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Time Series Forecast Using AR-Belief Approach

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Abstract : This paper aims at applying a recent new approach to predicting the growth rate of Thailand GDP. The new approach will provide uncertainty about predicted values solely from observed data without the need to supply some subjective prior distribution on unknown model parameters. This is achieved by building a belief function (i.e., a distribution of a random set) from the likelihood function given the observed data, and use it to assess prediction uncertainty. With our sampling model as an autoregressive time series model, we demonstrate empirically that this approach can provide a reliable confidence interval for predicted values.

Keywords : belief function; AR(p) model; growth rate; likelihood-based approach; Monte Carlo simulation.
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1 Introduction

In the context of prediction, it is highly desirable to have predicted intervals for predicted future quantities of interest to assess uncertainty about them. Usually,

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a Bayesian approach could be feasible, but the cost is the somewhat controversial supply of subjective prior distributions for unknown parameters in the assumed sampling model. The quest for a "Bayesian without priors" has led to an "empirical Bayes", i.e., supplying priors from data. Starting with Dempster [1] in the 1960's (but see Dempster [2]), and further developed mainly by Martin and Liu [3] (see also Martin and Liu [4]), as well as elaborated recently for some applications in Kanjanatarakul et al. [5], an approach to achieve predicted intervals without calling upon the existence of prior distributions (of parameters) is based on a mathematical theory of evidence (Shafer [6]) also known as theory of belief functions.

In this paper, with the prediction of the growth rate of Thailand GDP in mind, we will use as our sampling model an autoregressive model with Gaussian white noise. Predictions of the growth rate will be investigated using our AR-Belief model. Empirical results will be shown to illustrate the approach as well as an application to a real situation in economics.

The remainder of this paper is organized as follows. The so-called belief function theory is reviewed in section 2. Section 3 describes the inferential model and the steps in obtaining predictive random sets. In section 4, we apply the methodology to our autoregressive model with real data where empirical results will be displayed. Section 5 concludes our empirical studies among other advantages. The methodology used in this paper can provide exact confidence predicted intervals, even for small sample sizes, i.e., does not need to consider asymptotics.

2 Belief Functions and Random Sets

We describe in this section an approach to providing a quantitative measure of uncertainty, in parameter estimation or in prediction of future values of variables, originated from Dempster's early works.

The fundamental question in using statistics to make inferences in any situation is how to quantify epistemic uncertainty involved? This is so since we need to specify our uncertainty on any conclusions we draw (on numerical estimates, predicted values, hypotheses about parameters,...); without it, our conclusions are "incomplete". Usually, confidence regions require asymptotics, and Bayesian approach requires priors to fulfill this need, the approach we are going to elaborate (in the next section) seems useful for any sample sizes, and does not require a prior. It is based upon on the concept known as belief functions (Shafer, [6]) inspired from Dempster's early works.

First, for simplicity, consider a finite set Θ where we are interested in quantifying epistemic uncertainties in it, such as "what is our uncertainty about whether a "hypothesis", represented as a subset A of Θ, is true". A probability measure on Θ (prior or posterior) P is additive, so that it cannot be used to describe "ignorance". Weakening additivity of set functions on the power set of Θ, denoted as 2^{Θ}, could achieve this requirement. Thus,

Definition 2.1. A *belief function* $Bel(.)$ on Θ is a map from 2^{Θ} to the unit interval

$[0,1]$, such that

1. $Bel(\varnothing) = 0$, $Bel(\Theta) = 1$,
2. for any $k \geq 1$, and $A_1, A_2, ..., A_k$ subsets of Θ,

$$Bel(\cup_{i=1}^{k} A_i) \geq \sum_{\varnothing \neq I \subseteq \{1,2,...,k\}} (-1)^{|I|+1} Bel(\cap_{i \in I} A_i)$$

where $|I|$ denotes the cardinality of the set I.

For $A_1 = A \subseteq \Theta$, and $A_2 = A^c$ (the set complement of A), we have

$$Bel(A \cup A^c) = Bel(\Theta) = 1 \geq Bel(A) + Bel(A^c)$$

so that the interval $[Bel(A), Pl(A)]$, where the "plausibility" of A is defined as $Pl(A) = 1 - Bel(A^c)$, can be used to express the uncertainty about A, including the case of ignorance.

Now, for the case of finite Θ, any belief function can be written as

$$Bel(A) = \sum_{B \subseteq A} m(B)$$

where $m(.) : 2^{\Theta} \to [0,1]$ and $\sum_{A \subseteq \Theta} m(A) = 1$, i.e., the set function $m(.)$ is a bona fide probability density function, not on Θ, but on 2^{Θ}. It is given by the Mobius inverse of $Bel(.)$:

$$m(A) = \sum_{B \subseteq A} (-1)^{|A \setminus B|} F(B).$$

As such, there exists a random set S with values in 2^{Θ} admitting precisely $m(.)$ as its probability density function, and $Bel(.)$ as its "distribution function", in the sense that $m(A) = P(S = A)$, and $Bel(A) = P(S \subseteq A)$. For details, see Nguyen [7].

This interpretation (first pointed out in Nguyen [8], 1978) is essential for "up-to-day" developments of new statistics (see the recent book by Martin and Liu [4]).

In our subsequent econometric application, we are facing the case where Θ is our parameter space and which is a subset of Euclidean space. Thus, it suffices to define belief functions on (infinite) Hausdorff, locally compact and second countable topological spaces. In fact, by duality, it suffices to consider plausibility (set) functions which play the role of "distributions" of random closed sets (random sets taking values as closed subsets) in an analogy with distributions of random vectors via Lebesgue-Stieltje's theorem in real analysis.

An example of plausibility functions which is used in our subsequent analysis is this. Let $\varphi : \mathbb{R}^d \to [0,1]$ be an upper semi continuous (usc) function (i.e., for any $s \in \mathbb{R}$, the level sets $\{x \in \mathbb{R}^d : \varphi(x) \geq s\}$ are closed subsets). Then the set function $Y : \mathcal{K}(\mathbb{R}^d)$ (compact subsets)$\to [0,1]$, defined by $Y(K) = \max_{x \in K} \varphi(x)$

is a plausibility function, characterizing a random closed set S, i.e., $Y(K) = P(S \cap K \neq \varnothing)$, via Choquet 's theorem. Indeed, let $(\Omega, \mathcal{A}, P)$ be a probability space, and let $\alpha(.) : \Omega \to [0,1]$ be a uniformly distributed random variable, then the random (closed) set

$$S(\omega) = \{x \in \mathbb{R}^d : \varphi(x) \geq \alpha(\omega)\}$$

has $Y(K) = \max_{x \in K} \varphi(x)$ as its distribution. Note that the random set S is obtained by randomizing the level sets of the function φ (i.e., choosing its level sets as random). Moreover, this random set is "nested", i.e., for $\omega \neq \omega'$, either $S(\omega) \subseteq S(\omega')$ or $S(\omega) \supseteq S(\omega')$.

The point is this. In order to quantify an epistemic uncertainty on Θ, it suffices to create an appropriate random set S (possibly a function of observed data) on it, and use the interval (Bel, Pl), where $Pl(.)$ is the "capacity functional" $Y(.)$ and $Bel(A) = 1 - Pl(A^c)$. As we will see in the next section, in statistical inference, the important point is that such a random set is data-dependent (data driven), i.e., constructed from data and not a sort of "generalized Bayesian prior". The probabilistic inference involved is a posterior analysis but without priors. One interesting (and important) observation is this. While belief functions were developed (by Shafer [6]) to provide a sort of generalized priors (a belief function is a generalization of a probability measure/a conventional prior), axiomatic definitions made them bona fide distributions of random sets (as pointed out by Nguyen [8]). This interpretation turns out to be, not just a formal connection, an essential view of how to construct belief functions (from data): knowing that belief functions are distributions of random sets, one proceeds to obtain belief functions (for use in quantifying epistemic uncertainty) by simply constructing random sets and then take their distributions as desired belief functions. That is why it looks like the theory of random sets suffices for the statistical framework which we will elaborate next.

3 Inferential Models and Associated Inference

We describe, in this section, the recent advances on probabilistic inference without priors (Martin and liu [3,4]). Note that the frame work is parametric to start out. It is the basis for any further inference such as prediction. The crucial idea is the use of random sets in the analysis. Recall that previous attempts to handle situations such as small sample sizes, subjective priors, and exact confidence levels (of regions) seem to fail because of lack of innovative models and probabilistic new tools, such as random sets and their distributions. While random sets appeared early in the history of statistics, random sets are sampling designs: Hajek [9], Kendall [10], Robbins [11], it was not until 1975 when Matheron [12] provided a rigorous theory that random sets became bona fide random elements, from which some issues in classical statistics such as confidence regions, coarse data can be formulated properly. For a contemporary literature, see Molchanov [13]. For a more down-to-earth text, see Nguyen [7].

It could be said that belief functions viewed as distributions of random sets (pointed out by Nguyen [8] is crucial for statistics). The point of view is everything! This is somewhat similar to the situation of local time (occupation time) of Brownian motion (or more generally, for diffusion processes): While the concept of local time (since Paul Levy) is important in probability theory (e.g. for investigating sample path of Brownian motion and stochastic integral), it is only well-known in probability. It was Nguyen and Pham who pointed out first (Nguyen and Pham [14]) its possible use in statistics of stochastic processes, triggering follow-up research making local time an important ingredient in statistical analysis ever since.

Specifically, if a belief function is needed for some reasons, the question is: How to construct it? Now, since a belief function is the distribution of some random set, it suffices to look for a random set and take its distribution. This is precisely the framework of inferential models that we are describing now.

Consider a parametric sampling model to start out. Suppose we "believe" that our observed data $X = \mathbf{x}$ is generated by a model $f(x|\theta)$, $\theta \in \Theta$. Of course, the observed data $\mathbf{x}$ does provide some information about the localization of the true parameter θo in the parameter space Θ, just like in the spirit of the maximum likelihood principle. We could get more if we realize that there exists a specific relation between X and θ. Indeed, if $F_\theta(.)$ denotes the distribution of X, under θ, then $X = F_\theta^{-1}(U)$, where $F_\theta^{-1}(.)$ is the quantile function of X, and U is the random variable, uniformly distributed on $[0, 1]$. But unlike the setting of simulations (where knowing a distribution, we wish to generate simulated data from it, by sample U), here U is unobservable (the observable is X instead). However, such a relation provides a link between data and parameters like in an equation. This situation could be formulated in general as follows. Let $Y(X)$ be our statistic of interest, then $Y = a(\theta, U)$ where U is an unobservable random variable with the known distribution, say, P_U. Clearly, if $U = u$ is observed then we could locate θ in the set $\Theta_y(u) = \{\theta : y = a(\theta, u)\}$. But in fact, U is not observed, so that given the observed $Y = y$, we need to predict the unobservable u, a draw from U, by a (predictive) random set S on the range of U. Then a random set on Θ is constructed as $\Theta_y(S) = \cup_{u \in S}\Theta_y(u)$ which is the (random) set of candidates for θ. Thus, for any (measurable) $A \subseteq \Theta$, $P(\Theta_y(S) \subseteq A)$ represents our belief that θ is in A, whereas $P(\Theta_y(S) \cap A \neq \varnothing)$ is the plausibility that θ is in A. In the application section which will follow, the predictive random set S will be constructed as a nested random set whose distribution is obtained from a coverage function.

4 An Application to Forecasting Growth Rate of Thailand GDP

4.1 AR(p) - Belief Approach

This paper follows the belief function approach and some notation used in Kanjanatarakul et al. [5]. The AR model will be employed in this study. The

AR(p) model may be written as the following.

$$X_t = c + \sum_{i=1}^{p} \phi_i X_{t-i} + \varepsilon_t, \tag{4.1}$$

where $\{X_t, t = 1, 2, \ldots.., T\}$, i=*1,2,...,p*, c and ϕ_i are parameters of the model, $\varepsilon_t \sim\ N(0, \sigma^2)$, and the parameter $\theta = (c, \phi_1, \phi_2, \ldots., \phi_p, \sigma^2)$.

The log likelihood function for the AR model is denoted by $L(\theta; x)$ for a sample size of T, and may be written as Lee [15],

$$L(\theta; x) = log f_{X_1}(x_1; \theta) + \sum_{t=2}^{T} log f_{X_t|X_{t-1}}(x_t|x_{t-1}; \theta). \tag{4.2}$$

However, the likelihood approach needs the large sample properties while the belief function approach does not. The belief function does not need the prior density function as usually needed in the Bayesian approach. The belief function approach, therefore, has several advantages compared with the frequentist and also the Bayesian approaches (Kanjanatarakul et al.[5]).

Shafer[6] proposed an intuitive approach in which a belief function is built (see Kanjanatarakul et al. [5]). The likelihood-based belief function Bel_x^Θ on Θ is induced by x of which the contour function $(pl_x(\theta))$ could be specified as:

$$pl_x(\theta) = \varphi(\theta; x) = \frac{L(\theta; x)}{sup_{\theta' \in \Theta} L_x(\theta')}, \tag{4.3}$$

The corresponding plausibility function can be computed from pl_x by an expression as:

$$Pl_x^\Theta(A) = \sup_{\theta \in A} \varphi(\theta; x) = \frac{\sup_{\theta \in A} L(\theta; x)}{\sup_{\theta' \in \Theta} L(\theta'; x)}, \tag{4.4}$$

where Pl_x^Θ is the plausibility function and $\forall A \subseteq \Theta$.

The focal sets (which are random and closed) of Bel_x^Θ are the level sets of the contour function $\varphi(\theta; x)$ defined as follows:

$$S(\omega; x) = \{\theta \in \Theta | \varphi(\theta; x) \geq \omega\}$$

where $\omega \in [0, 1]$.

By Demspter [2] using the sampling model for inference to forecast the problem, the random variable of interest Y is the function of parameter θ. The unobserved auxiliary variable is $U \in \amalg$ which has the known probability distribution P_U independent from θ. An inference model can be written as:

$$Y = a(\theta, U), \tag{4.5}$$

where Y is continuous and some random quantity $Y \in \Upsilon$. U has a normal distribution. After observing a value x of random quantities $X \sim f_\theta(x)$, $f_\theta(x)$ is specified for the probability mass or density function. $a(\cdot)$ is defined in such a way that the distribution of Y fixed θ (see Kanjanatarakul et al. [5]) is a conditional distribution $h_{x,\theta}(y)$. Consequently, the inference model can be obtained as $Y = F^{-1}_{x,\theta}(U)$ in which $F_{x,\theta}$ is the conditional cdf of Y given $X = x$.

The definition of mapping can be illustrated according to Kanjanatarakul et al. [5]. We composed the original multi-valued mapping $S_x : [0,1] \to 2^{\Theta}$ with $a(\cdot)$ to get the new multi-valued mapping $S^{'}_x$ from $[0,1] \times \text{II}$ to 2^{Υ}. It can be described that $S^{'}_x : [0,1] \times \text{II} \to 2^{\Upsilon}$, so $(\omega, u) \to a\Big(S_x(\omega), u\Big)$. The function $a(\cdot)$ draws each (θ_0, u) to $y_0 = a(\theta_0, u)$. The set $S^{'}_x(\omega, u) = a\Big(S_x(\omega), u\Big)$ is therefore the set of every value $a(\theta_0, u)$ for θ_0 in $S_x(\omega)$.

The distribution of U is independent of θ. U and ω relative to Bel^{Θ}_x are also independent. The belief function Bel^{Θ}_x is influenced by the Lebesgue measure λ on [0,1] and the multi-valued mapping S_x from [0,1] to 2^{Θ}. Thus, the product measure $(\lambda \otimes P_U)$ on $[0,1] \times \text{II}$ and the multi-valued mapping $S^{'}_x$ cause predictive belief functions and predictive plausibility functions on Υ as follows:

$$Bel^{\Upsilon}_x(A) = (\lambda \otimes P_U)\Big((\omega, u) | a(S_x(\omega), u) \subseteq A\Big), \tag{4.6}$$

$$Pl^{\Upsilon}_x(A) = (\lambda \otimes P_U)\Big((\omega, u) | a(S_x(\omega), u) \cap A \neq \varnothing\Big), \tag{4.7}$$

where $A \subseteq \Upsilon$, particularly Υ is the real line. It will be used to define the lower and upper predictive cdfs of Y as following:

$$F^{\ell}_x(y) = Bel^{\Upsilon}_x((-\infty, y]), \tag{4.8}$$

$$F^{\wp}_x(y) = Pl^{\Upsilon}_x((-\infty, y]), \tag{4.9}$$

where $y \in \mathbb{R}$, F^{ℓ}_x is a lower predictive cdf, and $F^{\wp}_x$ is an upper predictive cdf. Eq.(4.6)-Eq.(4.9) can be analytically estimated by Monte Carlo (MC) simulation.

Example 4.1. Considering, X_t is growth rate of the GDP. $\{X_t,\ t = 1, 2, \ldots, T\}$ has a normal distribution $N(\mu, \sigma^2)$. The contour function on $\theta = (\mu, \sigma^2)$ given a value x of X is:

$$pl_x(\mu, \sigma^2) = \frac{(2\pi\sigma^2)^{-T/2} exp\Big(-\frac{1}{2\sigma^2}\Big)\sum_{t=1}^{T}(x_t - \mu)^2}{(2\pi\hat{\sigma}^2)^{-T/2} exp\Big(-\frac{1}{2\hat{\sigma}^2}\Big)\sum_{t=1}^{T}(x_t - \bar{x})^2}. \tag{4.10}$$

Resulting in,

$$pl_x(\mu, \sigma^2) = \Big(\frac{\hat{\sigma}^2}{\sigma^2}\Big) exp\Big(\frac{T}{2} - \frac{1}{2\sigma^2}\sum_{t=1}^{T}(x_t - \mu)^2\Big), \tag{4.11}$$

where $\bar{x}$ and $\hat{\sigma}^2$ are the sample mean and the sample variance, respectively.

Then, $Y \sim N(\mu, \sigma^2)$ is the predicted dataset which is not yet observed data and has the same distribution. The predicted data can be expressed as

$$Y = a(\theta, U) = \mu + \sigma U, \tag{4.12}$$

where $U \sim N[0, 1]$.

Incorporating the set of parameters $\theta = (c, \phi, \sigma^2)$ and the distribution of U into Eq.(4.12), Y can be forecasted in the future. From the AR(1) model, thus, the objective function in estimation can be written as:

$$Y_q = c + \phi Y_{q-1} + \sigma u. \tag{4.13}$$

For any (ω, u) in $[0, 1] \times \mathbb{R}$, the set $a\Big(S_x(\omega), u\Big)$ is the interval $\Big[y^\ell(\omega, u), y^\wp(\omega, u)\Big]$ defined by the lower and upper bounds for the values in the future. Specifying a set of random variables (ω_q, u_q), they are independent in a simulation "q" times which $q > t$. Hence, the lower and upper boundary is $\Big[y_q^\ell(\omega_q, u_q), y_q^\wp(\omega_q, u_q)\Big]$.

Therefore, the lower boundary and the upper boundary corresponding to Eq.(4.10)-Eq.(4.11) may be expressed as:

$$y_q^\ell(\omega_q, u_q) = \min_\theta [c + \phi y_{q-1} + \sigma u_q]] \text{subject to} [pl_x(\theta) \geq \omega_q] \tag{4.14}$$

and,

$$y_q^\wp(\omega_q, u_q) = \max_\theta [c + \phi y_{q-1} + \sigma u_q]] \text{subject to} [pl_x(\theta) \geq \omega_q]. \tag{4.15}$$

We draw 10,000 pairs of (ω_q, u_q) independently and obtain 10,000 intervals of $\Big[y_q^\ell(\omega_q, u_q), y_q^\wp(\omega_q, u_q)\Big]$. Consequently, the quantities $Bel_x^\Upsilon(A)$ and $Pl_x^\Upsilon(A)$ are defined by predictive belief and plausibility functions on Υ, such that,

$$\widehat{Bel}_x^\Upsilon(A) = \frac{1}{10,000} \sharp \Bigg\{ \big[q \in \{1, ..., 10,000\}\big] \big| [y_q^\ell(\omega_q, u_q), y_q^\wp(\omega_q, u_q)] \subseteq A \Bigg\}, \tag{4.16}$$

$$\widehat{Pl}_x^\Upsilon(A) = \frac{1}{10,000} \sharp \Bigg\{ \big[q \in \{1, ..., 10,000\}\big] \big| [y_q^\ell(\omega_q, u_q), y_q^\wp(\omega_q, u_q)] \cap A \neq \varnothing \Bigg\}. \tag{4.17}$$

We may also define the lower and upper predictive quantiles at level α for any $\alpha \in (0, 1)$ as:

$$q_\alpha^\ell = (F_x^\wp)^{-1}(\alpha), \tag{4.18}$$

$$q_\alpha^\wp = (F_x^\ell)^{-1}(\alpha). \tag{4.19}$$

Finally, the two cdfs of the belief and plausibility functions describe the knowledge of Y, given the observed data x completely.

4.2 Empirical Results

This study aims at forecasting the Thailand GDP growth rate using AR-based belief function model. First, we used the linear AR(p) model to analyse the past behavior of Thailand GDP growth rate, and the order p was decided in terms of AIC and BIC. Thereafter, the AR-based belief function model was applied to forecast Thailand GDP growth rate from 2015 to 2020. All the data, GDP annual growth rate in Thailand, are from the National Economic and Social Development (NESDB), and cover the period of 1952 to 2014.

The Thailand GDP growth rate is illustrated in Fig. 1. The GDP growth rates are all positive but a few. The Asian Financial Crisis in 1997 led to negative growth of Thailand GDP. Table 1 provides summary statistics on Thailand GDP Growth Rate. We found that the average of Thailand GDP Growth Rate is about 5.7%, and has a negative skewness (-0.3789). However, the result of the Jarque-Bera test statistic was not significant, thereby implying that Thailand GDP annual growth rate is normally distributed. Table 2 shows the results of AIC and BIC for AR(p) where $p = 1, ..., 5$. The results show that the AR(1) model has a better performance than the candidate models due to minimum value of AIC and BIC.

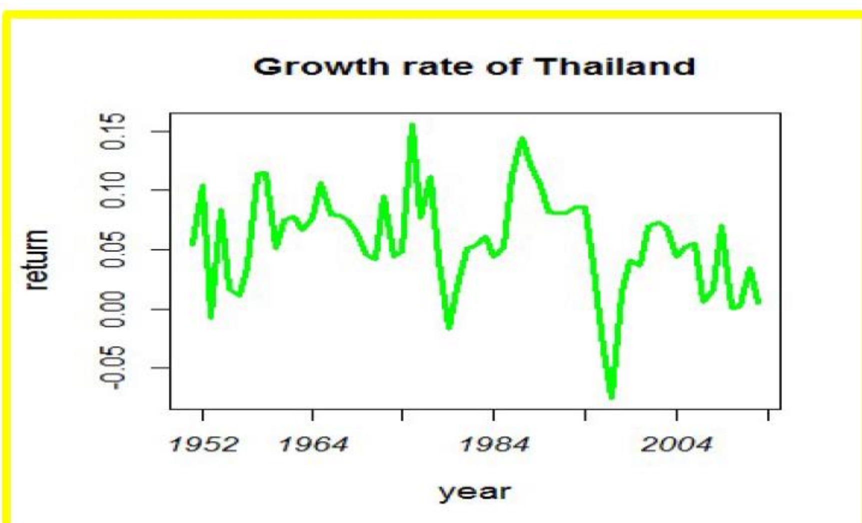


Figure 1: Thailand's growth rate form 1952-2014.

Table 1: Summary statistics for Thailand GDP Annual Growth Rate

Statistic Value	GDP growth rate
Mean	0.057454
Median	0.055223
Maximum	0.155751
Minimum	-0.074973
Std. Dev.	0.042079
Skewness	-0.378945
Kurtosis	3.694852
Jarque-Bera	2.775191
Probability	0.249675
Sum	3.619617
Sum Sq. Dev.	0.109778
Observations	63

Note: Computation at the confidential interval 90%.

Table 2: Akaike information criterion (AIC) and Bayesian information criterion (BIC)

	AR(1)	AR(2)	AR(3)	AR(4)	AR(5)
AIC	-230.6654	-228.7273	-226.7275	-225.9599	-224.0337
BIC	-224.2360	-220.1547	-216.0118	-213.1011	-209.0317

Table 3 presents the estimated results of AR(1) model. All the parameters are statistically significant at a 99% level of confidence. As expected, the first lag of the growth rate of Thailand GDP has a positive effect in domestic economic growth. The estimated ϕ equals to 0.4657, which implies Thailand GDP growth rates as having strong autocorrelation. The residuals of the model are consistent with the white noise series with $\varepsilon_t \sim \; N(0, 0.001407)$, such that σ is equal to 0.03751. These results show that the AR(1) process with normal distribution is the appropriate model in this study.

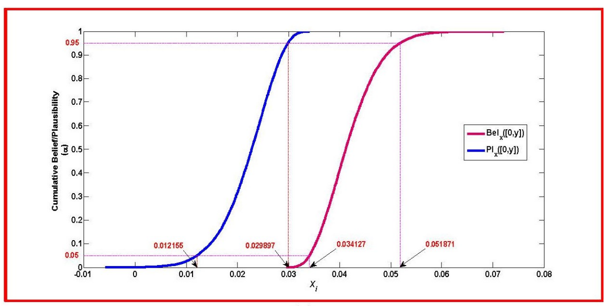
Table 3: Estimation of the selected AR(1) model for the GDP growth rate of the year 1952-2014

Parameter	value	std. error	z-test	p-value
c	0.05669	0.00859	6.59840	<0.0001***
ϕ	0.46565	0.11150	4.17600	<0.0001***

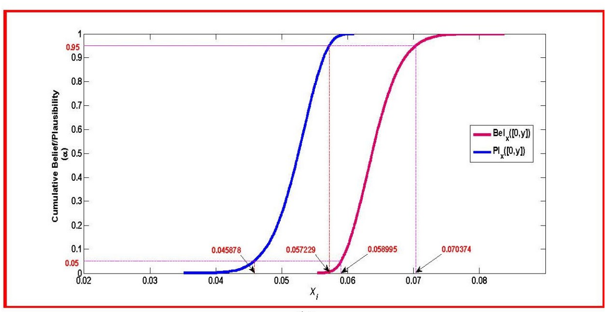
Note: "***" is the significance level at 0.01.

Now, we turn to the prediction of growth rates using AR(1)-belief approach. There are two steps in the prediction. First, the recursive method was used to estimate the mean values of growth rates, and then we computed α-quantile intervals of prediction and the predictive cdfs of belief and plausibility.

Fig.2.1 - Fig.2.3 show the forecasts of growth rates ranging from year 2015 to 2020 at the confidence level 90%. They exhibit the predictive lower and upper cdfs of Y or $F_x^{\ell}(y)$ and $F_x^{\wp}(y)$. From the figures, for example, Fig.2.1(b) illustrates that the growth rate range of year 2016 will have the quantile range of the plausibility and belief equal to (0.045878-0.058995) at $\alpha = 0.05$. On the above level, the growth rate range is equal to (0.057229-0.070374) at $\alpha = 0.95$. Other figures of growth rate intervals are interpreted similar to the year 2016.

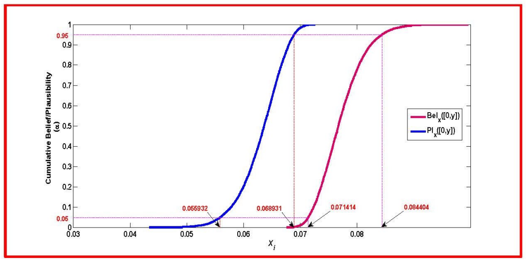


(a)

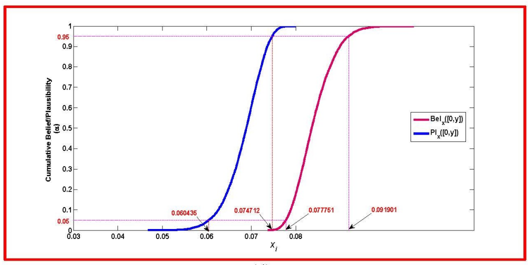


(b)

Figure 2.1: The predictive lower and upper cdfs of the plausibility and belief function: 2.1(a) year 2015 and 2.1(b) year 2016

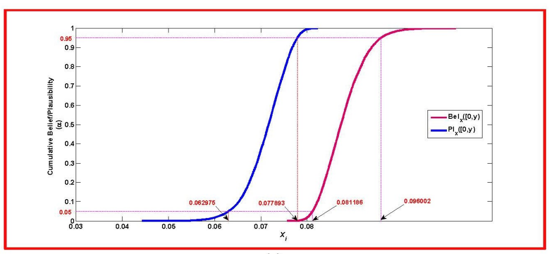


(c)

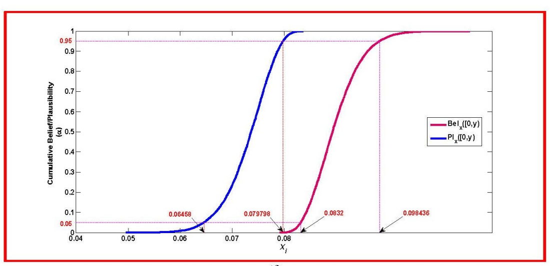


(d)

Figure 2.2: The predictive lower and upper cdfs of the plausibility and belief function: 2.2(c) year 2017 and 2.2(d) year 2018



(e)



(f)

Figure 2.3: The predictive lower and upper cdfs of the plausibility and belief function: 2.3(e) year 2019 and 2.3(f) year 2020

Table 4 thoroughly indicates lower and upper predictive intervals which are collected from Fig.2.1-Fig.2.3. The table can be explained as follows. Given $q_\alpha^\ell = (F_x^\wp)^{-1}(\alpha)$ and $q_\alpha^\wp = (F_x^\ell)^{-1}(\alpha)$ be the lower and upper predictive quantiles at level α for any $\alpha \in (0,1)$. The predictive plausibility function is thus $Pl_x^\Upsilon((-\infty, q_\alpha^\ell]) = \alpha$ and $Pl_x^\Upsilon((-\infty, q_{1-\alpha}^\ell]) = 1-\alpha$. The predictive belief function is also $Bel_x^\Upsilon((-\infty, q_\alpha^\wp]) = \alpha$ and $Bel_x^\Upsilon((-\infty, q_{1-\alpha}^\wp]) = 1-\alpha$.

Table 4: The α-quantile predictions of growth rates based on belief and plausibility functions

Cumulative	Predictive growth rate (years)		
Plausibility \ Belief	2015	2016	2017
$Pl_x^\Upsilon((-\infty, q_\alpha^\ell])$	0.01220	0.04590	0.05590
$Pl_x^\Upsilon((-\infty, q_{1-\alpha}^\ell])$	0.02990	0.05720	0.06890
$Bel_x^\Upsilon((-\infty, q_\alpha^\wp])$	0.03410	0.05900	0.07140
$Bel_x^\Upsilon((-\infty, q_{1-\alpha}^\wp])$	0.05190	0.07040	0.08440
Cumulative	Predictive growth rate (years)		
Plausibility \ Belief	2018	2019	2020
$Pl_x^\Upsilon((-\infty, q_\alpha^\ell])$	0.06040	0.06300	0.06460
$Pl_x^\Upsilon((-\infty, q_{1-\alpha}^\ell])$	0.07470	0.07790	0.07980
$Bel_x^\Upsilon((-\infty, q_\alpha^\wp])$	0.07780	0.08120	0.08320
$Bel_x^\Upsilon((-\infty, q_{1-\alpha}^\wp])$	0.09190	0.09600	0.09840

Note: the confidence level 90% and $\alpha = 0.05$.

Taking year 2015, for instance, we have quantile predictive ranges (0.01220-0.03410) and (0.02990-0.05190) at the confidence interval 90%, as well as other years. Concisely, the growth rate ranges for 6 years (2015-2020) will grow (1.22-5.19%), (4.59-7.04%), (5.59-8.44%), (6.04-9.19%), (6.30-9.60%), and (6.46-9.84%) subsequently.

5 Conclusion

In this paper, we proposed the standard AR(1) model with belief function framework for the GDP growth rate of Thailand. The reason is the AR(1) model is the best fit by the minimum value of AIC and BIC criteria for model selection. The uncertainty in this research was associated with the Dempster-Shafer belief function theory. The statistical forecasting relied on the historical data and the econometric model. There were some main steps in approximation. Firstly, the AR(1) model was used to extract the set of the parameter θ applied to the belief function built from the normalized likelihood-based function. Secondly, in the prediction step, the variable of interest Y was used to forecast as a function of the parameter θ and an auxiliary random variable U with the known distribution P_U. Lastly, the modified approach AR(p)-belief to predict the growth rate intervals obtained unobserved data in each year.

The statistical AR(1)-belief function is an alternative approach with several assumptions free to forecast the random variable of interest: the growth rate of Thailand in this study. The approach is more flexible than the conventional method (the AR model) because a number of real data does not matter for the computation. In other words, it can generate the small sample size to the large data by MC simulation to estimate the predictive cdfs of Y. The results show the predictive economic growth by those intervals between the lower and upper predictive boundary. We also illustrate step by step in predictions from year 2015 to 2020 displayed by Fig.2.1-Fig.2.3 and Table 4. The growth rate range of the lower and the upper anticipated bound is summarized as the range (1.22-9.84%) forecast from 2015 to 2020.

The outcomes from the belief function approach will be definitely extended in the future. We will apply a more efficient model with the belief function approach to predict a growth rate of GDP by ARMA(1,1)-beliefs with a related economic variable as the future work. The approach may be extended to include expert opinions from several government and private institutions into the advanced belief function.

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