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Capital Asset Pricing Model Through Quantile Regression: An Entropy Approach

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Abstract : This paper introduces the generalized maximum entropy(GME) approach, which was proposed by Golan, Judge and Miller in 1997 to estimate the quantile regression model for capital asset pricing because this information-theoretic estimator method is robust to multicolinearity and ill-posed problems inherent in CAPM. Monte Carlo simulations for quantile regression exhibited that the primal GME estimator outperforms several classical estimators such as least squares, maximum likelihood and Bayesian when the extreme quantile is considered. We describe statistical inference techniques for this estimator and demonstrate its usefulness in risk measurement through capital asset pricing model.

Keywords : quantile regression; generalized maximum entropy; CAPM; beta risk.

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1 Introduction

For many decades, Capital Asset Pricing Model (CAPM) has become a successful model in financial econometrics. The CAPM was proposed by [1] and [2] based on the pioneer work of Markowitz on modern portfolio diversification theory. CAPM revolves around the linear combination between expected return and risk. The concept of the CAPM is that an investor can expose him/herself to a reasonable amount of risk through CAPM equation. CAPM emphasizes that the formation of the optimal portfolio relies on the investor's evaluation of the future prospects of various assets. The formula for calculating the CAPM is as follows:

$$(r_t - r_f) = (r_M - r_f)\beta + \varepsilon_t, \tag{1.1}$$

where r_t is return of stock, r_f is risk free rate, r_M is return of market, and β is the beta risk parameter. This beta represents a marginal contribution to the risk of the whole market portfolio of risky assets. This implies that asset nominated with high beta coefficient bigger than one indicates that the asset's price is apparently more volatile than the market. On the contrary, a beta of smaller than 1 implies that the asset is less volatile than the market. In efficient market application of CAPM, risk premium and expected return of a security will vary in direct proportion to the beta coefficient. With this successful model, many applications of CAPM have been found such as the work of [3], [4], [5], [6], [7], [8].

In general, the least squares method with a normality assumption is applied to estimate the beta coefficient in the CAPM. Alternatively, in this paper, we consider a quantile regression using a maximum entropy approach to compute parameters in CAPM. This is due to prospect theory of [9], which postulates that: (i) investors exhibit "loss-aversion" in a gain; and (ii) investors exhibit "risk-seeking" in a loss. On the other hand, investors could prefer lower beta risk after they have experienced a gain and higher beta risk after they have experienced a loss. Thus, if we can distinguish different beta risks of the stocks in the market, it would be a useful information for an investor's decision. To achieve this aim, we replace the familiar concept of conditional expectation (mean) by the concept of conditional median, and more generally, conditional quantiles. A number of studies on quantile regression, have been undertaken such as [4], [6], [10], [11], [12].

In a sense, maximum entropy is a nonparametric estimation of probability density functions, consistent with data and prior information. We relate entropy to quantitative measure of uncertainty (information) through the beta in the CAPM. The difference between probability and entropy is this: probability measures are quantitative measures of chances of occurrences of events but entropy is a measure of a global uncertainty about a random variable (or stochastic system). Thus, by using a method of entropy in CAPM model may leads us to a new finding in the financial risk management field.

This paper is structured as follows. In Section 2 we give a review of quantile regression model. Section 3 presents the formulation of generalized maximum entropy estimation in quantile regression model. Section 4 presents simulation studies of finite sample performance and robustness of our proposed method. While, Section 5 discusses the results, and Section 6 concludes.

2 Review of Quantile Regression Model

Quantile regression is one of the most successful technique used in statistics and econometrics. In contrast to the linear regression method which estimates the conditional mean of the response variable given certain values of the predictor variables, quantile regression aims at estimating either the conditional median or other quantiles of the response variable. [13] mentioned that quantile regression is useful if conditional quantile functions are interested. The main advantage of quantile regression is that its estimation is more robust against outliers in the response measurements, compared to linear regression. Different measures of central tendency and statistical dispersion can be used to obtain a more comprehensive analysis of the relationship between variables. The model structure of quantile regression model can be written as follows:

$$y_t = x'_{i,t}\beta_i^\tau + \varepsilon_t \quad ; \quad i = 1, ..., k \quad \text{and} \quad t = 1, ..., n \tag{2.1}$$

where y_t is dependent variables, $x'_{i,t}$ is $(n \times k)$ independent variables, β_i^τ is $(1 \times k)$ vector of coefficients and ε_t is an the error term without any assumed distribution. Thus, $\tau^{th}(0 < \tau < 1)$ a conditional quantile of y_i given $x'_{i,t}$ is defined as

$$Q_y(\tau|x) = x'_{i,t}\beta_i^\tau \tag{2.2}$$

Why do we need to employ entropy estimator?

In the estimation context, the classical estimation of quantile regression model focused on least squares (LS) which is a general technique for estimating families of conditional quantile functions (see, [14]).The τ specific coefficient vector β^τ can be estimated by minimizing the loss function through check function $\rho^\tau(\varepsilon_t) = \varepsilon_t(\tau - I(\varepsilon_t < 0))$, thus

$$\hat{\beta}^\tau = \arg\min_{\beta^\tau} \sum_{j=1}^{n} \rho^\tau(y_t - x_{i,t}\beta_i^\tau) \tag{2.3}$$

The further extended estimation of this model is the Maximum likelihood estimation (MLE). It was investigated by [15] and proved that the MLE outperforms the competing conventional simplex(BR) of [16] and Lasso Penalized Quantile Regression (LPQR) of [17]. In this approach, the estimated β^τ is obtained by maximizing the likelihood based on the asymmetric Laplace density (ALD):

$$L(\beta^\tau, \sigma\,|y) = \frac{\tau^n(1-\tau)^n}{\sigma^n}\left(-\sum_{t=1}^{n}\rho^\tau\frac{(y_j - x_{i,t}\beta_i^\tau)}{\sigma}\right), \tag{2.4}$$

where σ is a sigma parameter. Note that, the maximization of the likelihood in (2.4) with respect to the parameter β^τ is equivalent to the minimization of the objective function in (2.3).

In contrast to the frequentist approach, an estimation method called Bayesian approach has been proposed to combine a prior density distribution to the frequentist approach have been proposed. In this direction, recent developments include, [18] who considered median regression and suggested non-parametric modeling for the error distribution based on either Plya tree or Dirichlet process priors. [19] who proposed the ALD and improper uniform priors to produce a proper joint posterior, and [20] who developed a Gibbs sampling method to estimate the quantile regression model based on a location-scale mixture representation of the ALD. In this Bayesian estimation, Markov Chain Monte Carlo (MCMC) algorithm is employed for sampling the conditional posterior distributions of unknown parameters β^{τ} and this provides a convenient way to incorporate a parameter uncertainty into predictive inferences. The posterior distribution of β^{τ} can be written as

$$P(\beta^{\tau},\sigma|y) \quad \propto \quad L(\beta^{\tau},\sigma|y)p(\beta^{\tau},\sigma), \tag{2.5}$$

where $p(\beta^{\tau},\sigma)$ is prior distribution of β^{τ} and σ. In general, we can choose a prior distribution depending on our belief to produce a proper conditional posterior. The further extension of the model estimation is carried out by [21]. The entropy estimation is proposed to estimate the quantile regression model by defining the information entropy of the distribution of probabilities as continuous function and maximizing entropy measure subject to two moment constraints :

$$f_{ME}(y) = \arg\max_{f} - \int f(y)\ln f(y)dy \tag{2.6}$$

subject to

$$E|y - x\beta^{\tau}| = c_1,$$

$$E(y - x\beta^{\tau}) = c_2,$$

where $f(y)$ is ALD and $\int f(y)dy = 1$; c_1 and c_2 are known constants. Even though the entropy estimation has already been proposed as estimator of quantile regression, it still adheres to the strong ALD assumption on the entropy measures. In the other word, the existing approach assumes that the distribution is ALD, but in practice, it may be different, so we need to make this method more flexible. Thus, it is greatly desirable to expand the flexibility of entropy estimation by relaxing the ALD in the objective function. Therefore, in this study, we also proposed to use a primal maximum entropy approach and add a quantile regression as a constraint in the Lagrangian method (see, [22]).

3 Methodology

3.1 Generalized Maximum Entropy Estimation

In this study, we applied a maximum entropy estimator to estimate the unknown parameters in equation (1). As this estimator for quantile regression and its statistical properties were already discussed, now it is the turn of the concept about the entropy approach. The maximum entropy concept consists of inferring the probability distribution that maximizes information entropy given a set of various constraints. Let p be a proper probability, [23] developed his information criteria and proposed a classical entropy as

$$H(p) = -\sum_{k=1}^{K} p_k \log p_k, \tag{3.1}$$

where $\sum_{k=1}^{K} p_k = 1$. The entropy measures the uncertainty of a distribution and reaches a maximum when p_k is uniform distribution [24]. This concept of entropy is applied in the present model by generalizing the maximum entropy as the inverse problem in the quantile regression framework. Rather than searching for the point estimates β_i^τ, we can estimate unknown parameter β_k^τ as the expectation of random variables with M support value $z_k = [\underline{z}_{k1}, ..., \overline{z}_{km}]$ and M dimension $p_k = [p_{k1}, ..., p_{kM}]$ for all $k = 1, ..., K$. Note that $\underline{z}$ and $\overline{z}$ denote the lower bound and upper bound, respectively, of each support z_k. Thus parameter can be computed by

$$\beta_i^\tau = \begin{bmatrix} \underline{z}_{11} & \dots & 0 & \dots & \overline{z}_{1m} \\ \underline{z}_{21} & \dots & 0 & \dots & \overline{z}_{2m} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \underline{z}_{k1} & \dots & 0 & \dots & \overline{z}_{km} \end{bmatrix} \begin{bmatrix} p_{11} & \cdots & \cdots & \dots & p_{1m} \\ p_{21} & \cdots & \cdots & \dots & p_{2m} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ p_{k1} & \cdots & \cdots & \dots & p_{km} \end{bmatrix} \tag{3.2}$$

$$\beta_k^\tau = \sum_m p_{km} z_{km}, \tag{3.3}$$

where p_{km} are the M dimensional estimated probability distribution defined on the set $z_{k,\cdot}$. Then, similar to the above computation, ε_t is also constructed as the expectation of random variables with M support value, $v_t = [v_{t1}, \cdots, v_{TM}]$, and M dimension proper probability weights $w_t = [w_{t1}, \cdots, w_{TM}]$. Thus error ε_t can be computed by

$$\varepsilon_t = \rho_\tau \begin{bmatrix} \underline{v}_{11} & \dots & 0 & \dots & \overline{v}_{1M} \\ \underline{v}_{21} & \dots & 0 & \dots & \overline{v}_{2M} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \underline{v}_{T1} & \dots & 0 & \dots & \overline{v}_{TM} \end{bmatrix} \begin{bmatrix} w_{11} & \cdots & \cdots & \dots & w_{1M} \\ w_{21} & \cdots & \cdots & \dots & w_{2M} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ w_{T1} & \cdots & \cdots & \dots & w_{TM} \end{bmatrix} \tag{3.4}$$

$$\varepsilon_t = \rho_\tau \sum_m w_{tm} v_{tm} \tag{3.5}$$

Using the reparameterized unknowns β_k^τ and ε_j, one can rewrite equation as

$$y_t = \sum_m p_{1m} z_{1m}(x'_{1,t}) + \cdots + \sum_m p_{Km} z_{Km}(x'_{K,t}) + \rho_\tau \sum_m w_{tm} v_{tm} \tag{3.6}$$

where the vector support z_k and v_t are convex set that is symmetric around zero with $2 \leq M \leq \infty$. And

$$\rho_\tau(\varepsilon_t) = \varepsilon_t(\tau - I(\varepsilon_t < 0)), \tag{3.7}$$

is the check function; this gives the τ^{th} sample quantile with its solution.

Then, the Generalized Maximum Entropy (GME) estimator for this model can be constructed as

$$H(p,w) = \arg\max_{p,w}\{H(p)+H(w)\} \equiv -\sum_k\sum_m p_{km}\log p_{km} - \sum_t\sum_m w_{tm}\log w_{tm} \quad (3.8)$$

subject to

$$y_t = \sum_m p_{1m}z_{1m}(x'_{1,t}) + \cdots + \sum_m p_{Km}z_{Km}(x'_{K,t}) + \rho_\tau\sum_m w_{tm}\nu_{tm} \quad (3.9)$$

$$\sum_m p_{km} = 1, \sum_m w_{tm} = 1, \quad (3.10)$$

where p, and w are on the interval [0,1]. To make it easy for derivation, let us consider quantile regression with one covariate ($k = 1$), then this optimization problem can be solved using the Lagrangian method

$$L = H(p,w) + \lambda'(y_t - \sum_m p_{1m}z_{1m}(x'_{1,t}) - \rho_\tau\sum_m w_{tm}\nu_{tm}) + a'(1-\sum_m p_{1m}) + b'(1-\sum_m w_{tm}) \quad (3.11)$$

where λ', a', b' are the vectors of Lagrangian multipliers. Thus, resulting in first-order conditions, we have

$$p_{1m} = \exp(-1-a)\exp(\sum_m \lambda_m z_{1m}(x'_{1,t})), \quad (3.12)$$

$$\text{and} \quad w_{tm} = \exp(-1-b_t)\exp(\sum_m \rho_t\lambda_m\nu_{tm}) \quad (3.13)$$

Since the constraint requires that $\sum_m p_{km} = 1, \sum_m w_{tm} = 1, \exp(-1-a)$ and $\exp(-1-b_t)$ is constant for a given parameter and error, respectively. Thus, solving the first order conditions yields

$$\hat{p}_{1m} = \frac{\exp(-z_{1m}\sum_t \hat{\lambda}x'_{1,t})}{\sum_m \exp(-z_{1m}\sum_t \hat{\lambda}(x'_{1,t}))} \quad (3.14)$$

$$\hat{w}_{tm} = \frac{\exp(\rho_\tau\hat{\lambda}\nu_{1m})}{\rho_\tau\sum_m \exp(\rho_\tau\hat{\lambda}_{tn}\nu_{1m})} \quad (3.15)$$

4 Simulation Study

In this section, we carry out several Monte Carlo experiments to compare the GME estimator and compare it with the classical estimations, including Least Squares (LS), Bayesian (BAY) and Maximum Likelihood estimation (MLE). We simulate the data from the quantile regression model where the error term is assumed to be asymmetric Laplace distribution (ALD), for three different quantile levels $\tau = (0.25, 0.50, 0.75)$. Hence, the simulation model takes the following form:

$$y_{1,t} = \beta_0^{\tau} + \beta_1^{\tau} x_{1,t} + \varepsilon_t^{\tau} \tag{4.1}$$

In the simulation, we set $\beta_0^{\tau} = 1$ and $\beta_1^{\tau} = 2$. We simulate the independent variables $x_{1,t}$ from $N(0,1)$. We evaluate the estimators in terms of the bias of the parameter estimates. We carry out all the experiments with sample size 20 and 40. For each sample size, we generated 100 datasets. Computations are performed in the R environment (R Development Core Team, 2012) using the package quantreg for LS, written by [25], and bayesQR written by [26]. For MLE, we follow the estimation technique of [15] and maximize the likelihood based ALD to obtain the parameters.

Table 1: Bias of quantile regressions

			Bias(%)					
N	Par.	true par	GME1	GME2	GME3	Bayes	MLE	LS
20	$\beta_0^{0.25}$	1	0.1089	0.1332	0.1438	0.1265	0.0761	0.0642
	$\beta_1^{0.25}$	2	0.2096	0.0361	0.0072	0.0857	0.1798	0.1603
40	$\beta_0^{0.25}$	1	0.0738	0.1438	0.1253	0.0156	0.0372	0.0265
	$\beta_1^{0.25}$	2	0.0826	0.0072	0.0252	0.0271	0.0217	0.0247
N	Par.	true par	GME1	GME2	GME3	Bayes	MLE	LS
20	$\beta_0^{0.50}$	1	0.0232	0.0498	0.0536	0.1799	0.1992	0.1992
	$\beta_1^{0.50}$	2	0.5524	0.3629	0.4121	0.0180	0.0151	0.0150
40	$\beta_0^{0.50}$	1	0.0250	0.0500	0.0601	0.0271	0.0362	0.0334
	$\beta_1^{0.50}$	2	0.0734	0.3989	0.4746	0.0093	0.0232	0.0244
N	Par.	true par	GME1	GME2	GME3	Bayes	MLE	LS
20	$\beta_0^{0.75}$	1	0.1762	0.1147	0.1337	0.2118	0.2749	0.2699
	$\beta_1^{0.75}$	2	0.0047	0.0195	0.0466	0.0401	0.1031	0.0863
40	$\beta_0^{0.75}$	1	0.0581	0.1188	0.1253	0.0769	0.1088	0.0991
	$\beta_1^{0.75}$	2	.01722	0.0974	0.0252	0.0293	0.0385	0.0431

Source: Calculation

Note: GME1, GME2, GME3 are the GME estimator using M=3, M=5, and M=7, respectively.

Table 1 reports the results of the Monte Carlo simulation. In all cases we compute the percentage relative bias with respect to $\beta_0^{\tau} = 1$ and $\beta_1^{\tau} = 2$. We also aim to examine whether or not the estimated parameter is sensitive to the number of support or not. Therefore, we estimate three different GME models using different number of supports $\{M = 3, 5, 7\}$. The support space of estimates β^{τ} and ε_t is specified as $z = [-10, \cdots, 10]$ and $\nu = [-5, \cdots, 5]$, respectively. Here, we specify the support values around the true values.

According to the results, we observe that GME estimation can perform well through this simulation study. The overall bias values of parameter at different quantile levels are lower than 10%. In addition, when the number of support is increased from 3 to 7, the biases of the estimated parameters are not quite stable. This result corresponds to the study of [27] which also found that when the number of support increase but the bias of the parameters are not quite stable. Nonetheless our results demonstrate that the GME performs well with accuracy in this simulation study. We expect that when support bound is precisely specified, the GME estimator has a smaller risk. Intuitively, if the support bound covers the true values, the estimated parameters are accurate at any numbers of support.

Comparing the GME and three other estimations at all quantile levels, we observe that the biases of the GME are mostly smaller than those of Bayesian, MLE and LS, when the number of observation T=20. However, the bias of the GME are mostly larger than those of Bayesian, MLE and LS when the number of observations is large, T=40. This result suggests that when the number of observation increases, GME performance seems to be a little better when compared with the conventional estimators. In addition, regarding the performance across quantiles, GME gave better estimates for lower and upper quantiles, $\tau = 0.25$ and $\tau = 0.75$.

As the simulation study has demonstrated, entropy approach to quantile regression modeling is effective and it generally outperforms Bayesian, MLE and LS when the number of observations is small. In addition, GME can obtain a low bias at the extreme quantile levels.

5 Empirical Results and Discussion

In this study, we apply our model to Capital Asset Pricing Model (CAPM) which has been intensively studied in financial economics in the last decade. The main contribution of this approach is to identify how the risk of a particular stock is related to the risk of the overall stock market using the risk measure Beta coefficient. If the relationship between individual stock's returns and market return exhibits heteroskedasticity, then the estimates of Beta for different quantiles of the relationship can be quite different. The study focus on AAPL which is one of the biggest and the most active stocks in NASDAQ market of United States. The data collected are from January 2008 to November 2015 which gives us 95 monthly data points. The stock data were obtained from Thomson Reuter Data Stream. In this study, we use Treasury bills as a proxy of the risk free rate. Table 2 gives the summary of our variables.

To illustrate the GME estimator introduced in Section 3, we consider the following CAPM quantile regression model

$$AAPL_t = \beta_0^\tau + \beta_1^\tau NASDAQ_t + \varepsilon_t \tag{5.1}$$

Note that we have focused on the relationship between NASDAQ and AAPL.

Table 2: Descriptive Statistics

	AAPL	NASDAQ
Mean	0.006	0.003
Median	0.008	0.007
Maximum	0.092	0.051
Minimum	-0.185	-0.085
Std. Dev.	0.045	0.024
Skewness	-1.123	-0.749
Kurtosis	6.273	4.010
Jarque-Bera	61.715***	12.778***
ADF-test	-8.917***	-8.252***

Source: Calculation
***Significant level at 0.05

In this data set we compare the entropy estimator to relevant benchmark estimators in terms of mean square error (MSE). We fit quantile regression using estimators in Section 4, namely LS (black thick line), MLE (red dashed line), and Bayesian (green dotted line), and Entropy (blue dashed-dotted line). Figure 1 presents the obtained results for each estimator. According to these results, it can be concluded that entropy estimator is slightly the better than the other estimator, with a few minor exceptions in the quantile level around the middle levels ($\tau = 0.4, 0.5$). However, this makes sense in light of the high performance of the GME in real data analysis.

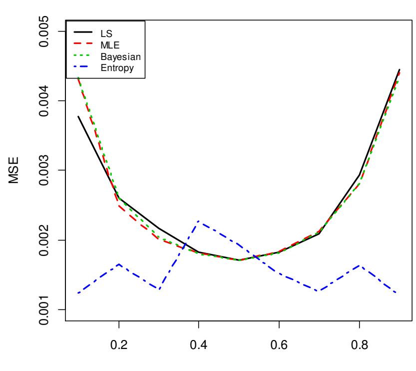
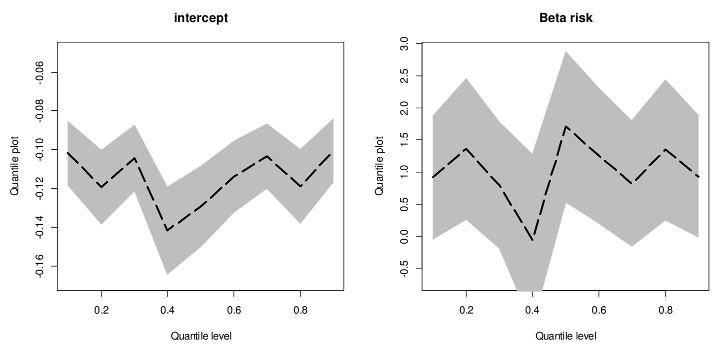


Figure 1: Model comparison for nine different quantile regression MSEs over the grid $\tau = \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.90\}$

Finally, we investigate the beta risk of the CAPM in each quantile level. As it can be observed in Figure 2, all of the estimated beta risks for CAPM are positive. This indicates a positive relationship between AAPL and NASDAQ returns in all quantile levels. Particularly, we find an evidence of high beta risk at quantile 0.2, 0.5, and 0.8. For the low quantile level, risk is strongly observed at quantile 0.4. This finding provides important evidence for clarifying the sign of the beta risk of stock returns.



(a) (b)

Figure 2: Quantile Regression Estimation (95%) CI

We then further illustrate the scatter plots between the AAPL and NASDAQ returns. In Figure 3, we present the quantile regression lines labeled $\tau = \{0.1, 0.2, \cdots, 0.9\}$. The quantile regression lines labeled in all qauntile levels display an upward slope, suggesting that as NASDAQ return increases, AAPL displays a greater return. However, the risk-return relation evolves into a fluctuated positive as the quantile increases.

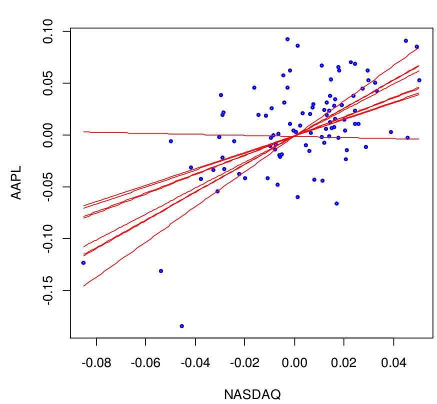


Figure 3: Data analysis: Fitted QR overlayed with nine different quantile regression lines over the grid $\tau = \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.90\}$

6 Conclusion

In this paper, we aim to distinguish different beta risks of the AAPL stock in the NASDAQ market and have proposed an entropy-based approach for the estimation of the qunatile regression model. By using quantile regression check function, we can cast the quantile regression problem into the primal maximum entropy framework. This estimator allows us to relax the assumption of parametric distributions in quantile regression, making easy the implementation of a Lagrangian method for obtaining the entropy estimates of the model probabilities subject to some useful constraints. The Generalized Maximum Entropy (GME) estimator is found to be a robust estimator that is resistant to multicollinearity and ill-posed problem such as limited, partial, or incomplete data. With Monte Carlo simulations, we have shown that the primal GME estimator is a better alternative to classical least squares, maximum likelihood, and Bayesian estimators especially in extreme quantile regime.

Last but not least, our model is applied to distinguish different beta risks of the AAPL returns in the NASDAQ market. We find that the beta risk evolves into fluctuated positive when the quantile increases. Finally, the proposed method can be extended to a more general framework, by employing a quantile regression to analyze the different data sets and also extending to dual maximum entropy which may be solved with simpler and more widely available unconstrained numerical methods. The dual algorithm involves fewer parameters in primal framework thus it will reduce the computation time of this model.

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