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Real Option Pricing Model Based on Mean Reversion Applied in a Wind Power Project

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Abstract : This paper evaluates the market value of a wind power project in China through a real option method which considers the uncertainty of on-grid electricity. The evaluating model assumes that the wind power project revenue follows a mean-reverting process of the Ornstein-Uhlenbeck (O-U) type and discusses the effect of cost and parameters of mean-reverting process on the project value. This study proposes to use Monte Carlo simulation method to price the wind power project market value and presents that this real option method can allow wind power project investors to decide whether to invest in many different scenarios.

Keywords : real options analysis; wind power; mean reverting process; Monte Carlo simulation.
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1 Introduction

Project valuation probably is the most important part of the investment process. It's well known that there are many methods to appraise a project value,

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such as discounted cash flow analysis, decision trees and others. As we known, most investment decisions share three important features which are irreversibility, uncertainty, and flexibility in varying degrees. However, the traditional methods usually ignore the uncertainties in the investment process that may influence the project value evaluation.

Real options analysis (ROA) as a new framework in the theory of investment decision has been recognized by more and more people in the past twenty years through many publications((Trigeorgis, 1996)[1]; (Buckley, 1998)[2]; (Mun, 2002) [3]; (Kodukula and Papudesu, 2006)[4]; (Guthrie, 2009)[5]; and (Damodaran, 2012)[6]). ROA can give flexibility to investors when making decisions about real assets, revealing uncertainty associated with cash-flows, and allowing investors to make decisions that positively influence the final project value.

ROA is useful in project appraisal when the project revenue streams resulting from the investment are uncertain and now ROA is widely used as a tool to help decision making in many fields(Trigeorgis, 1996)[1]. In the energy investment area, there have been a growing number of publications on real options analysis in energy investment in recent years, especially, in wind power project which we shall consider in this paper.

There are many authors use ROA model to evaluate the market value of a wind power project with different approaches. (Fleten and Maribu, 2004)[7] and (Cheng, Hou, and Wu, 2010)[8] evaluate the market value by PDE method while (Luna, Assuad, and Dyner, 2003)[9] and (Cheng et al., 2010)[8] evaluate the market value by binomial tree method. Moreover, (Yang, Nguyen, De T'Serclaes, and Buchner, 2010)[10] use simulation method and assume that the electricity price follows geometric Brownian motion. (Zhou et al., 2007)[11] also use simulation method but he assume that the electricity price follows the mean reversion process. However, to our knowledge, few authors study the problem of using simulation method with electricity output follow the mean-reverting of Ornstein-Uhlenbeck (O-U) type process which we shall consider here.

For more detail, we propose to employ a ROA model to evaluate the market value of a wind power projects in China by using Monte Carlo simulation method which the uncertainty of output (also called on-grid electricity) of the wind farm follows O-U process.

2 Background

Let us give a brief introduction of wind power investment environment in China. According to the global wind report 2015 (GWEC, 2016)[12], China added 30.8 GW of wind installed capacity in 2015 which alone accounted for 48% of total global installation and this makes the cumulative wind power installed capacity in China reached 145.4GW which accounted for about 33% of total global cumulative installation. Wind power has entered the large-scale development phase in China. There are some favorable and unfavorable factors in wind power investment. Such as according to Renewable Energy Law of China (MOFCOM, 2009)[13], a power grid company signs a long-term power purchase agreement with wind power project (WPP) investors and agrees to buy all electricity generated by the WPP within the coverage of their power grid.In addition, the Clean Development Mechanism (CDM) is one of the Flexible Mechanisms defined in the Kyoto Protocol (Solomon, 2007)[14] that provides for emissions reduction projects which generate Certified Emission Reduction units and may be traded in emissions trading schemes. If a CDM project invests in China, it may claim Certified Emission Reductions (CERs) from the project and may trade the CERs in industrialized countries to recover part of its investment cost or make a profit. Apart from the above, there still exist some unfavorable factors to investment income, such as abandoned wind power rationing. In March 2011, the State Electricity Regulatory Commission issued the “Wind Power and Photovoltaic Power Generation Regulatory Report” (Council, 2012)[15], which provided statistics regarding non-purchased wind power electricity during January-June 2010. The amount wind electricity which was curtailed in the north and northeast areas were the largest, accounting for 57.20% and 38.33% of the total nationwide, respectively. Abandoned wind rate has been an important factor to affect investment profits.

Because of the specific nature of investment in China, we consider the on-grid electricity price and CERs price as constants in our model.

3 Methodology

3.1 Modeling under Mean Reverting Process

We consider the revenue of a completed wind power project (WPP), and we suppose the WPP is a CDM project. Thus the revenue of the WPP comes from electricity output and carbon emission income. Let V be the revenue of the WPP, we have

$$V = G_e P_f + G_e F_e P_e \tag{3.1}$$

where G_e is electricity output of wind power farm, F_e is emission factor of carbon, P_f and P_e are on-grid electricity price and Certified Emission Reductions (CERs) prices, respectively.

For numerical calculation, we collected the historical data of monthly electricity fed to the grid from a wind power farm which named project No. 0689 (PDD.2006)[16]. Figure 1 shows the time series plot of the monthly revenue V.

After a brief analysis on the data by using ACF and PACF techniques, one can see that the time series V is a stationary process. The O-U process (3.2) is a continuous time mean reverting process and can be used to model a stationary series (Arratia, Cabana, and Cabana, 2012)[17]. Thus in this study, we suppose the WPP's revenue follows an Ornstein-Uhlenbeck process (also called one-factor mean-reverting process):

$$dV_t = q(\frac{p}{q} - V_t)dt + \sigma dz_t, \tag{3.2}$$

where $dz_t = \varepsilon_t\sqrt{dt}$,$\varepsilon_t \sim N(0,1)$ and z_t is a Brownian motion, q measures the speed of mean reversion, $\frac{p}{q}$ is the "long run mean" to which the process tends to revert, and σ is a measure of the process volatility.

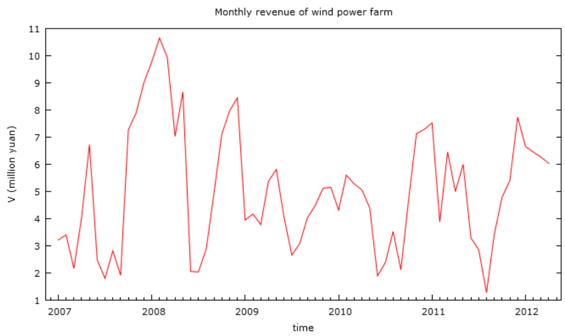

Figure 1: Historical data of $V = G_eP_f + G_eF_eP_e$

Now, our starting point is to consider the following problem: at what point is it optimal to pay a sunk cost I in return worth V for a WPP. Note that the WPP investment opportunity is equivalent to a perpetual call option: the right but not the obligation to buy a share of stock at a pre-specified price. Therefore, the decision to invest is equivalent to decide when to exercise such an option. Thus the investment decision can be viewed as a problem of option valuation. Alternatively, it can be viewed as a problem in dynamic programming. We will derive the optimal investment rule by using contingent claims methods. In what follows, we will denote the value of the investment opportunity, or equivalently, the value of the option to invest WPP by F where $F = F(V(t))$ is a function of V and t. Once we hold the option, we want a rule that maximize its expected present value (A.K. Dixit and R.S. Pindyck, 1994)[18]:

$$F(V_t) = \max E[e^{-r(T-t)}(V_T - I)^+], T \geq t. \tag{3.3}$$

Here T is the unknown time when the decision is made and r is the discount rate. To facilitate the application of ROA, we will use the Monte Carlo simulation method to solve this problem.

3.2 Solution and Parameters Estimation of O-U Process

3.2.1 The Explicit Solution of O-U Process

We go back to find the solution of equation (3.2). Let $f(V_t, t) = V_te^{qt}$ and by using Ito's lemma, one get

$$\begin{aligned} df(V_t,t) &= qV_te^{qt}dt + e^{qt}dV_t \\ &= qV_te^{qt}dt + e^{qt}[q(\tfrac{p}{q} - V_t)dt + \sigma dz_t] \\ &= pe^{qt}dt + \sigma e^{qt}dz_t. \end{aligned} \tag{3.4}$$

Integrate on the both sides of equation (3.4) from 0 to t, we have

$$V_t e^{qt} = V_0 + \int_0^t e^{qs} p ds + \int_0^t e^{qs} \sigma dz_s. \tag{3.5}$$

Thus we can get the explicit solution of O-U process,

$$\begin{aligned} V_t &= V_0 e^{-qt} + \tfrac{p}{q}(1 - e^{-qt}) + \textstyle\int_0^t e^{q(s-t)} \sigma dz_s \\ &= V_0 e^{-qt} + \tfrac{p}{q}(1 - e^{-qt}) + e^{-qt} \sigma \textstyle\int_0^t e^{qs} dz_s. \end{aligned} \tag{3.6}$$

Recall from the definition of the Ito stochastic integral that $\int_0^t e^{qs} dz_s$ (=W_t say)is the mean square limit of approximating Riemann-Stieltjes sums

$$S_n = \sum_{i=1}^{n} e^{qs_{i-1}} (z_{s_i} - z_{s_{i-1}}). \tag{3.7}$$

For a partition (τ_n) of $[0,t]$ with mesh $(\tau_n) \to 0$. The latter sum has a normal distribution with mean zero and variance

$$\sum_{i=1}^{n} e^{2qs_{i-1}} (s_i - s_{i-1}). \tag{3.8}$$

Note that (3.8) is the Riemann sum approximation to the integral

$$\int_0^t e^{2qs} ds = \frac{e^{2qt} - 1}{2q}. \tag{3.9}$$

Since the mean square convergence implies convergence in distribution. We may conclude that the mean square limit W_t of the normally distributed Riemann-Stieljes sums S_n is normally distributed with

$$E(W_t) = 0, \quad Var(W_t) = \frac{e^{2qt} - 1}{2q}. \tag{3.10}$$

Additionally, according to the properties of Brownian Motion $\{z_t\}$,we can get the mean and variance of V_t as follows:

$$E(V_t) = V_0 e^{-qt} + \frac{p}{q}(1 - e^{-qt}), \tag{3.11}$$

$$\begin{aligned} Var(V_t) &= Var(e^{-qt} \sigma \int_0^t e^{qs} dz_s) \\ &= \sigma^2 e^{-2qt} Var(W_t) \\ &= \frac{\sigma^2}{2q}(1 - e^{-2qt}). \end{aligned} \tag{3.12}$$

Hence, the O-U mean reverting model is a Gaussian model in the sense that, given V_0 and the time t, the process V_t is normally distributed,

$$V_t \sim N\left(V_0e^{-qt} + \frac{p}{q}(1-e^{-qt}), \frac{\sigma^2}{2q}(1-e^{-2qt})\right). \tag{3.13}$$

As time $t \to \infty$,we can see from the above equations that

$$\lim_{t\to\infty} E(V_t) := E(V_\infty) = \frac{p}{q}, \lim_{t\to\infty} Var(V_t) := Var(V_\infty) = \frac{\sigma^2}{2q}, \tag{3.14}$$

and O-U stochastic process converges in distribution to $N(\frac{p}{q}, \frac{\sigma^2}{2q})$ as time $t \to \infty$.

3.2.2 Parameter Estimation Method of O-U Process

Using Euler's method, we can first discretize the O-U process and then use the Maximum Likelihood Estimation method (MLE) to obtain the parameters p, q , and σ. According to Euler's discretization method and assume that the time-step is Δ, the discrete form of (3.6) is as follows:

$$V_{t+1} = V_te^{-q\Delta} + \frac{p}{q}(1-e^{-q\Delta}) + e^{-q\Delta}\sigma\int_0^\Delta e^{qs}dz_s. \tag{3.15}$$

It follows from equation (3.10) the random variable $\int_0^\Delta e^{qs}dz_s$ is normally distributed with its mean and variance as follows:

$$E(\int_0^\Delta e^{qs}dz_s) = 0, \quad Var(\int_0^\Delta e^{qs}dz_s) = \frac{e^{2q\Delta}-1}{2q}. \tag{3.16}$$

Then one can write

$$\int_0^\Delta e^{qs}dz_s = \sqrt{\frac{e^{2q\Delta}-1}{2q}}\varepsilon_t. \text{ where } \varepsilon_t \sim N(0,1). \tag{3.17}$$

Now, we rewrite the exact solution of the: equation (3.15) into discrete form as follows:

$$\begin{aligned} V_{t+1} &= V_te^{-q\Delta} + \tfrac{p}{q}(1-e^{-q\Delta}) + e^{-q\Delta}\sigma\sqrt{\tfrac{e^{2q\Delta}-1}{2q}}\varepsilon_t \\ &= V_te^{-q\Delta} + \tfrac{p}{q}(1-e^{-q\Delta}) + \sigma\sqrt{\tfrac{1-e^{-2q\Delta}}{2q}}\varepsilon_t \end{aligned} \tag{3.18}$$

and V_t is normally distributed with

$$V_t \sim N\left(V_te^{-q\Delta} + \frac{p}{q}(1-e^{-q\Delta}), \sigma^2\frac{1-e^{-2q\Delta}}{2q}\right). \tag{3.19}$$

We note that the probability density function $f(x)$ of a normal distribution $X \sim N(a, b^2)$ is:

$$f(x) = \frac{1}{\sqrt{2\pi}b}e^{-\frac{(x-a)^2}{2b^2}}. \tag{3.20}$$

So, by substituting $a = v_t e^{-q\Delta} + \frac{p}{q}(1 - e^{-q\Delta})$,$b^2 = \sigma^2 \frac{1-e^{-2q\Delta}}{2q}$ into (3,20) and let $\mu = \frac{p}{q}$ then we get the conditional probability density of an observation v_{i+1} condition on previous observation v_i is

$$f(v_i|v_{i-1}, \mu, q, \hat{\sigma}) = \frac{1}{\sqrt{2\pi\hat{\sigma}^2}} \exp\left[-\frac{(v_i - v_{i-1}e^{-q\Delta} - \mu(1 - e^{-q\Delta}))^2}{2\hat{\sigma}^2}\right], \tag{3.21}$$

where $\hat{\sigma}^2 = \sigma^2 \frac{1-e^{-2q\Delta}}{2q}$.

The log-likelihood function of the set of data $v_0, v_1, v_2, \cdots, v_n$ can be obtained from the following function:

$$\begin{aligned} L(\mu, q, \hat{\sigma}) &= \sum_{i=1}^{n} \ln f(v_{i+1}|v_i, \mu, q, \hat{\sigma}) \\ &= \frac{n}{2}\ln(2\pi) - n\ln(\hat{\sigma}) - \frac{1}{2\hat{\sigma}^2}\sum_{i=1}^{n}\left[v_i - v_{i-1}e^{-q\Delta} - \mu(1-e^{-q\Delta})\right]^2. \end{aligned} \tag{3.22}$$

In order to derive the maximum likelihood, we set all the partial derivatives equal to zero:

$$\begin{cases} \frac{\partial L(\mu,q,\hat{\sigma})}{\partial \mu} = 0 \\ \frac{\partial L(\mu,q,\hat{\sigma})}{\partial q} = 0 \\ \frac{\partial L(\mu,q,\hat{\sigma})}{\partial \hat{\sigma}} = 0 \end{cases} \Rightarrow \begin{cases} \frac{1}{\hat{\sigma}^2}\sum\limits_{i=1}^{n}\left[v_i - v_{i-1}e^{-q\Delta} - \mu(1 - e^{-q\Delta})\right] = 0 \\ -\frac{\Delta e^{-q\Delta}}{\hat{\sigma}^2}\sum\limits_{i=1}^{n}\left[(v_i - \mu)(v_{i-1} - \mu) - e^{-q\Delta}(v_{i-1} - \mu)\right] = 0 \\ \frac{n}{\hat{\sigma}} - \frac{1}{\hat{\sigma}^2}\sum\limits_{i=1}^{n}\left[v_i - v_{i-1}e^{-q\Delta} - \mu(1 - e^{-q\Delta})\right]^2 = 0. \end{cases} \tag{3.23}$$

Solving these equations, we obtain:

$$\begin{cases} \mu = \frac{\sum\limits_{i=1}^{n}(v_i - v_{i-1}e^{-q\Delta})}{n(1-e^{-q\Delta})}, \\ q = \frac{1}{\Delta}\ln\frac{\sum\limits_{i=1}^{n}[(v_i-\mu)(v_{i-1}-\mu)]}{\sum\limits_{i=1}^{n}(v_{i-1}-\mu)^2}, \\ \hat{\sigma}^2 = \frac{1}{n}\sum\limits_{i=1}^{n}\left[v_i - \mu - e^{-q\Delta}(v_{i-1} - \mu)\right]^2. \end{cases} \tag{3.24}$$

Let us denote

$$s_x = \sum_{i=1}^{n} v_{i-1}, s_y = \sum_{i=1}^{n} v_i, s_{xx} = \sum_{i=1}^{n} v_{i-1}^2, s_{xy} = \sum_{i=1}^{n} v_i v_{i-1}, s_{yy} = \sum_{i=1}^{n} v_i^2. \tag{3.25}$$

Thus we can rewrite the estimation of parameters as follows:

$$\mu = \frac{s_y s_{xx} - s_x s_{xy}}{n(s_{xx} - s_{xy}) - (s_x^2 - s_x s_y)}, \tag{3.26}$$

$$q = -\frac{1}{\Delta}\ln\frac{s_{xy} - \mu(s_x + s_y) + n\mu^2}{s_{xx} - 2\mu s_x + n\mu^2}, p = \mu q, \tag{3.27}$$

$$\hat{\sigma}^2 = \frac{1}{n}[s_{yy} - 2e^{-q\Delta}s_{xy} + e^{-2q\Delta}s_{xx} - 2\mu(1-e^{-q\Delta})(s_y - e^{-q\Delta}s_x) + n\mu^2(1-e^{-q\Delta})^2], \tag{3.28}$$

and $\sigma^2 = \hat{\sigma}^2 \dfrac{2q}{1-e^{-2q\Delta}}$.

3.3 Monte Carlo Simulation Procedure

For the purpose of pricing the real option, we will solve the problem by simulation. Simulation is not an analytical method but is meant to imitate a real-life system, especially when other analyses are too mathematically complex or too difficult to reproduce. A simulation calculates numerous scenarios of a model by repeatedly picking values from the probability distribution for the uncertain variables and using those values for the event. One type of simulation is Monte Carlo simulation which randomly generates values for uncertain variables over and over to simulate a real-life model. In recent years researchers have begun to apply the Monte Carlo simulation method to the pricing of real options. In this study, the following basic steps are involved such calculations, and more details will be shown in the next section.

Step 1. Generate a random revenue path $V_i = (V_{i1}, V_{i2}, V_{i3}, \cdots, V_{in})$ which follows the O-U process, where $i = 1, 2, \cdots, m$ denote the simulation times.

Step 2. Use equation (3.3) and the simulation of revenue paths to calculate the value of option F_i.

Step 3. Repeating the above two steps to get a large number of samples $V_1, V_2, V_3, \cdots, V_m$ and $F_1, F_2, F_3, \cdots, F_m$.

Step 4. Calculating the average of $F_1, F_2, F_3, \cdots, F_m$, we obtain the option value $F = \frac{\sum_{i=1}^{m} F_i}{m}$.

3.4 Numerical Calculation

3.4.1 Monte Carlo Data Simulation

Mean reversion is the theory suggesting that prices and returns eventually move back towards their mean or average. This mean or average can be the historical average of the price, return, or another relevant average. We shall simulate annual data of on-grid electricity by using monthly on-grid electricity data. Table 1 shows a histogram data of monthly on-grid electricity (OGE) historical data from 2007 to 2011. (The data is collected from the Monitoring report forms of project No.0689 in the CDM database [19]).

One can see from Table 1 that the data presents a seasonal feature. We suppose that each monthly output follows one normal distribution and then simulate the annual data as follows.

Table 1: Monthly on-grid electricity from 2007 to 2011

Month(Gwh)	2007	2008	2009	2010	2011
January	5.31901	16.21047	6.53894	7.14620	12.47605
February	5.62989	17.67216	6.90734	9.28281	6.41836
March	3.57051	16.48416	6.25526	8.74791	10.69763
April	6.71283	11.65560	8.92105	8.37510	8.27833
May	11.15392	14.36952	9.63379	7.28175	9.94163
June	4.09018	3.39240	6.68772	3.12184	5.43568
July	2.96721	3.36336	4.39220	3.93134	4.71288
August	4.64902	4.76256	5.09862	5.83857	2.09069
September	3.16417	8.19446	6.67822	3.49657	5.67635
October	12.06622	11.78033	7.42845	7.81239	7.94024
November	13.09409	13.21852	8.47918	11.81753	8.95553
December	14.95098	14.01494	8.54082	12.08499	12.83414

Step 1. Let X_t denote the OGE of the month and assume that $X_i \sim N(\mu_i, \sigma_i^2)$,$i = 1, 2, 3, \cdots, 12$. We denote $x_{ij}(i = 1, 2, \cdots, 12; j = 1, 2, \cdots, n)$ for the sample data from the distribution $N(\mu_i, \sigma_i^2)$. Substituting the historical monthly data from Table 1 into the formula $\hat{\mu}_i = \frac{\sum_{j=1}^{5} x_{ij}}{5}$ and $\hat{\sigma}_i^2 = \frac{\sum_{j=1}^{5} (x_{ij}-\hat{\mu}_i)^2}{5-1}(i = 1, 2, \cdots, 12)$, we obtain the parameters estimation of mu_i, and σ_i^2.

Step 2. Use MATLAB program to generate random number $\hat{x}_i$ from the normal distribution $N(\mu_i, \sigma_i^2)$, $(i = 1, 2, \cdots, 12)$. Let $\hat{G}_e = \sum_{i=1}^{12} \hat{x}_i$, thus we obtain an annual data of OGE. By simulating 60 times, we can get a simulation data set $\left\{\hat{G}_{e1}, \hat{G}_{e2}, \hat{G}_{e3}, \cdots, \hat{G}_{e60}\right\}$. We get the simulation annual data set shown in Figure 2.

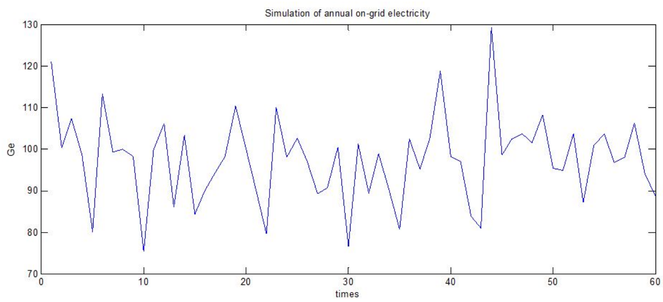

Figure 2: Simulation path of annual on-grid electricity

3.4.2 Parameters Estimation

According to the system requirements previously described, suppose we take 100 simulations to get a data set $\left\{\hat{G}_{e1}, \hat{G}_{e2}, \hat{G}_{e3}, \cdots, \hat{G}_{e100}\right\}$. Calculating revenue by using the formula $V = G_e P_f + G_e F_e P_e$, which follows the O-U process as the described in section 3.2. We can estimate the parameters by using the simulated data and obtain the O-U process $dV_t = 30.722(58.531 - V_t)dt + 46.539dz_t$.

Next, parameters description table which will be used in section 3.4.2 and 3.4.3 is shown in Table 2. In this table, some data in the table is collected from the project design document (PDD) of projects No. 0689 in the CDM database, some parameters are estimated by using formula, and some parameters is estimated according to the government or global agency reports.

Table 2: Parameters description

Parameters	Representation	Value(Unit)
P_f	On-grid electricity price (PDD, 2006)[16]	0.545 (Yuan/Kwh)
P_e	CERs price(PDD, 2006)[16]	7 (EUR/tCO2 e)
F_e	Baseline emission factor of carbon (PDD, 2006)[16]	1.024 (Ton/Mwh)
r	Risk-free rate	0.05%
μ	Estimation of long run mean of O-U process	58.531
q	Estimation of the speed of mean reversion	30.722
σ	Estimation of the process volatility	46.539
I	Annual average cost (Including static costC_s and cost of operation and maintenance C_o, and supposing$C_o = 3\%C_s$) (PDD, 2006)[16] and (IRENA, 2015)[20]	49.7785 (Million Yuan)
T	Time to invest	1,2,...,5
V_0	Annual average revenue from 2007 to 2011 By using equation (3.1)	59.3234 (Million Yuan)

3.4.3 Real Option Value

We shall price the real option F according to the steps described in section 3.3. In order to get the simulation paths of O-U process with the initial value V_0 and the year to invest T, we divide the interval $[0, T]$ into n time periods with the time subinterval is $dt = T/n$ By using the parameters p, q, σ as in Table 2, one can get simulation paths of V with different simulated times (denoted by npath) as shown in Figure 3

Figure 3 shows four simulation results with different simulated times. Figure 3 (a), 3(b), 3(c), and 3(d) show the simulation paths which were generated one time (npath=1), 10 times (npath=10), 100 times, and 10000 times respectively.

Following with the steps described in section 3.3, using the parameters shown in Table 2, and inputting the simulation step number $n = 60$ and times $m = 10000$, we call the MATLAB code to obtain the value of option F. Table 3 shows F when we change the investment time T.

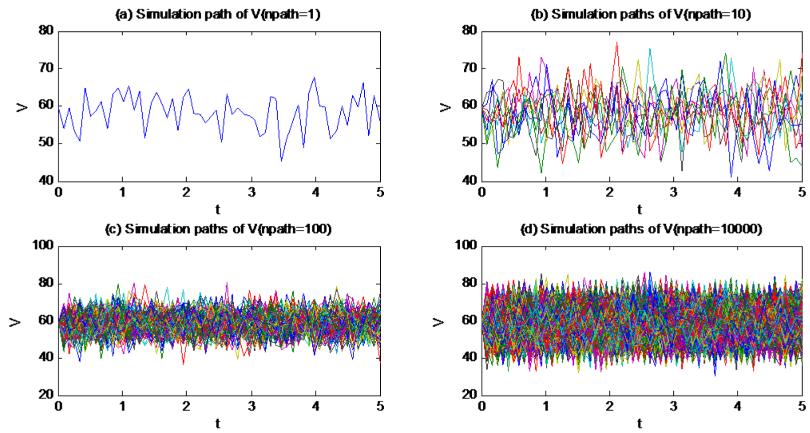

Figure 3: Simulation paths of annual revenue

Table 3: Option values F(million Yuan) of various time.

V_0	I	σ	r	q	μ	T	F
59.3234	49.7785	46.539	0.05	30.722	58.531	5	6.8289
59.3234	49.7785	46.539	0.05	30.722	58.531	4	7.1715
59.3234	49.7785	46.539	0.05	30.722	58.531	3	7.5500
59.3234	49.7785	46.539	0.05	30.722	58.531	2	7.9490
59.3234	49.7785	46.539	0.05	30.722	58.531	1	8.3448

One can see from the Table 3 that the investment opportunity value (or option value F) will be decreased as the time goes on. If the investor invests the WPP in the first year, this investment opportunity value is worth 8.34 million Yuan, but if he (or she) invests in the last year of development right, the opportunity value is worth 6.83 million Yuan.

3.4.4 Critical Value

As we know, if the real option (or investment opportunity) value F is positive the project is worthwhile to invest. If it is negative the project should be abandoned. If F equals to zero then the investment opportunity almost worthless. Now we need to calculate the critical value to help the investor for making a decision. In order to reach this aim, we consider the effect of crucial parameters on the project value.

Firstly, we consider the effect of cost on option value. By fixing the others parameters and changing the cost value (I), we obtain the option values shown in Figure 4. From the simulated results, the WPP investment opportunity value F will equal zero when the critical cost is $I^* = 61$ million Yuan. Thus if the annual average cost including static investment cost and operation and maintenance cost is higher than 61 million Yuan, the WPP is worthless to invest.

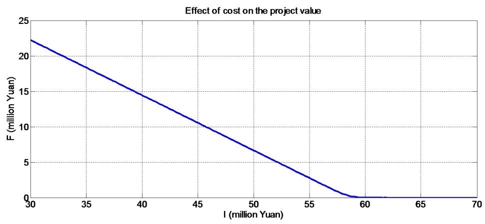

Figure 4: Effect of cost on the WPP investment opportunity value F.

Next, we fix the others parameters and change only the parameter μ. We shall consider the effect of long-run mean μ on the option values. By inputting various values of $\mu = \frac{p}{q}$ into equation (3.2) and (3.3), a simulated path results of F has been shown in the Figure 5. According to the calculated results, with the increase of long-run mean of the revenue, the WPP value increases gradually. From the numerical results the WPP value will equals zero when the long-run mean of the revenue goes to 47 million Yuan.

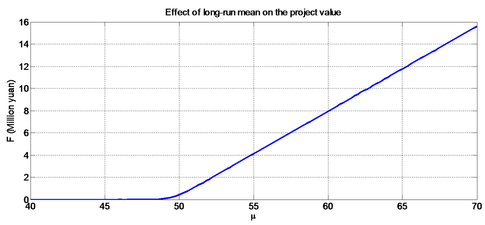

Figure 5: Effect of long-run mean on the WPP value.

Now, we move to consider the effect of volatility. Similarly as the description above, we fix others parameters and only to change volatility σ in the interval [0,70].

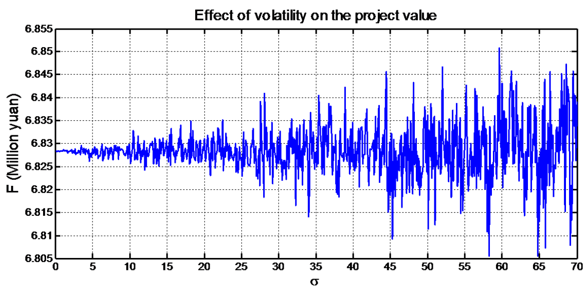

Figure 6: Effect of long-run volatility on the WPP value.

We get simulated path of F shown in Figure 6. Although the volatility σ changes greatly, the WPP value F changes not so much. The variation range of F is between about 6.8 million Yuan and 6.85 million Yuan. This means the volatility of the O-U process influences the project value but the effect was not so significance.

3.5 Scenario Analysis under O-U Process

By considering the uncertainties of the future policy and of wind power consumption in China, the scenario analysis in this study will focus on effect of on-grid electricity price and abandoned wind rate (AWR) to options value. On the other hand, with the gradual establishment of China carbon market trading system, investors are faced with great opportunities. The price of CERs will also become one of the important factors affecting the profit of WPP, so the scenario analyses also focus on the CERs price.

3.5.1 Case 1: Vary On-Grid Electricity Price P_f

Firstly, we shall consider the case of the on-grid electricity price (OGE) price change. Suppose that the P_f increase 5% from $P_f = 0.436$ to $P_f = 0.654$ Yuan/Kwh, then the option value will increase from $F = 0.001$ to $F = 14.919$ million Yuan as shown in Table 4. This means for a WPP investor, if his (or her) expected return is over 6.827 million Yuan, he(or she) can invest when the OGE price $P_f \geq 0.545$ Yuan/Kwh, otherwise he may give up to invest.

Table 4: Option values F(million Yuan) of various time.

P_f	V_0	I	σ	q	μ	F
0.436	59.323	49.779	38.117	30.722	47.940	0.001
0.463	59.323	49.779	40.223	30.722	50.588	0.780
0.491	59.323	49.779	42.328	30.722	53.236	2.772
0.518	59.323	49.779	44.434	30.722	55.884	4.798
0.545	59.323	49.779	46.539	30.722	58.532	6.827
0.572	59.323	49.779	48.644	30.722	61.180	8.844
0.600	59.323	49.779	50.750	30.722	63.828	10.889
0.627	59.323	49.779	52.855	30.722	66.476	12.913
0.654	59.323	49.779	54.961	30.722	69.124	14.919

3.5.2 Case 2: Vary Abandoned Wind Rate

From Table 5, the project value (F) will be increased with the Abandoned Wind Rate (AWR) level decreased. At the current level of AWR, the project value is 6.82 million Yuan. In this simulated calculation, the project value will reach to 15.78 million Yuan after AWR decreasing about 20% from the current level.

Table 5: Option values F(million Yuan) of various time.

AWR level	σ	μ	F
-20%	55.847	70.238	15.776
-15%	53.520	67.312	13.538
-10%	51.193	64.385	11.295
-5%	48.866	61.459	9.067
Current level	46.539	58.532	6.820
5%	44.212	55.605	4.583
10%	41.885	52.679	2.351
15%	39.558	49.752	0.282
20%	37.231	46.826	0.000

For a WPP investor, if his (or her) expected return is more than 6.827 million Yuan, he (or she) should pay attention to the changes of AWR according the government report and compare with the current level of AWR. Similarly if the AWR level increase from 5% to 20%, the expected value will decrease from $F = 6.82$ to $F = 0.0$ million Yuan and it is not worth to invest.

3.5.3 Case 3: Vary Certified Emission Reductions (CERs) Price

At last, we consider the effect of CERs price (P_e) on the project value. According to the results in Table 6, if we start from the initial value $P_e = 56$ Yuan/tco2 and we reduce Pe from 5 to 20 percent then F changes from 6.61 million Yuan to 5.992 million Yuan. On the other hand if we increase P_e from 5 to 20 percent then F changes from 7.036 million Yuan to 7.684 million Yuan. One can see that, although Pe changes greatly, the project value F does not change so much. The project value just changes from 5.992 million Yuan to 7.684 million Yuan. Similar to case 2, if the investor expected return is more than 6.817 million Yuan , he (or she) should not invest if $P_e \leq 56$ Yuan/tco2.

4 Conclusions and Limitations

In this paper, we consider the revenue of a completed wind power farm that follows an O-U process and obtain the project value through Monte Carlo simulation. After the modeling process, we carry out an empirical analysis with the actual data of WPP No.0689 in CDM database.One can see from the frameworks that real options analysis can predict a dynamic series of future decisions. ROA allows an investor or managing person to have a lot of flexibility in acting and can adjust to those changes taking place in the economy.

At the end, we note that there are also some limitations of this paper. Those limitations are as follows.

(i) The model only considers the primary factors relevant to the wind energy project and economic evaluation. In the real world, a WPP faces more uncertainties, such as investment cost, tax, policy, technology, etc.

Table 6: Option values F(million Yuan) of various time.

P_e(Yuan/tco2)	σ	μ	F
44.8	45.653	57.418	5.992
47.6	45.874	57.696	6.186
50.4	46.096	57.975	6.408
53.2	46.317	58.253	6.610
56.0	46.539	58.532	6.817
58.8	46.760	58.811	7.036
61.6	46.982	59.089	7.251
64.4	47.204	59.368	7.465
67.2	47.425	59.646	7.684

(ii) The option considered in this paper is simplistic, in the reality, usually investment projects are composed of a set of a large number of related options.

Because of the great uncertainty of the development of renewable energy, the investment projects of renewable energy have increased complexity and uncertainty. So the ROA frameworks employed in this study need a lot of works to improve in the future.

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