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Suzuki-Type Fixed Point Results for E-Contractive Maps in Uniform Spaces

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Abstract : Suzuki's fixed point results from [T. Suzuki, A generalized Banach contraction principle that characterizes metric completeness, Proc. Amer. Math. Soc. 136 (2008) 1861-1869] and [T. Suzuki, A new type of fixed point theorem in metric spaces, Nonlinear Anal. TMA, 71 (2009) 5313-5317] are extended to the case of Hausdorff uniform space. Examples are given to distinguish our results from the known ones. Some more general results are also obtained in Hausdorff uniform space.

Keywords : E-contractive maps; common fixed points; Uniform space.
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1 Introduction and Preliminaries

Let X be a nonempty set and let ϑ be a nonempty family of subsets of $X \times X$. The pair (X,ϑ) is called a uniform space if it satisfies the following properties:

(i) if G is in ϑ, then G contains the diagonal $\{(x,x)|x \in X\}$;

(ii) if G is in ϑ and H is a subset of $X \times X$ which contains G, then H is in ϑ;

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(iii) if G and H are in ϑ, then $G \cap H$ is in ϑ;

(iv) if G is in ϑ, then there exists H in ϑ, such that, whenever (x, y) and (y, z) are in H, then (x, z) is in G;

(v) if G is in ϑ, then $\{(y, x)|(x, y) \in G\}$ is also in ϑ.

ϑ is called the *uniform structure* of X and its elements are called *entourages* or *neighbourhoods* or *surroundings*. In Bourbaki [1] and Zeidler [2], (X,ϑ) is called a *quasiuniform space* if property (v) is omitted. Some authors such as Berinde [3, 4], Jachymski [5], Kada et al [6], Kang [7], Rhoades [8]], Rus [9, 10], Wang [11] and Zeidler [2], Sintunavarat et al [12, 13, 14, 15] studied the theory of fixed point or common fixed point for contractive selfmappings in complete metric spaces or Banach spaces in general.

Later, Aamri and El Moutawakil [16] proved some common fixed point theorems for some new contractive or expansive maps in uniform spaces by introducing the notions of an A-distance and an E-distance. Diagonal uniformity introduced by Weil, this approach was largely developed and pursued by Bourbaki [1].

For any set X, the diagonal $\{(x, x)|x \in X\}$ will be denoted by Δ where confusion might occur. If $V, W \in X \times X$, then $V \circ W = \{(x, y)| \quad there \quad exists \quad z \in X : (x, z) \in W \quad and \quad (z, y) \in V\}$ and $V^{-1} = \{(x, y)|(y, x) \in V\}$.

If $V \in \vartheta$ and $(x, y) \in V, (y, x) \in V$, x and y are said to be V -close, and a sequence $\{x_n\}$ in X is a Cauchy sequence for ϑ, if for any $V \in \vartheta$, there exists $N \geq 1$ such that x_n and x_m are V -close for $n, m \geq N$. A uniformity ϑ defines a unique topology $\tau(\vartheta)$ on X for which the neighborhoods of $x \in X$ are the sets $V(x) = \{y \in X|(x, y) \in V\}$ when V runs over ϑ.

A sequence $\{x_n\}$ in X is convergent to x for ϑ, if for any $V \in \vartheta$, there exists $n_0 \in \mathbb{N}$ such that $x_n \in V(x)$ for every $n \geq n_0$ and denote by $\lim_{n\to\infty} x_n = x$. A uniform space (X, ϑ) is said to be Hausdorff if and only if the intersection of all the $V \in \vartheta$ reduces to the diagonal Δ of X, i.e., if $(x, y) \in V$ for all $V \in \vartheta$ implies $x = y$. This guarantees the uniqueness of limits of sequences. $V \in \vartheta$ is said to be symmetrical if $V = V^{-1}$. Since each $V \in \vartheta$ contains a symmetrical $W \in \vartheta$ and if $(x, y) \in W$ then x and y are both W and V -close, then for our purpose, we assume that each $V \in \vartheta$ is symmetrical. When topological concepts are mentioned in the context of a uniform space (X, ϑ), they always refer to the topological space $(X, \tau(\vartheta))$.

2 Main Result

Now, we introduce the concept of A-distance, E-distance and prove many fixed point theorem in these uniform spaces which are a nice generalization of the known results in metric spaces.

Definition 2.1. Let (X, ϑ) be a uniform space. A function $p : X \times X \longrightarrow \mathbb{R}^+$ is said to be an *A-distance* if for any $V \in \vartheta$ there exists $\delta > 0$ such that if $p(z, x) \leq \delta$ and $p(z, y) \leq \delta$ for some $z \in X$, then $(x, y) \in V$.

Definition 2.2. Let (X, ϑ) be a uniform space. A function $p : X \times X \longrightarrow \mathbb{R}^+$ is said to be an *E-distance* if

(p_1) p is an A-distance,

(p_2) $p(x, y) \leq p(x, z) + p(z, y)$. $\forall x, y, z \in X$.

Example 2.3. Let us give some examples of A and E-distance.

1. Let (X, ϑ) be a uniform space and let d be a distance on X: Clearly (X, ϑ_d) is a uniform space where ϑ_d is the set of all subsets of $X \times X$ containing a "band" $B_\epsilon = \{(x, y) \in X^2 | d(x, y) < \epsilon\}$ for some $\epsilon > 0$. Moreover, if $\vartheta \subseteq \vartheta_d$, then d is an E-distance on (X, ϑ).
2. Recently, J.R. Montes and J.A. Charris introduced the concept of W-distance on uniform spaces. Every W-distance p is an E-distance since it satisfies (p_1), (p_2) and the following condition: for all $x \in X$, the function $p(x, .)$ is lower semi-continuous. That is, if there exist a sequence $\{y_n\}$ in X such that $y_n \longrightarrow y \in X$ this implies that $p(x, y) \leq \liminf\limits_{n \longrightarrow \infty} p(x, y_n)$.
3. Let $X = [0, \infty)$ and $d(x, y) = |x - y|$ the usual metric. Consider the function p defined as follows

$$p(x, y) = \begin{cases} y & \text{if } y \in [0, 1), \\ 2y. & \text{if } y \in [1, \infty) \end{cases}$$

It is easy to see that the function p is an E-distance on (X, ϑ_d) but it is not an W-distance on (X, ϑ_d) since the function $p(x, .) : X \longrightarrow \mathbb{R}^+$ is not lower semi-continuous at 1.

The following Lemma contain some useful properties of A-distances. It is stated in [6] for metric spaces and in [17] for uniform spaces. The proof is straightforward.

Lemma 2.4. *Let (X, ϑ) be a Hausdorff uniform space and p be an A-distance on X. Let $\{x_n\}$ and $\{y_n\}$ be sequences in X and $\{\alpha_n\}$, $\{\beta_n\}$ be sequences in $\mathbb{R}^+$ converging to 0. Then, for $x, y, z \in X$, the following holds:*

(a) If $p(x_n, y) \leq \alpha_n$ and $p(x_n, z) \leq \beta_n$ for all $n \in \mathbb{N}$, then $y = z$. In particular, if $p(x, y) = 0$ and $p(x, z) = 0$, then $y = z$,

(b) if $p(x_n, y_n) \leq \alpha_n$ and $p(x_n, z) \leq \beta_n$ for all $n \in \mathbb{N}$, then $\{y_n\}$ converges to z,

(c) if $p(x_n, x_m) \leq \alpha_n$ for all $n, m \in \mathbb{N}$ with $m > n$, then $\{x_n\}$ is a Cauchy sequence in (X, ϑ).

Let (X, ϑ) be a uniform space with an A-distance p. A sequence in X is p-Cauchy if it satisfies the usual metric condition. That is, for every $\epsilon > 0$ there exists $n_0 \in \mathbb{N}$ such that $p(x_n, x_m) < \epsilon$ for all $n, m \geq n_0$. There are several concepts of completeness in this setting

Definition 2.5. Let (X, ϑ) be a uniform space and p be an A-distance on X.

(1) X is S-complete if every p-Cauchy sequence $\{x_n\}$, there exists x in X with $\lim_{n\to\infty} p(x_n, x) = 0$.

(2) X is p-Cauchy complete if every p-Cauchy sequence $\{x_n\}$, there exists x in X with $\lim_{n\to\infty} x_n = x$ with respect to $\tau(\vartheta)$.

Remark 2.6. *Let (X, ϑ) be a Hausdorff uniform space and let $\{x_n\}$ be a p-Cauchy sequence. Suppose that X is S-complete, then there exists $x \in X$ such that $\lim_{n\to\infty} p(x_n, x) = 0$. Lemma 2.4(b) then gives $\lim_{n\to\infty} x_n = x$ with respect to the topology $\tau(\vartheta)$. Therefore S-completeness implies p-Cauchy completeness.*

Definition 2.7. Let (X, ϑ) be a Hausdorff uniform space and p be an A-distance on X. Two selfmappings f and g of X are said to be *weak compatible* if they commute at their coincidence points, that is, $fx = gx$ implies that $fgx = gfx$.

Theorem 2.8. *Let (X, ϑ) be a S-complete Hausdorff uniform space such that p be an E-distance and continuous on X. Let $T : X \longrightarrow X$ be a selfmap and $\theta =: [0, 1) \longrightarrow (\frac{1}{2}, 1]$ be defined by*

$$\theta(r) = \begin{cases} 1, & 0 \leq r \leq \frac{\sqrt{5}-1}{2} \\ \frac{1-r}{r^2}, & \frac{\sqrt{5}-1}{2} \leq r \leq \frac{1}{\sqrt{2}} \\ \frac{1}{1+r}, & \frac{1}{\sqrt{2}} \leq r < 1. \end{cases} \tag{2.1}$$

If there exists $r \in [0, 1)$ such that for each $x, y \in X$, satisfying the condition

$$\begin{aligned} &\theta(r) \min\{p(x, Tx), p(Tx, x)\} \leq \max\{p(x, y), p(y, x)\} \\ \Longrightarrow\ &\max\{p(Tx, Ty), p(Ty, Tx)\} \leq r \min\{p(x, y), p(y, x)\}. \end{aligned} \tag{2.2}$$

Then T has a unique fixed point $z \in X$ and for each $x \in X$, the sequence $\{T^n x\}$ converges to z.

Proof. Putting $y = Tx$ in (2.2). Hence from

$$\theta(r) \min\{p(x, Tx), p(Tx, x)\} \leq \max\{p(x, Tx), p(Tx, x)\},$$

it follows

$$\max\{p(Tx, T^2x), p(T^2x, Tx)\} \leq r \min\{p(x, Tx), p(Tx, x)\} \tag{2.3}$$

for every $x \in X$. Thus,

$$p(Tx, T^2x) \leq rp(x, Tx). \tag{2.4}$$

Let $x_0 \in X$ be arbitrary and form the sequence $\{x_n\}$ by $x_1 = Tx_0$ and $x_{n+1} = Tx_n$ for $n \in \mathbb{N} \cup \{0\}$. By (2.4), we have

$$\begin{aligned} p(x_n, x_{n+1}) &= p(Tx_{n-1}, T^2x_{n-1}) \\ &\leq rp(x_{n-1}, Tx_{n-1}) \\ &\ \vdots \\ &\leq r^n p(x_0, x_1) \longrightarrow 0. \end{aligned}$$

Also, by definition E-distance, we have

$$\begin{aligned} p(x_n, x_m) &\leq p(x_n, x_{n+1}) + p(x_{n+1}, x_{n+2}) + \cdots + p(x_{m-1}, x_m) \\ &\leq r^n p(x_0, x_1) + r^{n+1} p(x_0, x_1) + \cdots + r^{m-1} p(x_0, x_1) \\ &= \frac{r^n - r^m}{1-r} p(x_0, x_1) \\ &< \frac{r^n}{1-r} p(x_0, x_1) \longrightarrow 0. \end{aligned}$$

Hence, $\{x_n\}$ is a p-Cauchy sequence.
Since X is S-complete, there exists $z \in X$ such that $x_n \to z$, as $n \to \infty$. That is,

$$\lim_{n\to\infty} p(x_{n+1}, z) = \lim_{n\to\infty} p(Tx_n, z) = 0. \tag{2.5}$$

Putting $x = T^{n-1}z$ in (2.3), we get that

$$\begin{aligned} & \max\{p(T^n z, T^{n+1} z), p(T^{n+1} z), T^n z\} \\ \leq\; & r \min\{p(T^{n-1} z, T^n z), p(T^n z, T^{n-1} z)\} \\ \leq\; & r \max\{p(T^{n-1} z, T^n z), p(T^n z, T^{n-1} z)\} \\ & \vdots \\ \leq\; & r^n \min\{p(Tz, z), p(z, Tz)\}, \end{aligned} \tag{2.6}$$

holds for each $n \in \mathbb{N}$ (where $T^0 z = z$). It follows by induction that

$$p(T^n z, T^{n+1} z) \leq r^n p(z, Tz). \tag{2.7}$$

Let us prove now that

$$\max\{p(z, Tx), p(Tx, z)\} \leq r \min\{p(z, x), p(x, z)\}, \tag{2.8}$$

holds for each $x \neq z$. First, since $x \neq z$ we have
$\max\{p(x, z), p(z, x)\} \neq 0$ and

$$\begin{aligned} 0 < \max\{p(x, z), p(z, x)\} &= \lim_{n\to\infty} \max\{p(x_n, x), p(x, x_n)\} \\ &= \max\{\lim_{n\to\infty} p(x_n, x), \lim_{n\to\infty} p(x, x_n)\}. \end{aligned}$$

On the other hand we have

$$\lim_{n\to\infty} \min\{p(x_n, Tx_n), p(Tx_n, x_n)\} = 0,$$

it follows that there exists a $n_0 \in \mathbb{N}$ such that

$$\theta(r) \min\{p(x_n, Tx_n), p(Tx_n, x_n)\} \leq \max\{p(x_n, x), p(x, x_n)\},$$

holds for every $n \geq n_0$. Assumption (2.2) implies that for such n

$$\max\{p(Tx_n, Tx), p(Tx, Tx_n) \leq r \min\{p(x_n, x), p(x, x_n)\},$$

thus as $n \to \infty$ (and continuity of p), we get that

$$\max\{p(z,Tx),p(Tx,z)\} \le r\min\{p(z,x),p(x,z)\}.$$

We will prove that

$$p(T^n z,z) \le p(Tz,z), \tag{2.9}$$

for each $n \in \mathbb{N}$. For $n = 1$ this relation is obvious. Suppose that it holds for some $m \in \mathbb{N}$. If $T^m z = z$, then $T^{m+1}z = Tz$ and $p(T^{m+1}z,z) = p(Tz,z) \le p(Tz,z)$. If $T^m z \ne z$, then we can apply (2.8) and the induction hypothesis, we get that

$$\begin{aligned} p(T^{m+1}z,z) &\le \max\{p(T^{m+1}z,z),p(z,T^{m+1}z)\} \\ &\le r\min\{p(T^m z,z),p(z,T^m z)\} \\ &\le rp(T^m z,z) \le rp(Tz,z) \le p(Tz,z), \end{aligned}$$

and similarly we have $p(z,T^n z) \le p(z,Tz)$. Hence (2.9) is proved by induction.

We consider two possible cases.

Case I. $0 \le r < \frac{1}{\sqrt{2}}$ (and hence $\theta(r) \le \frac{1-r}{r^2}$). We will prove first that

$$\max\{p(Tz,T^n z),p(T^n z,Tz)\}) \le r\min\{p(z,Tz),p(Tz,z)\} \tag{2.10}$$

for $n \ge 2$. For $n = 2$ it follows from (2.7). Suppose that (2.10) holds for some $n > 2$. Then we prove that

$$\max\{p(Tz,T^{n+1}z),p(T^{n+1}z,Tz)\} \le r\min\{p(z,Tz),p(Tz,z).$$

Then we can apply (2.6) and the induction hypothesis, we get that

$$\begin{aligned} &\theta(r)\min\{p(T^n z,T^{n+1}z),p(T^{n+1}z,T^n z)\} \\ \le\ & \frac{1-r}{r^n}\min\{p(T^n z,T^{n+1}z),p(T^{n+1}z,T^n z)\} \\ \le\ & (1-r)\min\{p(z,Tz),p(Tz,z)\} \\ \le\ & (1-r)p(Tz,z) \\ \le\ & \max\{p(z,T^n z),p(T^n z,z)\}, \end{aligned}$$

the last inequality is hold, because

$$\begin{aligned} p(Tz,z) &\le p(Tz,T^n z) + p(T^n z,z) \\ &\le rp(Tz,z) + p(T^n z,z), \end{aligned}$$

hence $(1-r)p(Tz,z) \le p(T^n z,z) \le \max\{p(T^n z,z),p(z,T^n z)\}$.
Assumption (2.2) implies that

$$\begin{aligned} \max\{p(Tz,T^{n+1}z),p(T^{n+1}z,Tz)\} &\le r\min\{p(z,T^n z),p(T^n z,z)\} \\ &\le r\min\{p(z,Tz),p(Tz,z)\}, \end{aligned}$$

that is relation (2.10) is proved by induction.

Now, if $Tz \neq z$ then (2.10) implies that $T^n z \neq z$ for each $n \geq 2$. Hence, (2.6) imply that

$$\begin{aligned} p(z, T^{n+1}z) &\leq \max\{p(z, T^{n+1}z), p(T^{n+1}z, z)\} \\ &\leq r \min\{p(z, T^n z), p(T^n z, z)\} \\ &\leq rp(z, T^n z) \\ &\vdots \\ &\leq r^n p(z, Tz) \longrightarrow 0. \end{aligned}$$

Thus $\lim_{n\to\infty} p(z, T^{n+1}z) = 0$ and $\lim_{n\to\infty} p(z, z) = \lim_{n\to\infty} p(x_n, x_{n+1}) = 0$, implies that $\lim_{n\to\infty} T^{n+1}z = z$. Again from (2.10), as $n \to \infty$ we get that

$$\max\{p(Tz, z), p(z, Tz)\} \leq r \min\{p(z, Tz), p(Tz, z)\},$$

hence we have $p(z, Tz) \leq rp(z, Tz)$ which is a contradiction. Hence $Tz = z$.

Case II. $\frac{1}{\sqrt{2}} \leq r < 1$ (and so $\theta(r) = \frac{1}{1+r}$). We will prove that there exists a subsequence $\{x_{n_k}\}$ of $\{x_n\}$ such that

$$\theta(r) \min\{p(x_{n_k}, Tx_{n_k}), p(Tx_{n_k}, x_{n_k})\} \leq \max\{p(x_{n_k}, z), p(z, x_{n_k})\} \tag{2.11}$$

holds for each $k \in \mathbb{N}$. From (2.3) we know that

$$\max\{p(x_n, x_{n+1}), p(x_{n+1}, x_n)\} \leq r \min\{p(x_{n-1}, x_n), p(x_n, x_{n-1})\}$$

holds for each $n \in \mathbb{N}$. Suppose that

$$\frac{1}{1+r} \min\{p(x_{n-1}, x_n), p(x_n, x_{n-1})\} > \max\{p(x_{n-1}, z), p(z, x_{n-1})\},$$

and

$$\frac{1}{1+r} \min\{p(x_n, x_{n+1}), p(x_{n+1}, x_n)\} > \max\{p(x_n, z), p(z, x_n)\},$$

holds for some $n \in \mathbb{N}$. Then

$$\begin{aligned} &\min\{p(x_{n-1}, x_n), p(x_n, x_{n-1})\} \\ \leq\ &\min\{p(x_{n-1}, z) + p(z, x_n), p(x_n, z) + p(z, x_{n-1})\} \\ \leq\ &\max\{p(x_{n-1}, z), p(z, x_{n-1})\} + \max\{p(x_n, z), p(z, x_n)\} \\ <\ &\frac{1}{1+r} \min\{p(x_{n-1}, x_n), p(x_n, x_{n-1})\} + \frac{1}{1+r} \min\{p(x_n, x_{n+1}), p(x_{n+1}, x_n)\} \\ <\ &\frac{1}{1+r} \min\{p(x_{n-1}, x_n), p(x_n, x_{n-1})\} + \frac{r}{1+r} \min\{p(x_n, x_{n-1}), p(x_{n-1}, x_n)\} \\ =\ &\min\{p(x_{n-1}, x_n), p(x_n, x_{n-1})\} \end{aligned}$$

which is impossible. Hence one of the following holds for each n:

$$\theta(r) \min\{p(x_{n-1}, x_n), p(x_n, x_{n-1}) \leq \max\{p(x_{n-1}, z), p(z, x_{n-1})\},$$

or

$$\theta(r)\min\{p(x_n, x_{n+1}), p(x_{n+1}, x_n)\} \leq \max\{p(x_n, z), p(z, x_n)\}.$$

In particular,

$$\theta(r)\min\{p(x_{2n-1}, x_{2n}), p(x_{2n}, x_{2n-1})\} \leq \max\{p(x_{2n-1}, z), p(z, x_{2n-1})\},$$

or

$$\theta(r)\min\{p(x_{2n}, x_{2n+1}), p(x_{2n+1}, x_{2n})\} \leq \max\{p(x_{2n}, z), p(z, x_{2n})\}.$$

In other words, there is a subsequence $\{x_{n_k}\}$ of $\{x_n\}$ such that (2.11) holds for each $k \in \mathbb{N}$. But then assumption (2.2) implies that

$$\max\{p(Tx_{n_k}, Tz), p(Tz, Tx_{n_k})\} \leq r\min\{p(x_{n_k}, z), p(z, x_{n_k})\},$$

or

$$\max\{p(Tx_{n_{k-1}}, Tz), p(Tz, Tx_{n_{k-1}})\} \leq r\min\{p(x_{n_{k-1}}, z), p(z, x_{n_{k-1}})\}.$$

Passing to the limit when $k \longrightarrow \infty$ we get that $\max\{p(z, Tz), p(Tz, z)\} \leq 0$, which is possible only if $Tz = z$.

Thus, we have proved that z is a fixed point of T. The uniqueness of the fixed point follows easily from (2.2). Indeed, if y, z are two fixed points of T,

$$\theta(r)\min\{p(z, y), p(y, z)\} \quad \leq \quad \max\{p(z, y), p(y, z)\},$$

then (2.2) implies that

$$\max\{p(Tz, Ty), p(Ty, Tz)\} \leq r\min\{p(z, y), p(y, x)\},$$

that is $p(z, y) \leq \max\{p(z, y), p(y, z)\} \leq r\min\{p(z, y), p(y, x)\} < p(z, y)$, whereform $y = z$. ☐

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