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Some Characterizations of (m, μ)-Precontinuous Functions[1]

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Abstract : The aim of this paper is to introduce weak froms of (m, μ)-continuous functions, which are called (m, μ)-precontinuous, almost (m, μ)-precontinuous and weakly (m, μ)-precontinuous, as funtions from an m-space into a generalized topological space. Several characterizations of functions are obtained.

Keywords : m-space; m-preopen set; generalized topological space; (m, μ)-continuous function; (m, μ)-precontinuous function; almost (m, μ)-precontinuous function; weakly (m, μ)-precontinuous function.
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1 Introduction

The concepts of m-spaces and M-continuity were introduced by Popa and Noiri in [1]. Later, Császár introduced the notions of generalized topological spaces and generalized continuity in [2]. Such spaces are the generalization of topological spaces. In [3] and [4], Boonpok introduced the concepts of continuity, almost

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continuity and weakly continuity from a generalized topological space into an m-space and investigated some their characterizations. After that, the nontions of continuity, almost continuity and weakly continuity from an m-space into a generalized topological space were studied by Phodee et al. in [5]. In this paper, we introduce the weak forms continuity from an m-space into a generalized topological space by using m-preopen sets which were introduced by Rasas [6]. Furthermore, some their characterizations are obtained.

2 Preliminaries

We begin this section by introducing the notion of a m-space in [7]. Let X be a nonempty set and $\mathscr{P}(X)$ the power set of X. A subfamily m of $\mathscr{P}(X)$ is called a *minimal structure* (briefly *m-structure*) on X if $\emptyset \in m$ and $X \in m$. The pair (X, m) is called an *m-space*. Each member of m is called *m-open* and the complement of an m-open set is called *m-closed*. For a m-structure m on X and $A \subset X$, the closure of A on m, denoted by m-Cl(A), is the intersection of all m-closed sets containing A, i.e., $m\text{-Cl}(A) = \bigcap\{F : X - F \in m \text{ and } A \subset F\}$, and the interior of A on m, denoted by m-Int(A), is the union of all m-open stes contained in A, i.e., $m\text{-Int}(A) = \bigcup\{U : U \in m \text{ and } U \subset A\}$. A subset A of an m-space (X, m) is said to be *m-preopen* [6] if $A \subset m\text{-Int}(m\text{-Cl}(A))$. The complement of an m-preopen set is called *m-preclosed*. It can verify that A is m-preclosed if and only if $m\text{-Cl}(m\text{-Int}(A)) \subset A$. For a subset A of an m-space (X, m), the m-pre-closure of A, denoted by m-pCl(A), and the m-pre-interior of A, denoted by m-pInt(A), defined as follows:

$$m\text{-pCl}(A) = \bigcap\{F : F \text{ is } m\text{-preclosed and } A \subset F\},$$

and

$$m\text{-pInt}(A) = \bigcup\{U : U \text{ is } m\text{-preopen and } U \subset A\}.$$

We see that $m\text{-pInt}(A) \subset A \subset m\text{-pCl}(A)$ and if $A \subset B \subset X$, then $m\text{-pCl}(A) \subset m\text{-pCl}(B)$ and $m\text{-pInt}(A) \subset m\text{-pInt}(B)$. It is easy to prove that $x \in m\text{-pCl}(A)$ if and only if $W \cap A \neq \emptyset$ for every m-preopen set W containing x. Furthermore, $m\text{-pCl}(X - A) = X - m\text{-pInt}(A)$ and $m\text{-pInt}(X - A) = X - m\text{-pCl}(A)$. It is easy to observe that if A_γ is m-preopen for all $\gamma \in J$, then $\bigcup_{\gamma \in J} A_\gamma$ is m-preopen, and if A_γ is m-preclosed for all $\gamma \in J$, then $\bigcap_{\gamma \in J} A_\gamma$ is m-preclosed. As a consequence of the previous fact, we obtain the following properties: m-pInt(A) is m-preopen; m-pCl(A) is m-preclosed; A is m-preopen if and only if $A = m\text{-pInt}(A)$; A is m-preclosed if and only if $A = m\text{-pCl}(A)$; $m\text{-pInt}(m\text{-pInt}(A)) = m\text{-pInt}(A)$; $m\text{-pCl}(m\text{-pCl}(A)) = m\text{-pCl}(A)$.

Now, we recall some notions of generalized topological spaces in [5]. A subcollection μ of subsets of a nonempty set Y is called a *generalized topology* (briefly, *GT*) on Y if $\emptyset \in \mu$ and any union of elements of μ belongs to μ. In this case, (Y, μ) is called a *generalized topological space* (briefly, *GTS*). A subset A of Y is called *μ-open* if $A \in \mu$. The complement of a μ-open set is called a *μ-closed* set. For a GTS (Y, μ) and $A \subset Y$, $c_\mu(A)$ is the intersection of all μ-closed sets containing A, i.e., the smallest μ-closed set containing A, and $i_\mu(A)$ is the union of all μ-open sets contained in A, i.e., the largest μ-open set contained in A. Clearly, $i_\mu(A) \subset A \subset c_\mu(A)$. It is easy to verify that c_μ and i_μ are idempotent (i.e., if $A \subset Y$, then $c_\mu(A) = c_\mu(c_\mu(A))$ and $i_\mu(A) = i_\mu(i_\mu(A))$) and monotonic (i.e., if $A \subset B \subset Y$, then $c_\mu(A) \subset c_\mu(B)$ and $i_\mu(A) \subset i_\mu(B)$). Moreover, $c_\mu(Y - A) = Y - i_\mu(A)$ and $i_\mu(Y - A) = Y - c_\mu(A)$. It is well known that $x \in c_\mu(A)$ if and only if $x \in V \in \mu$ implies $V \cap A \neq \emptyset$.

A subset A of a GTS (Y, μ) is said to be *μr-open* (resp. *μ-semi-open*, *μ-preopen*, *μ-α-open*, *μ-β-open*) if $A = i_\mu(c_\mu(A))$ (resp. $A \subset c_\mu(i_\mu(A))$, $A \subset i_\mu(c_\mu(A))$, $A \subset i_\mu(c_\mu(i_\mu(A)))$, $A \subset c_\mu(i_\mu(c_\mu(A)))$). The complement of a μr-open (resp. μ-semi-open, μ-preopen, μ-α-open, μ-β-open) set is said to be *μr-closed* (resp. *μ-semi-closed*, *μ-preclosed*, *μ-α-closed*, *μ-β-closed*). Clearly, A is μr-closed if and only if $A = c_\mu(i_\mu(A))$. Let (X, m) be an m-space and (Y, μ) a GTS. A function $f : (X, m) \to (Y, \mu)$ is said to be *(m, μ)-continuous* (resp. *almost (m, μ)-continuous*, *weakly (m, μ)-continuous*) *at a point $x \in X$* if for each μ-open set V containing $f(x)$, there exists an m-open set U containing x such that $f(U) \subset V$ (resp. $f(U) \subset i_\mu(c_\mu(V))$, $f(U) \subset c_\mu(V)$). A function $f : (X, m) \to (Y, \mu)$ is said to be *(m, μ)-continuous* (resp. *almost (m, μ)-continuous*, *weakly (m, μ)-continuous*) if f is (m, μ)-continuous (resp. *almost (m, μ)-continuous*, *weakly (m, μ)-continuous*) at every point in X.

3 Main Results

In this section, we shall introduce some weak forms continuity from an m-space into a GTS and study some of their characterizations. Throughout this section, let (X, m) and (Y, μ) be an m-space and a GTS, respectively.

Definition 3.1. A function $f : (X, m) \to (Y, \mu)$ is said to be *(m, μ)-precontinuous at a point $x \in X$* if for each μ-open set V containing $f(x)$, there exists an m-preopen set U containing x such that $f(U) \subset V$. A function $f : (X, m) \to (Y, \mu)$ is said to be *(m, μ)-precontinuous* if f is (m, μ)-precontinuous at every point in X.

Remark 3.2. *Clearly, every (m, μ)-continuous function is (m, μ)-precontinuous but the converse is not true as the following example.*

Example 3.3. Let $X = \{1, 2, 3, 4\}$, $m = \{\emptyset, \{1, 2\}, \{1, 3\}, X\}$ and $Y = \{a, b, c, d\}$, $\mu = \{\emptyset, \{a, b\}, \{a, c\}, \{a, b, c\}\}$. Define $f : (X, m) \to (Y, \mu)$ as follows: $f(1) = a$, $f(2) = b$, $f(3) = b$, $f(4) = b$. Then f is (m, μ)-precontinuous but it is not (m, μ)-continuous.

Theorem 3.4. *A function $f : (X, m) \to (Y, \mu)$ is (m, μ)-precontinuous if and only if $f^{-1}(V)$ is m-preopen in X for every μ-open set V in Y.*

Proof. Let V be a μ-open set in Y and let $x \in f^{-1}(V)$. Since f is (m, μ)-precontinuous, there exists an m-preopen set U in X containing x such that

$f(U) \subset V$. Then $x \in U \subset f^{-1}(V)$, and so $x \in m\text{-pInt}(f^{-1}(V))$. Thus $f^{-1}(V) = m\text{-pInt}(f^{-1}(V))$. Hence, $f^{-1}(V)$ is m-preopen in X.

Conversely, let $x \in X$ and V a μ-open set in Y containing $f(x)$. Then $x \in f^{-1}(V)$. By assumption, $x \in m\text{-pInt}(f^{-1}(V))$. Thus there exists an m-preopen set U in X such that $x \in U \subset f^{-1}(V)$. Hence, $f(U) \subset V$, and so f is (m,μ)-precontinuous at x. This implies f is (m,μ)-precontinuous. □

Theorem 3.5. *For a function $f : (X,m) \to (Y,\mu)$, the following properties are equivalent:*

(1) *f is (m,μ)-precontinuous;*

(2) *$f(m\text{-pCl}(A)) \subset c_\mu(f(A))$ for every subset A of X;*

(3) *$m\text{-pCl}(f^{-1}(B)) \subset f^{-1}(c_\mu(B))$ for every subset B of Y;*

(4) *$f^{-1}(i_\mu(B)) \subset m\text{-pInt}(f^{-1}(B))$ for every subset B of Y;*

(5) *$f^{-1}(F)$ is m-preclosed in X for every μ-closed set F in Y.*

Proof. (1)⇒(2) Let A be a subset of X and let $x \in m\text{-pCl}(A)$. Let V be a μ-open set in Y containing $f(x)$. By (1), there exists an m-preopen set U in X such that $x \in U \subset f^{-1}(V)$. Since $x \in m\text{-pCl}(A)$, $U \cap A \neq \emptyset$. Then $\emptyset \neq f(U \cap A) \subset f(U) \cap f(A) \subset V \cap f(A)$. This implies $f(x) \in c_\mu(f(A))$, and so $x \in f^{-1}(c_\mu(f(A)))$. Thus $m\text{-pCl}(A) \subset f^{-1}(c_\mu(f(A)))$. Hence, $f(m\text{-pCl}(A)) \subset c_\mu(f(A))$.

(2)⇒(3) Let B be a subset of Y. By (2), $f(m\text{-pCl}(f^{-1}(B))) \subset c_\mu(f(f^{-1}(B)))$. Hence, $m\text{-pCl}(f^{-1}(B)) \subset f^{-1}(c_\mu(B))$.

(3)⇒(4) Let B be a subset of Y. By (3), $m\text{-pCl}(f^{-1}(Y-B)) \subset f^{-1}(c_\mu(Y-B))$. Hence, $f^{-1}(i_\mu(B)) \subset m\text{-pInt}(f^{-1}(B))$.

(4)⇒(5) Let F be a μ-closed set in Y. Then $Y - F = i_\mu(Y - F)$. By (4), $f^{-1}(Y - F) \subset m\text{-pInt}(f^{-1}(Y - F))$. This implies $m\text{-pCl}(f^{-1}(F)) \subset f^{-1}(F)$. Hence, $m\text{-pCl}(f^{-1}(F)) = f^{-1}(F)$, and so $f^{-1}(F)$ is m-preclosed in X

(5)⇒(1) It follows from Theorem 3.4. □

Next, we shall introduce a weak form of (m,μ)-precontinuous functions and study some of their characterizations.

Definition 3.6. A function $f : (X,m) \to (Y,\mu)$ is said to be *almost* (m,μ)-*precontinuous at a point* $x \in X$ if for each μ-open set V containing $f(x)$, there exists an m-preopen set U containing x such that $f(U) \subset i_\mu(c_\mu(V))$. A function $f : (X,m) \to (Y,\mu)$ is said to be *almost* (m,μ)-*precontinuous* if f is almost (m,μ)-precontinuous at every point in X.

Remark 3.7. *It is clear that every (m,μ)-precontinuous function is almost (m,μ)-precontinuous but the converse is not true as the following example.*

Example 3.8. Let $X = \{1,2,3,4\}$, $m = \{\emptyset, \{1,2\}, \{1,3\}, X\}$ and $Y = \{a,b,c,d\}$, $\mu = \{\emptyset, \{a,b\}, \{a,c\}, \{a,b,c\}\}$. Define $f : (X,m) \to (Y,\mu)$ as follows: $f(1) = c$, $f(2) = a$, $f(3) = b$, $f(4) = c$. Then f is almost (m,μ)-precontinuous but it is not (m,μ)-precontinuous.

Theorem 3.9. *For a function $f : (X, m) \to (Y, \mu)$, the following properties are equivalent:*

(1) *f is almost (m, μ)-precontinuous;*

(2) *$f^{-1}(V) \subset m\text{-pInt}(f^{-1}(i_\mu(c_\mu(V))))$ for every μ-open set V in Y;*

(3) *$m\text{-pCl}(f^{-1}(c_\mu(i_\mu(F)))) \subset f^{-1}(F)$ for every μ-closed set F in Y;*

(4) *$m\text{-pCl}(f^{-1}(c_\mu(i_\mu(c_\mu(B))))) \subset f^{-1}(c_\mu(B))$ for every subset B of Y;*

(5) *$f^{-1}(i_\mu(B)) \subset m\text{-pInt}(f^{-1}(i_\mu(c_\mu(i_\mu(B)))))$ for every subset B of Y;*

(6) *$f^{-1}(V)$ is m-preopen in X for every μr-open set V in Y;*

(7) *$f^{-1}(F)$ is m-preclosed in X for every μr-closed set F in Y.*

Proof. (1)⇒(2) Let V be a μ-open set in Y and let $x \in f^{-1}(V)$. Then $f(x) \in V$. By (1), there exists an m-preopen set U in X containing x such that $f(U) \subset i_\mu(c_\mu(V))$. Thus $x \in m\text{-pInt}(f^{-1}(i_\mu(c_\mu(V))))$. Now, it obtain that $f^{-1}(V) \subset m\text{-pInt}(f^{-1}(i_\mu(c_\mu(V))))$.

(2)⇒(3) Let F be a μ-closed set in Y. Then $Y - F$ is μ-open in Y. By (2), $f^{-1}(Y-F) \subset m\text{-pInt}(f^{-1}(i_\mu(c_\mu(Y-F))))$. This implies $m\text{-pCl}(f^{-1}(c_\mu(i_\mu(F)))) \subset f^{-1}(F)$.

(3)⇒(4) Let B be a subset of Y. Then $c_\mu(B)$ is μ-closed set in Y. By (3), $m\text{-pCl}(f^{-1}(c_\mu(i_\mu(c_\mu(B))))) \subset f^{-1}(c_\mu(B))$.

(4)⇒(5) Let B be a subset of Y. By (4), $m\text{-pCl}(f^{-1}(c_\mu(i_\mu(c_\mu(Y - B))))) \subset f^{-1}(c_\mu(Y - B))$. This implies $f^{-1}(i_\mu(B)) \subset m\text{-pInt}(f^{-1}(i_\mu(c_\mu(i_\mu(B)))))$.

(5)⇒(6) Let V be a μr-open set in Y. Then $i_\mu(V) = V = i_\mu(c_\mu(i_\mu(V)))$. By (5), $f^{-1}(V) \subset m\text{-pInt}(f^{-1}(V))$. This implies $f^{-1}(V)$ is m-preopen in X.

(6)⇒(7) It is clear.

(7)⇒(1) Let $x \in X$ and V a μ-open set in Y containing $f(x)$. Then $Y - i_\mu(c_\mu(V))$ is μr-closed in Y and $x \in f^{-1}(i_\mu(c_\mu(V)))$. By (7), $f^{-1}(Y - i_\mu(c_\mu(V)))$ is m-preclosed in X. Thus $f^{-1}(i_\mu(c_\mu(V)))$ is m-preopen in X. Set $U = f^{-1}(i_\mu(c_\mu(V)))$. Then U is m-preopen containing x such that $f(U) \subset i_\mu(c_\mu(V))$. Hence, f is almost (m, μ)-precontinuous at x. This implies f is almost (m, μ)-precontinuous. □

Theorem 3.10. *For a function $f : (X, m) \to (Y, \mu)$, the following properties are equivalent:*

(1) *f is almost (m, μ)-precontinuous;*

(2) *$m\text{-pCl}(f^{-1}(U)) \subset f^{-1}(c_\mu(U))$ for every μ-β-open set U in Y;*

(3) *$m\text{-pCl}(f^{-1}(U)) \subset f^{-1}(c_\mu(U))$ for every μ-semi-open set U in Y;*

(4) *$f^{-1}(U) \subset m\text{-pInt}(f^{-1}(i_\mu(c_\mu(U))))$ for every μ-preopen set U in Y.*

Proof. (1)⇒(2) Let U be a μ-β-open set in Y. Then $c_\mu(U)$ is μr-closed in Y. By (7) in Theorem 3.9, $f^{-1}(c_\mu(U))$ is m-preclosed in X. Hence, $m\text{-pCl}(f^{-1}(U)) \subset f^{-1}(c_\mu(U))$.

(2)⇒(3) It follows from the fact that every μ-semi-open set in Y is μ-β-open.

(3)⇒(1) Let F be a μr-closed set in Y. Then $c_\mu(F) = F = c_\mu(i_\mu(F))$, and so F is μ-semi-open in Y. By (3), m-pCl$(f^{-1}(F)) \subset f^{-1}(F)$. Hence, $f^{-1}(F)$ is m-preclosed in X. By (7) in Theorem 3.9, f is almost (m, μ)-precontinuous.

(1)⇒(4) Let U be a μ-preopen set in Y. Then $U \subset i_\mu(c_\mu(U))$ and $i_\mu(c_\mu(U))$ is μr-open in Y. By (6) in Theorem 3.9, $f^{-1}(i_\mu(c_\mu(U)))$ is m-preopen in X. Hence, $f^{-1}(U) \subset m$-pInt$(f^{-1}(i_\mu(c_\mu(U))))$.

(4)⇒(1) Let U be a μr-open set in Y. Then $U = i_\mu(c_\mu(U))$ and U is μ-preopen in Y. By (4), $f^{-1}(U) \subset m$-pInt$(f^{-1}(U))$. Hence, $f^{-1}(U)$ is m-preopen in X. By (6) in Theorem 3.9, f is almost (m, μ)-precontinuous. □

Finally, we shall introduce a weak form of almost (m, μ)-precontinuous functions and study some of their characterizations.

Definition 3.11. A function $f : (X, m) \to (Y, \mu)$ is said to be *weakly* (m, μ)-*precontinuous at a point* $x \in X$ if for each μ-open set V containing $f(x)$, there exists an m-preopen set U containing x such that $f(U) \subset c_\mu(V)$. A function $f : (X, m) \to (Y, \mu)$ is said to be *weakly* (m, μ)*-precontinuous* if f is weakly (m, μ)-precontinuous at every point in X.

Remark 3.12. *It is obvious that every almost* (m, μ)*-precontinuous function is weakly* (m, μ)*-precontinuous but the converse is not true as the following example.*

Example 3.13. Let $X = \{1, 2, 3, 4\}, m = \{\emptyset, \{1, 2\}, \{1, 3\}, X\}$ and $Y = \{a, b, c, d\}$, $\mu = \{\emptyset, \{a, b\}, \{a, c\}, \{a, b, c\}\}$. Define $f : (X, m) \to (Y, \mu)$ as follows: $f(1) = d$, $f(2) = a$, $f(3) = b$, $f(4) = d$. Then f is weakly (m, μ)-precontinuous but it is not almost (m, μ)-precontinuous.

Theorem 3.14. *A function* $f : (X, m) \to (Y, \mu)$ *is weakly* (m, μ)*-precontinuous if and only if* $f^{-1}(V) \subset m$-pInt$(f^{-1}(c_\mu(V)))$ *for every* μ*-open set* V *in* Y.

Proof. Let V be a μ-open set in Y and let $x \in f^{-1}(V)$. Since f is weakly (m, μ)-precontinuous, there exists an m-preopen set U in X containing x such that $f(U) \subset c_\mu(V)$. Then $x \in U \subset f^{-1}(c_\mu(V))$, and so $x \in m$-pInt$(f^{-1}(c_\mu(V)))$. Hence, $f^{-1}(V) \subset m$-pInt$(f^{-1}(c_\mu(V)))$.

Conversely, let $x \in X$ and V a μ-open set in Y containing $f(x)$. Then $x \in f^{-1}(V)$. By assumption, $x \in m$-pInt$(f^{-1}(c_\mu(V)))$. Thus there exists an m-preopen set U in X such that $x \in U \subset f^{-1}(c_\mu(V))$. Hence, $f(U) \subset c_\mu(V)$, and so f is weakly (m, μ)-precontinuous at x. This implies f is weakly (m, μ)-precontinuous. □

Theorem 3.15. *For a function* $f : (X, m) \to (Y, \mu)$*, the following properties are equivalent:*

(1) *f is weakly (m, μ)-precontinuous;*

(2) m-pCl$(f^{-1}(i_\mu(F))) \subset f^{-1}(F)$ *for every μ-closed set F in Y;*

(3) m-pCl$(f^{-1}(i_\mu(c_\mu(B)))) \subset f^{-1}(c_\mu(B))$ *for every subset B of Y;*

(4) $f^{-1}(i_\mu(B)) \subset m\text{-pInt}(f^{-1}(c_\mu(i_\mu(B))))$ *for every subset* B *of* Y*;*

(5) $m\text{-pCl}(f^{-1}(V)) \subset f^{-1}(c_\mu(V))$ *for every* μ*-open set* V *in* Y*.*

Proof. (1)⇒(2) Let F be a μ-closed set in Y. Then $Y - F$ is μ-open in Y. By Theorem 3.14, $f^{-1}(Y - F) \subset m\text{-pInt}(f^{-1}(c_\mu(Y - F)))$. Hence, $m\text{-pCl}(f^{-1}(i_\mu(F))) \subset f^{-1}(F)$.

(2)⇒(3) It is clear.

(3)⇒(4) Let B be a subset of Y. By (3), we have $m\text{-pCl}(f^{-1}(i_\mu(c_\mu(Y-B)))) \subset f^{-1}(c_\mu(Y - B))$. This implies $f^{-1}(i_\mu(B)) \subset m\text{-pInt}(f^{-1}(c_\mu(i_\mu(B))))$.

(4)⇒(1) Let V be a μ-open set in Y. Then $V = i_\mu(V)$. By (4), $f^{-1}(V) \subset m\text{-pInt}(f^{-1}(c_\mu(V)))$. By Theorem 3.14, f is weakly (m, μ)-precontinuous.

(2)⇔(5) It is easy to verify. □

Theorem 3.16. *For a function* $f : (X, m) \to (Y, \mu)$*, the following properties are equivalent:*

(1) f *is weakly* (m, μ)*-precontinuous;*

(2) $m\text{-pCl}(f^{-1}(i_\mu(F))) \subset f^{-1}(F)$ *for every* μr*-closed set* F *in* Y*;*

(3) $m\text{-pCl}(f^{-1}(i_\mu(c_\mu(G)))) \subset f^{-1}(c_\mu(G))$ *for every* μ*-*β*-open set* G *in* Y*;*

(4) $m\text{-pCl}(f^{-1}(i_\mu(c_\mu(G)))) \subset f^{-1}(c_\mu(G))$ *for every* μ*-semi-open set* G *in* Y*.*

Proof. (1)⇒(2) Let F be a μr-closed set in Y. Then F is μ-closed in Y. By (2) in Theorem 3.15, $m\text{-pCl}(f^{-1}(i_\mu(F))) \subset f^{-1}(F)$.

(2)⇒(3) Let G be a μ-β-open set in Y. Then $c_\mu(G)$ is μr-closed in Y. By (2), $m\text{-pCl}(f^{-1}(i_\mu(c_\mu(G)))) \subset f^{-1}(c_\mu(G))$.

(3)⇒(4) It follows from the fact that every μ-semi-open set in Y is μ-β-open.

(4)⇒(1) Let V be a μ-open set in Y. Then V is μ-semi-open in Y. By (4), $m\text{-pCl}(f^{-1}(V)) \subset f^{-1}(c_\mu(V))$. By (5) in Theorem3.15, f is weakly (m, μ)-precontinuous. □

Theorem 3.17. *For a function* $f : (X, m) \to (Y, \mu)$*, the following properties are equivalent:*

(1) f *is weakly* (m, μ)*-precontinuous;*

(2) $m\text{-pCl}(f^{-1}(i_\mu(c_\mu(G)))) \subset f^{-1}(c_\mu(G))$ *for every* μ*-preopen set* G *in* Y*;*

(3) $m\text{-pCl}(f^{-1}(G)) \subset f^{-1}(c_\mu(G))$ *for every* μ*-preopen set* G *in* Y*;*

(4) $f^{-1}(G) \subset m\text{-pInt}(f^{-1}(c_\mu(G)))$ *for every* μ*-preopen set* G *in* Y*.*

Proof. (1)⇒(2) Let G be a μ-preopen set in Y. Then $c_\mu(G)$ is μr-closed in Y. By (2) in Theorem 3.16, $m\text{-pCl}(f^{-1}(i_\mu(c_\mu(G)))) \subset f^{-1}(c_\mu(G))$.

(2)⇒(3) It follows from the definition of the μ-preopen set in a GTS.

(3)⇒(4) Let G be a μ-preopen set in Y. Then $G \subset i_\mu(c_\mu(G))$ and $i_\mu(Y - G)$ is μ-preopen in Y. By (3), $m\text{-pCl}(f^{-1}(i_\mu(Y - G))) \subset f^{-1}(c_\mu(i_\mu(Y - G)))$. This implies $f^{-1}(G) \subset m\text{-pInt}(f^{-1}(c_\mu(G)))$.

(4)⇒(1) It follows from fact that every μ-open set in Y is μ-preopen and Theorem 3.14. □

Now, we recall that a GTS (Y, μ) is said to be *μ-regular* [8] if for each μ-closed set F of Y not containing y, there exist disjoint μ-open set V and W such that $y \in V$ and $F \subset W$. It can verify that a GTS (Y, μ) is μ-regular if and only if for each $y \in Y$ and each $V \in \mu$ containing y, there exists $W \in \mu$ such that $y \in W \subset c_\mu(W) \subset V$.

Theorem 3.18. *For a function $f : (X, m) \to (Y, \mu)$, where (Y, μ) is μ-regular, the following are equivalent:*

(1) *f is (m, μ)-precontinuous;*

(2) *f is almost (m, μ)-precontinuous;*

(3) *f is weakly (m, μ)-precontinuous.*

Proof. It is clear that (1)⇒(2) and (2)⇒(3).

(3)⇒(1) Let $x \in X$ and V a μ-open set in Y containing $f(x)$. Since (Y, μ) is μ-regular, there exists $W \in \mu$ such that $f(x) \in W \subset c_\mu(W) \subset V$. By (3), there exists an m-preopen set U containing x such that $f(U) \subset c_\mu(W)$. Then $f(U) \subset V$. Hence, f is (m, μ)-precontinuous at x. This implies f is (m, μ)-precontinuous. □

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