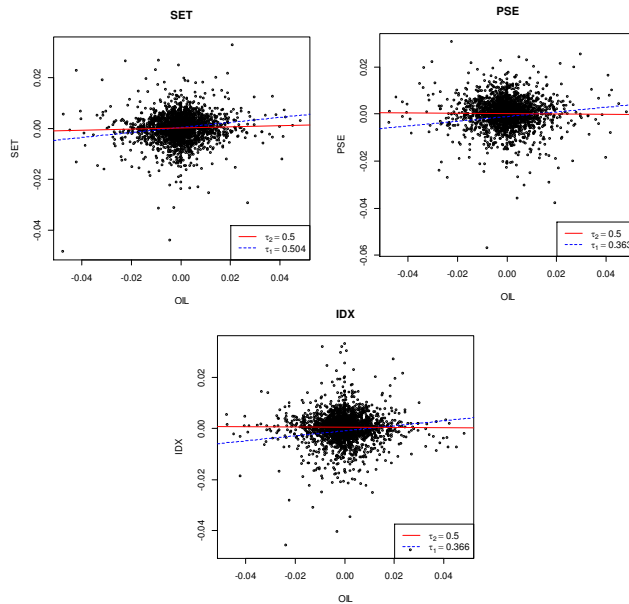


Figure 4: Quantile curves

## 6 Final Remarks

In this paper, we consider the quantile level in quantile regression as the estimated parameter. This is because researchers often face difficulty selecting an appropriate quantile level if not estimating a model using all of the various quantile levels. Thus, we consider the quantile level to be an unknown parameter and estimate it jointly with other parameters in the model. This quantile model with unknown quantile will provide more flexibility to explore the relationship between dependent and independent variables in the model. We use the primal generalized maximum entropy (GME) developed here to estimate the unknown parameters in the quantile regression model which includes quantile as one of the unknown parameters.

Although the GME estimation has already been proposed. It still adheres to the strong Asymmetric Laplace Distribution (ALD) assumption on the entropy measures. Thus, it is greatly desirable to expand the flexibility of entropy estimation by relaxing the ALD assumption in the objective function. Thus, another main contribution of this study is to develop an entropy estimation for quantile regression model without assuming the ALD. In this study, both simulation and application studies are employed to measure the performance of our estimation.

In the simulation study, the entropy estimation is compared for performance with the conventional estimations, Ordinary Least Squares, and Maximum likelihood methods. The results show that entropy estimation not only performs well but outperforms those conventional estimations, particularly when there exists in the model an asymmetric relation between dependent and independent variables.

We subsequently apply our model to investigate the relationship between oil price and three emerging stock markets in ASEAN consisting Thailand, Indonesia, and the Philippines. In this application, it is interesting to compare the proposed model and the linear regression. Therefore, the SSE method is conducted to compare the performance of our model with linear regression and numerical evidence shows that the proposed model outperforms the linear regression model.

## References

- [1] S. Tu, M. Wang, X. Sun, Bayesian variable selection and estimation in maximum entropy quantile regression, *Journal of Applied Statistics* 44 (2) (2017) 253–269.
- [2] A. Golan, G. Judge, D. Miller, *Maximum Entropy Econometrics: Robust Estimation with Limited Data*, (1997).
- [3] R. Koenker, *Quantile regression* (No. 38), Cambridge university press, (2005).
- [4] B. L. Snchez, H. V. Lachos, V. F. Labra, Likelihood based inference for quantile regression using the asymmetric Laplace distribution, *Journal of Statistical Computation and Simulation* 81 (2013) 1565–1578.
- [5] R. C. Mittelhammer, G. G. Judge, D. J. Miller, *Econometric Foundations*, Cambridge University Press, Cambridge (2000), 313–336.
- [6] C. E. Shannon, A mathematical theory of communication *Bell Syst, Tech. J.* 27 (1948) 379–423.
- [7] E. T. Jaynes, Information theory and statistical mechanics, *Physical review* 106 (4) (1957) 620–630.
- [8] V. Pipitpojanakarn, P. Maneejuk, W. Yamaka, S. Sriboonchitta, How does economic growth affect the well-being in Asia? *International Journal of Economic Research* 14 (6) (2017) 231–244.

(Received 31 August 2017)

(Accepted 30 October 2017)