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Intuitionistic Fuzzy α-Generalized Semi Closed Sets

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Abstract : In this paper, we introduce the concepts of intuitionistic fuzzy α-generalized semi closed sets and intuitionistic fuzzy α-generalized open sets. Further, we study some of their properties.

Keywords : intuitionistic fuzzy topological space; intuitionistic fuzzy α-generalized semi closed set; intuitionistic fuzzy α-generalized semi open set.
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1 Introduction

Zadeh [1] introduced the concept of fuzzy sets and later Atanassov [2] generalized this idea to the new class of intuitionistic fuzzy sets using the notion of fuzzy sets. On the other hand Coker [3] introduced intuitionistic fuzzy topological spaces using the notion of intuitionistic fuzzy sets. In this paper we introduce the concepts of intuitionistic fuzzy α-generalized semi closed sets and intuitionistic fuzzy α-generalized semi open sets. We obtain their properties and relationships.

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2 Preliminaries

Definition 2.1 ([2])**.** Let X be a non empty fixed set. An *intuitionistic fuzzy set*(IFS in short) A in X is an object having the form

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle |\, x \in X\}$$

where the function $\mu_A(x) : X \to [0,1]$ denotes the degree of membership(namely μ_A(x)) and the function $\nu_A(x) : X \to [0,1]$ denotes the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A, respectively and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$.

IFS(X) denote the set of all intuitionistic fuzzy sets in X.

Definition 2.2 ([2])**.** Let A and B be IFSs of the form
$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle |\, x \in X\}$ and $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle |\, x \in X\}$. Then

1. $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$,
2. $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$,
3. $A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle |\, x \in X\}$,
4. $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle |\, x \in X\}$,
5. $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)\rangle |\, x \in X\}$.

For the sake of simplicity, we shall use the notation $A = \langle x, \mu_A, \nu_A\rangle$ instead of $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in X\}$.

Definition 2.3 ([2])**.** The intuitionistic fuzzy sets $0_\sim = \{\langle x, 0, 1\rangle : x \in X\}$ and $1_\sim = \{\langle x, 1, 0\rangle : x \in X\}$ are the empty set and the whole set of X respectively.

Definition 2.4 ([3])**.** An *intuitionistic fuzzy topology* (IFT in short) on X is a family τ of IFSs in X satisfying the following axioms.

1. $0_\sim, 1_\sim \in \tau$,
2. $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,
3. $\cup G_i \in \tau$ for any family $\{G_i |\, i \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called an *intuitionistic fuzzy topological space*(IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set(IFOS in short) in X.

The complement A^c of an IFOS A in an IFTS (X, τ) is called an *intuitionistic fuzzy closed set*(IFCS in short) in X.

Definition 2.5 ([3])**.** Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \nu_A\rangle$ be an IFS in X. Then the intuitionistic fuzzy interior and the intuitionistic fuzzy closure are defined as follows:

1. $int(A) = \cup\{G |\, G$ is an IFOS in X and $G \subseteq A\}$,

2. $cl(A) = \cap\{K|\, K$ is an IFCS in X and $A \subseteq K\}$.

Proposition 2.6 ([3]). *For any IFSs A and B in (X, τ), we have*

1. *$int(A) \subseteq A$,*
2. *$A \subseteq cl(A)$,*
3. *$A \subseteq B \Rightarrow int(A) \subseteq int(B)$ and $cl(A) \subseteq cl(B)$,*
4. *$int(int(A)) = int(A)$,*
5. *$cl(cl(A)) = cl(A)$,*
6. *$cl(A \cup B) = cl(A) \cup cl(B)$,*
7. *$int(A \cap B) = int(A) \cap int(B)$.*

Proposition 2.7 ([3]). *For any IFS A in (X, τ), we have*

1. *$int(0_\sim) = 0_\sim$ and $cl(0_\sim) = 0_\sim$,*
2. *$int(1_\sim) = 1_\sim$ and $cl(1_\sim) = 1_\sim$,*
3. *$(int(A))^c = cl(A^c)$,*
4. *$(cl(A))^c = int(A^c)$.*

Proposition 2.8 ([4]). *If A is an IFCS in X then $cl(A) = A$ and if A is an IFOS in X then $int(A) = A$. The arbitrary union of IFCSs is an IFCS in X.*

Definition 2.9 ([5]). An IFS $A = \langle x, \mu_A, \nu_A \rangle$ in an IFTS (X, τ) is said to be an

1. *intuitionistic fuzzy regular closed set*(IFRCS in short) if $A = cl(int(A))$ [4],
2. *intuitionistic fuzzy α-closed set*(IFαCS in short) if $cl(int(cl(A))) \subseteq A$ [5],
3. *intuitionistic fuzzy semi closed set*(IFSCS in short) if $int(cl(A)) \subseteq A$ [4],
4. *intuitionistic fuzzy pre closed set*(IFPCS in short) if $cl(int(A)) \subseteq A$ [4],
5. *intuitionistic fuzzy γ-closed set*(IFγCS in short) if $cl(int(A)) \cap int(cl(A)) \subseteq A$ [6].

Definition 2.10. An IFS $A = \langle x, \mu_A, \nu_A \rangle$ in an IFTS (X, τ) is said to be an

1. *intuitionistic fuzzy regular open set*(IFROS in short) if $A = int(cl(A))$ [4],
2. *intuitionistic fuzzy α-open set*(IFαOS in short) if $A \subseteq int(cl(int(A)))$ [5],
3. *intuitionistic fuzzy semiopen set*(IFSOS in short)if $A \subseteq cl(int(A))$ [4],
4. *intuitionistic fuzzy preopen set* (IFPOS in short) if $A \subseteq int(cl(A))$ [4],
5. *intuitionistic fuzzy γ-open set*(IFγOS in short) if $A \subseteq int(cl(A)) \cup cl(int(A))$ [6].

Definition 2.11 ([7]). An IFS A of an IFTS (X, τ) is an

1. *intuitionistic fuzzy semipreopen set*(IFSPOS in short) if there exists an IF-POS B such that $B \subseteq A \subseteq cl(B)$,
2. *intuitionistic fuzzy semipreclosed set*(IFSPCS in short) if there exists an IFPCS B such that $int(B) \subseteq A \subseteq B$.

Definition 2.12 ([4])**.** Let A be an IFS in (X, τ), then *semi interior* of $A(sint(A)$ in short) and *semi closure* of $A(scl(A)$ in short) are defined as

1. $sint(A) = \cup\{K|\, K$ is an IFSOS in X and $K \subseteq A\}$,
2. $scl(A) = \cap\{K|\, K$ is an IFSCS in X and $A \subseteq K\}$.

Definition 2.13 ([8])**.** Let A be an IFS in (X, τ), then *semipre interior* of $A(spint(A)$ in short) and *semipre closure* of $A(spcl(A)$ in short) are defined as

1. $spint(A) = \cup\{G|\, G$ is an IFSPOS in X and $G \subseteq A\}$,
2. $spcl(A) = \cap\{K|\, K$ is an IFSPCS in X and $A \subseteq K\}$.

Definition 2.14 ([9])**.** Let A be an IFS of an IFTS $(X,\, \tau)$. Then

1. $\alpha cl(A) = \cap\{K|\, K$ is an IFαCS in X and $A \subseteq K\}$,
2. $\alpha int(A) = \cup\{K|\, K$ is an IFαOS in X and $K \subseteq A\}$.

Definition 2.15. An IFS A of an IFTS (X,τ) is an

1. *intuitionistic fuzzy generalized closed set*(IFGCS in short) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in X [10],
2. *intuitionistic fuzzy generalized semiclosed set*(IFGSCS in short) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in X [11],
3. *intuitionistic fuzzy generalized semipreclosed set*(IFGSPCS in short) if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in X [8],
4. *intuitionistic fuzzy alpha generalized closed set*(IFαGCS in short) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in X [9],
5. *intuitionitic fuzzy generalized alpha closed set*(IFGαCS in short) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFαOS in X [12].

The complements of the above mentioned intuitionistic fuzzy closed sets are called their respective open sets.

Remark 2.16 ([4])**.** *Let A be an IFS in $(X,\, \tau)$. Then*

1. *$scl(A) = A \cap int(cl(A))$,*
2. *$sint(A) = A \cup cl(int(A))$.*

If A is an IFS of X then $scl(A^c) = (sint(A))^c$.

Remark 2.17 ([9])**.** *Let A be an IFS in $(X,\, \tau)$. Then*

1. $\alpha cl(A) = A \cup cl(int(cl(A)))$,

2. $\alpha int(A) = A \cap int(cl(int(A)))$.

Definition 2.18 ([10]). Two IFSs are said to be *q-coincident*(AqB in short) if and only if there exists an element $x \in X$ such that $\mu_A(x) > \nu_B(x)$ or $\nu_A(x) < \mu_B(x)$. For any two IFSs A and B of $X, A\bar{q}B$ if and only if $A \subseteq B^c$.

3 Intuitionistic Fuzzy α-Generalized Semi-Closed Sets

In this section we introduce intuitionistic fuzzy α-generalized semi-closed sets and study some of its properties.

Definition 3.1. An IFS A in (X, τ) is said to be an *intuitionistic fuzzy α-generalized semi-closed set* (IFαGSCS in short) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFSOS in (X, τ) and the family of all IFαGSCS of an IFTS (X, τ) is denoted by $IF\alpha GSC(X)$.

Example 3.2. Let $X = \{a, b\}$. Let τ=$\{0_\sim, G, 1_\sim\}$ be an IFT on X,where $G = \langle x, (0.6, 0.7), (0.4, 0.3)\rangle$. Here $\mu_G(a) = 0.6$, $\mu_G(b) = 0.7$, $\nu_G(a) = 0.4$ and $\nu_G(b) = 0.3$. Let us consider the IFS $A = \langle x, (0.1, 0.2), (0.9, 0.8)\rangle$. We have IFSOS = $\{0_\sim, G, S, 1_\sim\}$ where $S = \langle x, (\ell_1, m_1), (\ell_2, m_2) \rangle$ and $\ell_1 \in (0.6, 1)$, $m_1 \in (0.7, 1)$, $\ell_2 \in (0, 0.4)$ and $m_2 \in (0, 0.3)$. Since $\alpha cl(A) = A$, A is IFαGSCS in (X, τ).

Theorem 3.3. *Every IFCS in (X, τ) is an IFαGSCS, but not conversely.*

Proof. Assume that A is an IFCS in (X, τ). Let us consider an IFS $A \subseteq U$ and U is an IFSOS in X. Since $\alpha cl(A) \subseteq cl(A)$ and A is an IFCS in X, $\alpha cl(A) \subseteq cl(A) = A \subseteq U$ and U is IFSOS. That is $\alpha cl(A) \subseteq U$. Therefore A is IFαGSCS in X. □

Example 3.4. Let $X = \{a, b\}$. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x, (0.4, 0.2), (0.6, 0.7)\rangle$. Then the IFS $A = \langle x, (0.7, 0.8), (0.3, 0.2)\rangle$ is IFαGSCS but not IFCS. Since $\alpha cl(A) = 1_\sim$ and possible $U = 1_\sim$.

Theorem 3.5. *Every IFαCS in (X, τ) is an IFαGSCS in (X, τ) but not conversely.*

Proof. Let A be an IFαCS in X. Let us consider an IFS $A \subseteq U$ and U be an IFSOS in (X, τ). Since A is an IFαCS, $\alpha cl(A) = A$. Hence $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is IFSOS. Therefore A is an IFαGSCS in X. □

Example 3.6. Let $X = \{a, b\}$. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x, (0.4, 0.5), (0.6, 0.5)\rangle$. Consider an IFS $A = \langle x, (0.8, 0.6), (0.2, 0.4)\rangle$ is IFαGSCS but not IFαCS. Since $cl(int(cl(A))) = 1_\sim \nsubseteq A$.

Theorem 3.7. *Every IFRCS in (X, τ) is an IFαGSCS in (X, τ), but not conversely.*

Proof. Let A be an IFRCS in (X, τ). Since every IFRCS is an IFCS, A is an IFCS in X. Hence by Theorem 3.3, A is an IFαGSCS in X. □

Example 3.8. Let $X = \{a, b\}$. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x, (0.8, 0.7), (0.2, 0.3)\rangle$. Consider an IFS $A = \langle x, (0, 0.2), (0.9, 0.8)\rangle$ which is an IFαGSCS but not IFRCS in X as $cl(int(A)) = 0_\sim \neq A$.

Remark 3.9. *An IFG closedness is independent of an IFαGS closedness.*

Example 3.10. Let $X = \{a, b\}$. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x, (0.4, 0.7), (0.5, 0.3)\rangle$.Then the IFS $A = \langle x, (0.3, 0.2), (0.6, 0.8)\rangle$ is an IFαGSCS but not IFGCS in X as $cl(A) \nsubseteq G$ eventhough $A \subseteq G$ and G is an IFSOS in X.

Example 3.11. Let $X = \{a, b\}$. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x, (0.6, 0.8), (0.4, 0.2)\rangle$. Then the IFS $A = \langle x, (0.7, 0.9), (0.3, 0.1)\rangle$ is an IFGCS but not IFαGSCS since $\alpha cl(A) = 1_\sim \nsubseteq B = \langle x, (0.8, 0.9), (0.2, 0)\rangle$ whenever $A \subseteq B$ and B is an IFSOS in X.

Theorem 3.12. *Every IFαGSCS in (X, τ) is an IFGSCS in (X, τ), but its converse may not be true in general.*

Proof. Assume that A is an IFαGSCS in (X, τ). Let an IFS $A \subseteq U$ and U be an IFOS in X. By hypothesis $\alpha cl(A) \subseteq U$, that is $A \cup cl(int(cl(A))) \subseteq U$. This implies $A \cup int(cl(A)) \subseteq U$. But $scl(A) = A \cup int(cl(A))$. Therefore $scl(A) = A \cup int(cl(A)) \subseteq U$ whenever $A \subseteq U$ and U is IFOS. Hence A is IFGSCS. □

Example 3.13. Let $X = \{a, b\}$. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x, (0.7, 0.8), (0.3, 0.1)\rangle$. Then the IFS $A = \langle x, (0.8, 0.8), (0.2, 0)\rangle$ is an IFGSCS but not an IFαGSCS as $\alpha cl(A) = 1_\sim \nsubseteq B = \langle x, (0.9, 0.9), (0.1, 0)\rangle$ whenever $A \subseteq B$ and B is an IFSOS in X.

Theorem 3.14. *Every IFαGSCS in (X, τ) is an IFGSPCS. But its converse need not be true in general.*

Proof. Assume that A is an IFαGSCS in (X, τ). Let an IFS $A \subseteq U$ and U be an IFOS. By hypothesis $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is IFSOS. This implies $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is IFOS. Since every IFαCS is an IFSPCS in (X, τ). We have $spcl(A) \subseteq \alpha cl(A)$. This implies $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is IFOS. Hence A is IFGSPCS in (X, τ). □

Example 3.15. Let $X = \{a, b\}$. Let $\tau = \{0_\sim, G, 1_\sim\}$ where $G = \langle x, (0.3, 0.1), (0.6, 0.8)\rangle$. An IFS $A = \langle x, (0.2, 0), (0.7, 0.8)\rangle$ is IFGSPCS but not IFαGSCS as $\alpha cl(A) \nsubseteq G$ even though $A \subseteq G$ and G is IFSOS.

Remark 3.16. *An IFP closedness is independent of IFαGS closedness.*

Example 3.17. Let $X = \{a, b\}$. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x, (0.4, 0.3), (0.6, 0.5)\rangle$. Then the IFS $A = \langle x, (0.2, 0.1), (0.7, 0.8)\rangle$ is IFPCS but not IFαGSCS. Since $\alpha cl(A) \nsubseteq G$ even though $A \subseteq G$ and G is IFSOS.

Example 3.18. Let $X = \{a, b\}$. Let $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ be an IFT on X, where $G_1 = \langle x, (0.3, 0.4), (0.7, 0.6)\rangle$ and $G_2 = \langle x, (0.4, 0.5), (0.6, 0.5)\rangle$. Then the IFS $A = \langle x, (0.3, 0.6), (0.7, 0.4)\rangle$ is IFαGSCS. Since $cl(int(A)) \nsubseteq A$, A is not an IFPCS.

Remark 3.19. *IFSP closedness is independent of an IFαGS closedness.*

Example 3.20. Let $X = \{a, b\}$. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x, (0.3, 0.4), (0.6, 0.5)\rangle$. Then an IFS $A = \langle x, (0.4, 0.3), (0.5, 0.6)\rangle$ is IFSPCS but not IFαGSCS. Since $\alpha cl(A) \nsubseteq B$, $B = \langle x, (0.5, 0.4), (0.4, 0.5)\rangle$ where $A \subseteq B$ and B is IFSOS.

Example 3.21. Let $X = \{a, b\}$. Let $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ be an IFT on X, where $G_1 = \langle x, (0.3, 0.4), (0.7, 0.6)\rangle$ and $G_2 = \langle x, (0.4, 0.5), (0.6, 0.5)\rangle$. Then the IFS $A = \langle x, (0.3, 0.6), (0.7, 0.4)\rangle$ is an IFαGSCS but not IFSPCS as $int(cl(int(A))) \nsubseteq A$.

Remark 3.22. *IFγclosedness is independent of IFαGS closedness.*

Example 3.23. Let $X = \{a, b\}$. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x, (0.2, 0.4), (0.8, 0.6)\rangle$. Then the IFS $A = \langle x, (0.6, 0.5), (0.4, 0.5)\rangle$ is IFγCS but not IFαGSCS. Since $\alpha cl(A) \nsubseteq B = \langle x, (0.7, 0.6), (0.3, 0.4)\rangle$ where $A \subseteq B$ and B is IFSOS in X.

Example 3.24. Let $X = \{a, b\}$. Let $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ be an IFT on X, where $G_1 = \langle x, (0.3, 0.4), (0.7, 0.6)\rangle$ and $G_2 = \langle x, (0.4, 0.5), (0.6, 0.5)\rangle$. Then the IFS $A = \langle x, (0.3, 0.6), (0.7, 0.4)\rangle$ is an IFαGSCS but not IFγCS as $int(cl(A)) \cap cl(int(A)) \nsubseteq A$.

Theorem 3.25. *Every IFαGSCS in (X, τ) is an IFαGCS in (X, τ) but not conversely.*

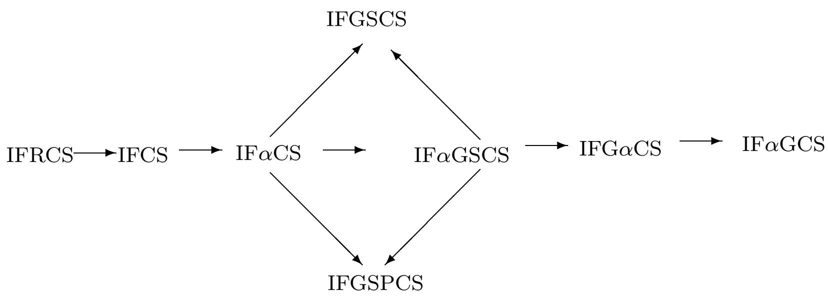
Proof. Assume that A is an IFαGSCS in (X, τ). Let us consider an IFS $A \subseteq U$ and U is IFOS in (X, τ). By hypothesis $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is IFSOS. This implies $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is IFOS. Therefore A is an IFαGCS in (X, τ). □

Example 3.26. Let $X = \{a, b\}$. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x, (0.1, 0.3), (0.7, 0.6)\rangle$. Then the IFS $A = \langle x, (0.3, 0.4), (0.6, 0.5)\rangle$ is IFαGCS but not IFαGSCS. Since $\alpha cl(A) \nsubseteq B = \langle x, (0.4, 0.4), (0.5, 0.4)\rangle$ eventhough $A \subseteq B$ and B is IFSOS.

Theorem 3.27. *Every IFαGSCS in (X, τ) is an IFGαCS in (X, τ) but its converse may not be true in general.*

Proof. Assume that A is an IFαGSCS in $(X,\,\tau)$. Let $A \subseteq U$ and U be IFαOS in X. By hypothesis $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is IFSOS. This implies $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and IFαOS. Therefore A is an IFGαCS in $(X,\,\tau)$. □

Example 3.28. Let $X = \{a, b\}$. Let $\tau = \{0_\sim,\, G,\, 1_\sim\}$ be an IFT on X, where $G = \langle x,\, (0.4,\, 0.2),\, (0.6,\, 0.8)\rangle$. Then the IFS $A = \langle x,\, (0.5,\, 0.6),\, (0.5,\, 0.4)\rangle$ is IFGαCS but not IFαGSCS. Since $\alpha cl(A) \nsubseteq B = \langle x,\, (0.55,\, 0.7),\, (0.45,\, 0.3)\rangle$ eventhough $A \subseteq B$ and B is IFSOS.
The relations between various types of intuitionistic fuzzy closed sets are given in the following diagram.



The reverse implications are not true in general.

Remark 3.29. *The intersection of any two IFαGSCS is not an IFαGSCS in general as seen from the following example.*

Example 3.30. Let $X = \{a, b\}$. Let $\tau = \{0_\sim,\, G,\, 1_\sim\}$ be an IFT on X, where $G = \langle x,\, (0.2,\, 0.3),\, (0.7,\, 0.6)\rangle$. Then the IFS $A = \langle x,\, (0.1,\, 0.8),\, (0.7,\, 0.2)\rangle$, $B = \langle x,\, (0.8,\, 0.2),\, (0.1,\, 0.7)\rangle$ are IFαGSCS. Now $A \cap B = \langle x,\, (0.1,\, 0.2),\, (0.7,\, 0.7)\,\rangle$. Since $\alpha cl(A \cap B) \nsubseteq G$ eventhough $A \subseteq G$ and G is IFSOS in X, $A \cap B$ is not an IFαGSCS in X.

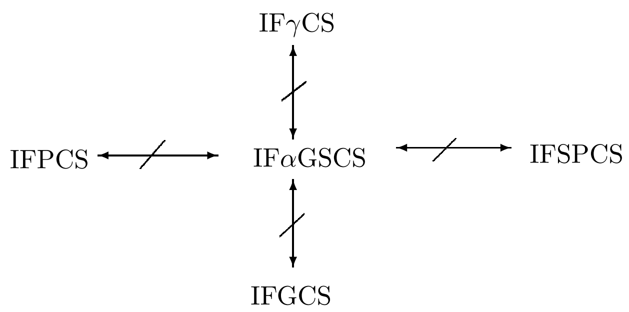
Theorem 3.31. *Let (X, τ) be an IFTS. Then for every $A \in$ IFαGSC(X) and for every $B \in$ IFS(X), $A \subseteq B \subseteq \alpha cl(A)$ implies $B \in$ IFαGSC(X).*

Proof. Let an IFS $B \subseteq U$ and U be an IFSOS in X. Since $A \subseteq B$, $A \subseteq U$ and A is IFαGSCS, $\alpha cl(A) \subseteq U$. By hypothesis, $B \subseteq \alpha cl(A)$, $\alpha cl(B) \subseteq \alpha cl(A) \subseteq U$. Therefore $\alpha cl(B) \subseteq U$. Hence B is IFαGSCS of X. □

Theorem 3.32. *If A is both IFSOS and IFαGSCS in (X, τ), then A is an IFαCS in X.*

Proof. Let A be an IFSOS in X. Since $A \subseteq A$, by hypothesis $\alpha cl(A) \subseteq A$. But $A \subseteq \alpha cl(A)$. Therefore $\alpha cl(A) = A$. Hence A is an IFαCS in X. □

The independent relations between various types of intuitionistic fuzzy closed sets are given in the following diagram.



Theorem 3.33. *The union of two IFαGSCS is an IFαGSCS in (X, τ), if they are IFCS in (X, τ).*

Proof. Assume that A and B are IFαGSCS in (X, τ). Since A and B are IFCS in X, $cl(A) = A$ and $cl(B) = B$. Let $A \cup B \subseteq U$ and U is IFSOS in X. Then $cl(int(cl(A \cup B))) = cl(int(A \cup B)) \subseteq cl(A \cup B) = A \cup B \subseteq U$, that is $\alpha cl(A \cup B) \subseteq U$. Therefore $A \cup B$ is IFαGSCS. □

Theorem 3.34. *Let (X, τ) be an IFTS and A be an IFS in X. Then A is an IFαGSCS if and only if A $\overline{q}$ F implies $\alpha cl(A)$ $\overline{q}$ F for every IFSCS F of X.*

Proof. Necessary Part: Let F be an IFSCS in X and let $A \ \overline{q} \ F$. Then $A \subseteq F^c$, where F^c is an IFSOS in X. Therefore by hypothesis $\alpha cl(A) \subseteq F^c$. Hence $\alpha cl(A)$ $\overline{q} \ F$.

Sufficient Part: Let F be an IFSCS in X and let A be an IFS in X. By hypothesis, $A \ \overline{q} \ F$ implies $\alpha cl(A) \ \overline{q} \ F$. Then $\alpha cl(A) \subseteq F^c$ whenever $A \subseteq F^c$ and F^c is an IFSOS in X. Hence A is an IFαGSCS in X. □

4 Intuitionistic Fuzzy α-Generalized Semi-Open Sets

In this section we introduce intuitionistic fuzzy α-generalized semi-open sets and study some of its properties.

Definition 4.1. An IFS A is said to be *intuitionistic fuzzy α-generalized semi-open set*(IFαGSOS in short) in (X, τ), if the complement A^c is an IFαGSCS in X.

The family of all IFαGSOS of an IFTS (X, τ) is denoted by $IF\alpha GSO(X)$.

Theorem 4.2. *For any IFTS (X, τ), every IFOS is an IFαGSOS, but not conversely.*

Proof. Let A be an IFOS in X. Then A^c is an IFCS in X. by Theorem 3.3, A^c is an IFαGSCS in X. Hence A is an IFαGSOS in X. □

Example 4.3. Let $X = \{a, b\}$. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x, (0.2, 0.1), (0.7, 0.8)\rangle$. Then the IFS $A = \langle x, (0.1, 0), (0.8, 0.9)\rangle$. Since A^c is an IFαGSCS, A is an IFαGSOS, but not IFOS.

Theorem 4.4. *In any IFTS (X, τ) every IFαOS is an IFαGSOS but not conversely.*

Proof. Let A be an IFαOS in X. Then A^c is an IFαCS in X. by Theorem 3.5, A^c is an IFαGSCS in X. Hence A is an IFαGSOS in X. □

Example 4.5. Let $X = \{a, b\}$. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x, (0.4, 0.5), (0.6, 0.5)\rangle$. Then the IFS $A = \langle x, (0.2, 0.4), (0.8, 0.6)\rangle$ is an IFαGSOS in X, but A is not an IFαOS in X.

Theorem 4.6. *For any IFTS (X, τ), every IFROS is an IFαGSOS but not conversely.*

Proof. Let A be an IFROS in X. Then A^c is an IFRCS in X. by Theorem 3.7, A^c is an IFαGSCS in X. Hence A is an IFαGSOS in X. □

Example 4.7. Let $X = \{a, b\}$. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x, (0.6, 0.7), (0.3, 0.1)\rangle$. Then the IFS $A = \langle x, (0.7, 0.8), (0.2, 0)\rangle$ is an IFαGSOS in X, but A is not an IFROS in X.

Remark 4.8. *IFαGSOS and IFGOS are independent in general.*

Example 4.9. Let $X = \{a, b\}$. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x, (0.4, 0.7), (0.5, 0.3)\rangle$. Then the IFS $A = \langle x, (0.6, 0.8), (0.3, 0.2)\rangle$ is an IFαGSOS in X, but A is not an IFGOS in X.

Example 4.10. Let $X = \{a, b\}$. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x, (0.6, 0.8), (0.4, 0.2)\rangle$. Then the IFS $A = \langle x, (0.3, 0.1), (0.7, 0.9)\rangle$ is an IFGOS in X, but A is not an IFαGSOS in X.

Theorem 4.11. *Every IFαGSOS in (X, τ) is an IFGSOS in (X, τ) but it converse may not be true in general.*

Proof. Let A be an IFαGSOS in X. Then A^c is IFαGSCS in X. by Theorem 3.12, A^c is IFGSCS in X. Hence A is an IFGSOS in X. □

Example 4.12. Let $X = \{a, b\}$. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x, (0.7, 0.8), (0.3, 0.1)\rangle$. Then the IFS $A = \langle x, (0.2, 0), (0.8, 0.8)\rangle$ is an IFGSOS in X, but A is not an IFαGSOS in X.

Remark 4.13. *IFSPOS is independent of IFαGSOS.*

Example 4.14. Let $X = \{a, b\}$. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x, (0.3, 0.4), (0.6, 0.5)\rangle$. Then the IFS $A = \langle x, (0.5, 0.6), (0.4, 0.3)\rangle$ is an IFSPOS in X, but A is not an IFαGSOS in X.

Example 4.15. Let $X = \{a, b\}$. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G_1 = \langle x, (0.3, 0.4), (0.7, 0.6)\rangle$ and $G_2 = \langle x, (0.4, 0.5), (0.6, 0.5)\rangle$ Then the IFS $A = \langle x, (0.7, 0.4), (0.3, 0.6)\rangle$ is an IFαGSOS in X, but A is not an IFSPOS in X.

Theorem 4.16. *Every IFαGSOS in (X, τ) is an IFαGOS in (X, τ), but its converse may not be true in general.*

Proof. Let A be an IFαGSOS in X.Then A^c is an IFαGSCS in X. by Theorem 3.25, A^c is an IFαGCS in X. Hence A is an IFαGOS in X. □

Example 4.17. Let $X = \{a, b\}$. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x, (0.1, 0.3), (0.7, 0.6)\rangle$. Then the IFS $A = \langle x, (0.6, 0.5), (0.3, 0.4)\rangle$ is IFαGOS in X, but not IFαGSOS in X.

Theorem 4.18. *Every IFαGSOS in (X, τ) is an IFGαOS in (X, τ), but its converse may not be true in general.*

Proof. Let A be an IFαGSOS in X. Then A^c is an IFαGSCS in X. by Theorem 3.27, A^c is an IFGαCS in X. Hence A is an IFGαOS in X. □

Example 4.19. Let $X = \{a, b\}$. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x, (0.4, 0.2), (0.6, 0.8)\rangle$. Then the IFS $A = \langle x, (0.5, 0.4), (0.5, 0.6)\rangle$ is IFGαOS in X, but not IFαGSOS in X.

Theorem 4.20. *Let (X, τ) be an IFTS. If A is an IFS of X then the following properties are equivalent:*

1. *$A \in$ IFαGSO(X).*
2. *$V \subseteq int(cl(int(A)))$ whenever $V \subseteq A$ and V is an IFSCS in X.*
3. *There exists IFOS G_1 such that $G_1 \subseteq V \subseteq int(cl(G))$ where $G = int(A)$; $V \subseteq A$ and V is an IFSCS in X.*

Proof. (1) ⇒ (2) Let $A \in$ IFαGSO(X). Then A^c is an IFαGSCS in X. Therefore $\alpha cl(A^c) \subseteq U$ whenever $A^c \subseteq U$ and U is an IFSOS in X. That is $cl(int(cl(A^c))) \subseteq U$. Taking complement on both sides, we get $(cl(int(cl(A^c))))^c = int(int(cl(A^c)))^c = int(cl(cl(A^c))^c) = int(cl(int(A^c)^c)) = int(cl(int(A))) \supseteq U^c$. This implies $U^c \subseteq int(cl(int(A)))$ whenever $U^c \subseteq A$ and U^c is an IFSCS in X. Replace U^c by V, $V \subseteq int(cl(int(A)))$ whenever $V \subseteq A$ and V is an IFSCS in X.

(2) ⇒ (3) Let $V \subseteq int(cl(int(A)))$ whenever $V \subseteq A$ and V is an IFSCS in X. Hence $int(V) \subseteq V \subseteq int(cl(int(A)))$. Then there exists IFOS G_1 in X such that $G_1 \subseteq V \subseteq int(cl(G))$ where $G = int(A)$ and $G_1 = int(V)$.

(3)⇒ (1) Suppose that there exists IFOS G_1 such that $G_1 \subseteq V \subseteq int(cl(G))$ where $G = int(A)$; $V \subseteq A$ and V is an IFSCS in X. It is clear that $(int(cl(G)))^c \subseteq V^c$. That is $(int(cl(int(A))))^c \subseteq V^c$. This implies $cl(cl(int(A)))^c \subseteq V^c$. Therefore $cl(int(cl(A^c))) \subseteq V^c$, $A^c \subseteq V^c$ and V^c is IFSOS in X. Hence $\alpha cl(A^c) \subseteq V^c$. That is A^c is an IFαGSCS in X. This implies $A \in$ IFαGSO(X). □

Theorem 4.21. *Let (X, τ) be an IFTS. Then for every $A \in$ IFαGSO(X) and for every $B \in$ IFS(X), $\alpha int(A) \subseteq B \subseteq A$ implies $B \in$ IFαGSO(X).*

Proof. By hypothesis $\alpha int(A) \subseteq B \subseteq A$. Taking complement on both sides, we get $A^c \subseteq B^c \subseteq (\alpha int(A))^c$. Let $B^c \subseteq U$ and U be an IFSOS in X. Since $A^c \subseteq B^c$, $A^c \subseteq U$. Since A^c is an IFαGSCS, $\alpha cl(A^c) \subseteq U$. Also $B^c \subseteq (\alpha int(A))^c = \alpha cl(A^c)$. Therefore $\alpha cl(B^c) \subseteq \alpha cl(A^c) \subseteq U$. Hence B^c is an IFαGSCS in X. This implies B is an IFαGSOS in X. That is $B \in$IFαGSO(X). □

Remark 4.22. *The union of any two IFαGSOS in (X, τ) is not an IFαGSOS in (X, τ).*

Example 4.23. Let $X = \{a, b\}$. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x, (0.3, 0.2), (0.6, 0.7)\rangle$. Then the IFS $A = \langle x, (0.6, 0.1), (0.2, 0.8)\rangle$ and $B = \langle x, (0.2, 0.8), (0.7, 0.1)\rangle$ are IFαGSOS in X but $A \cup B = \langle x, (0.6, 0.8), (0.2, 0.1) \rangle$ is not an IFαGSOS in X.

Theorem 4.24. *An IFS A of an IFTS (X, τ) is an IFαGSOS if and only if $F \subseteq \alpha int(A)$ whenever $F \subseteq A$ and F is an IFSCS in X.*

Proof. Necessary Part: Suppose A is an IFαGSOS in X. Let F be an IFSCS in X and $F \subseteq A$. Then F^c is an IFSOS in X such that $A^c \subseteq F^c$. Since A^c is an IFαGSCS, we have $\alpha cl(A^c) \subseteq F^c$. Hence $(\alpha int(A))^c \subseteq F^c$. Therefore $F \subseteq \alpha int(A)$.
Sufficient Part: Let A be an IFS in X and let $F \subseteq \alpha int(A)$ whenever F is an IFSCS in X and $F \subseteq A$. Then $A^c \subseteq F^c$ and F^c is an IFSOS. By hypothesis, $(\alpha int(A))^c \subseteq F^c$, which implies $\alpha cl(A^c) \subseteq F^c$. Therefore A^c is an IFαGSCS in X. Hence A is an IFαGSOS in X. □

5 Conclusion

In this paper we have introduced intuitionistic fuzzy α-generalized semi closed sets and intuitionistic fuzzy α-generalized semi open sets. Also we have studied some of its basic properties and the relationships between other existing intuitionistic fuzzy closed and open sets.

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