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Iterative Scheme of Strongly Nonlinear General Nonconvex Variational Inequalities Problem[1]

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Abstract : In this work, we suggest and analyze an iterative scheme for solving the strongly nonlinear general nonconvex variational inequalities by using projection technique and Wiener-Hopf technique. We prove strong convergence of iterative scheme to the solution of the strongly nonlinear general nonconvex variational inequalities requires to the modified mapping T which is Lipschitz continuous but not strongly monotone mapping. Our result can be viewed and improvement the result of E. Al-Shemas [E. Al-Shemas, General nonconvex Wiener-Hopf Equations and general nonconvex variational inequalities, Journal of Mathematical Sciences: Advances and Applications 19 (1) (2013) 1–11].

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1 Introduction

The theory of variational inequalities is a branch of the mathematical sciences dealing with general equilibrium problems. It has a wide range of applications in economics, operations research, industry, physical, and engineering sciences. Many research papers have been written lately, both on the theory and applications of this field. Important connection with main areas of pure and applied science have been made, see for example [1–3] and the references cited therein.

Variational inequalities theory, which was introduce by Stampacchia [4], provides us with a simple, natural general and unified framework to study a wide class of problems arising in pure and applied science. The development of variational inequality theory can be viewed as the simultaneous pursuit of two different lines of research. On the one hand, it reveals the fundamental facts on the qualitative aspects of the solutions to important classes of problems. On the other hand, it also enables us to develop highly efficient and powerful new numerical methods for solving, for example, obstacle, unilateral, free, moving, and complex equilibrium problems.

Moreover, Noor [5], Moudafi [6] and Pang et al. [7] have also considered the variational inequality problems over these nonconvex sets. In [5,8], Noor has shown that the projection technique can be extended to nonconvex variational inequalities and has established the equivalence between the nonconvex variational inequalities and fixed point problems by using the projection technique.

In this work we consider necessary and sufficient condition for proof the iterative scheme which modified a mapping T with is Lipschitz continuous but not strongly monotone mapping and proof the strong convergence of iterative schemes to the solution of the strongly nonlinear general nonconvex variational inequalities.

2 Preliminaries

Let C be a closed subset of a real Hilbert space H with inner product $\langle \cdot, \cdot \rangle$ and norm $\| \cdot \|$ respectively. Let us recall the following well-known definitions and some auxiliary results of nonlinear convex analysis and nonsmooth analysis.

Definition 2.1. Let $u \in H$ be a point not lying in C. A point $v \in C$ is called a *closest point or a projection of u onto C* if $d_C(u) = \|u - v\|$ when d_C is a usual distance. The set of all such closest points is denoted by $P_C(u)$; that is,

$$P_C(u) = \{v \in C : d_C(u) = \|u - v\|\}. \tag{2.1}$$

Definition 2.2. Let C be a subset of H. The *proximal normal cone to C at x* is given by

$$N_C^P(x) = \{z \in H : \exists \rho > 0; x \in P_C(x + \rho z)\}. \tag{2.2}$$

The following characterization of $N_C^P(x)$ can be found in [9].

Lemma 2.3. *Let C be a closed subset of a Hilbert space H. Then*

$$z \in N_C^P(x) \text{ if and only if } \exists \sigma > 0, \langle z, y - x\rangle \leq \sigma \|y - x\|^2, \quad \forall y \in C. \tag{2.3}$$

Clark et al. [10] and Poliquin et al. [11] have introduced and studied a new class of nonconvex sets, which are called uniformly prox-regular sets. This class or uniformly prox-regular sets has played an important part in many nonconvex applications such as optimization, dynamic systems, and differential inclusions.

Definition 2.4. For a given $r \in (0, +\infty]$, a subset C of H is said to be *uniformly prox-regular with respect to* r if, for all $\overline{x} \in C$ and for all $0 \neq z \in N_C^P(x)$, one has

$$\langle \frac{z}{\|z\|}, x - \overline{x}\rangle \leq \frac{1}{2r}\|x - \overline{x}\|^2, \quad \forall x \in C. \tag{2.4}$$

It is well known that a closed subset of a Hilbert space is convex if and only if it is proximally smooth of radius $r > 0$. Thus, in Definition 2.4, in the case of $r = \infty$, the uniform r-prox-regularity C is equivalent to convexity of C. Then, it is clear that the class of uniformly prox-regular sets is sufficiently large to include the class p-convex sets, $C^{1,1}$ submanifolds (possibly with boundary) of H, the images under a $C^{1,1}$ diffeomorphism of convex sets, and many other nonconvex sets; see [10, 11].

Let C_r be a uniformly r-prox-regular(nonconvex) set. For given nonlinear mappings $T, A, g : C_r \to H$, we consider the problem of finding $u \in C_r : g(u) \in C_r$ such that

$$\langle Tu, g(v) - g(u)\rangle + \lambda\|g(v) - g(u)\|^2 \geq \langle A(u), g(v) - g(u)\rangle, \forall v \in C_r : g(v) \in C_r. \tag{2.5}$$

which is called *the strongly nonlinear general nonconvex variational inequality (SNGNVI)*, introduced by Eman [12].

It is worth mentioning that if $g = I$, the identity mapping, then problem (2.5) is equivalent to finding $u \in C_r$ such that

$$\langle Tu, v - u\rangle + \lambda\|v - u\|^2 \geq \langle A(u), v - u\rangle, \forall v \in C_r, \tag{2.6}$$

which is known as *strongly nonlinear nonconvex variational inequality* introduced and studied by Noor [8]. If $A(u) \equiv 0$, then problem (2.5) is equivalent to finding $u \in C_r$ such that

$$\langle Tu, g(v) - g(u)\rangle + \lambda\|g(v) - g(u)\|^2 \geq 0, \forall v \in C_r : g(v) \in C_r, \tag{2.7}$$

which is called the *nonconvex variational inequality.* If $g = I$ and $\lambda = 0$, then problem (2.7) is equivalent to finding $u \in C_r$ such that

$$\langle Tu, v - u\rangle \geq 0, \forall v \in C_r, \tag{2.8}$$

is called *general nonconvex variational inequality* introduce by Bounkhel et. al. [13] and Noor [5, 8].

It is known that problem (2.5) is equivalent to finding $u \in C_r$ such that

$$0 \in Tu - A(u) + N^P_{C_r} g(u), \tag{2.9}$$

which $N^P_{C_r} g(u)$ denote the normal cone of C_r at $g(u)$. The problem (2.9) is called the *the nonconvex variational inclusion problem associated with nonconvex variational inequalities* (2.5).

Let C be a closed subset of a real Hilbert space H. A mapping $T : C \to H$ is called $\gamma -$ *strongly monotone* if there exists a constant $\gamma > 0$ such that

$$\langle Tx - Ty, x - y\rangle \geq \gamma \|x - y\|^2, \tag{2.10}$$

for all $x, y \in C$. A mapping T is called $\mu -$ *Lipschitz continuous* if there exists a constant $\mu > 0$ such that

$$\|Tx - Ty\| \leq \mu \|x - y\|, \tag{2.11}$$

for all $x, y \in C$.

Lemma 2.5 ([14]). *Let C be a nonempty closed subset of H, $r \in (0, +\infty]$ and set $C_r ;= \{x \in H : d(x, C) < r\}$. If C is uniform r-prox-regular, then the following hold:*

(1) for all $x \in C_r, P_C(x) \neq \emptyset$,
(2) for all $s \in (0, r), P_C$ is Lipschitz continuous with constant $t_s = \frac{r}{r-s}$ on C_s,
(3) the proximal normal cone is closed as a set-valued mapping.

Lemma 2.6. *In a real Hilbert space H, there holds the inequality*

1. $\|x+y\|^2 \leq \|x\|^2 + 2\langle y, x+y\rangle \quad x, y \in H$ *and* $\|x-y\|^2 = \|x\|^2 - 2\langle x, y\rangle + \|y\|^2$,
2. $\|tx + (1-t)y\|^2 = t\|x\|^2 + (1-t)\|y\|^2 - t(1-t)\|x-y\|^2, \forall t \in [0, 1]$.

3 Main Results

In this section we first establish the equivalent between the strongly nonlinear general nonconvex variational inequalities (2.5) and the fixed point problem with the projection technique.

Lemma 3.1 ([12]). *For given $x^* \in C_r$ is a solution of the strongly nonlinear general nonconvex variational inequalities (2.5), if and only if*

$$g(x^*) = P_C[g(x^*) - \rho T x^* + \rho A(x^*)], \tag{3.1}$$

where P_C is the projection of H onto the uniformly prox-regular set C_r.

Proof. Let $x^* \in C_r$ be a solution of (2.5) , from (2.9) and for a constant $\rho > 0$, we have

$$0 \in g(x^*) + \rho N^P_{C_r} g(x^*) - (g(x^*) - \rho(Tx^* - A(x^*)) = (I + \rho N^P_{C_r})g(x^*) - (g(x^*) - \rho T x^* + \rho A(x^*))$$

if and only if

$$g(x^*) = (I + \rho N^P_{C_r})^{-1}[g(x^*) - \rho Tx^* + \rho A(x^*)] = P_C[g(x^*) - \rho Tx^* - \rho A(x^*)],$$

where we have used the well-known fact that $P_C = (I + \rho N^P_{C_r})^{-1}$. □

We now consider the problem of solving the nonconvex Wiener-Hopf equations. To be more precise, let P_C be the projection of H onto nonconvex set C and $Q_C = I - P_C$, where I is identity mapping. For given nonlinear operators T, A, g consider the problem of finding $z \in H$ such that

$$Tg^{-1}P_Cz + \rho^{-1}Q_Cz = A(g^{-1}P_Cz), \tag{3.2}$$

where we have used the fact that g^{-1} exists. Equation (3.2) is called the *strongly nonlinear nonconvex Wiener-Hopf equation.*

Lemma 3.2 ([12]). *The nonconvex Wiener-Hopf equation (3.2) has a solution $x^* \in H$ if and only if strongly nonlinear general nonconvex variational inequality (2.5) has a solution $u \in C_r$, provided*

$$\begin{aligned} u &= g^{-1}P_Cx^*, \\ x^* &= g(u) - \rho(Tu - A(u)), \end{aligned} \tag{3.3}$$

where P_C is the projection of H onto the closed nonconvex set C_r.

In this paper we introduce a mapping with define by $T = T_1 + T_2$ where T_1 is a Lipschitz continuous and strongly monotone mapping, T_2 is a Lipschitz continuous mapping. Then we have a mapping T is a Lipschitz continuous mapping but not strongly monotone mapping and we have the following algorithm.

Algorithm 3.1. *For arbitrarily chosen initial points $x_0 \in C_r$, $T_1, T_2 : C \to H$ with $T = T_1 + T_2$, the sequence $\{x_{n+1}\}$ defined by*

$$\begin{aligned} g(u_n) &= P_Cx_n, n = 0, 1, 2, 3, ... \\ x_{n+1} &= (1 - \alpha_n)x_n + \alpha_n[g(u_n - \rho Tu_n + \rho A(u_n)], n = 0, 1, 2, 3, ..., \end{aligned} \tag{3.4}$$

where $\{\alpha_n\}$ is a sequence in $[0, 1]$.

Now, we suggest and analyze the algorithm (3.1) for solving the strongly nonlinear general nonconvex variational inequalities (2.5). Thus, from now on, without loss of generality we will always assume that $\mu_2 + \beta < \mu_1$.

Theorem 3.3. *Let C be a uniformly r-prox-regular closed subset of a Hilbert space H, and let $T_1, T_2, g, A : C \to H$ be such that T_1 is a μ_1-Lipschitz continuous and γ-strongly monotone mapping, T_2 is a μ_2-Lipschitz continuous mapping, g is a σ-Lipschitz continuous and η-strongly monotone and A is a β-Lipschitz continuous. If $T = T_1 + T_2$ and there exists constant $\rho > 0$ and $s \in (M^{\rho,\eta}\delta_{T(C)}, \xi)$, such that*

$$\frac{\gamma t_s - (1-k)(\mu_2 + \beta)t_s}{t_s(\mu_1^2 - (\mu_2 + \beta)^2)} - \triangle_{t_s} < \rho < \frac{\gamma t_s - (1-k)(\mu_2 + \beta)t_s}{t_s(\mu_1^2 - (\mu_2 + \beta)^2)} + \triangle_{t_s}, \tag{3.5}$$

where $t_s = \frac{r}{r-s}, s \in (0,r),\ \triangle_{t_s} = \frac{\sqrt{(\gamma t_s-(1-k)(\mu_2+\beta)t_s)^2-(\mu_1^2-(\mu_2+\beta)^2)(t_s^2-(1-kt_s)^2)}}{t_s(\mu_1^2-(\mu_2+\beta)^2)}$,
$\gamma t_s\rho > 1, h < 1$ *with* $k = \sqrt{1-2\eta+\sigma^2}$
and $\gamma t_s > (1-k)(\mu_2+\beta)t_s + \sqrt{(\mu_1^2-(\mu_2+\beta)^2(t_s^2-(1-kt_s)^2)}$*. If the sequence of positive real number* $\alpha_n \in [0,1]$ *with* $\Sigma_{n=0}^{\infty}\alpha_n = \infty$*, then the sequences* $\{x_n\}$ *obtained from Algorithm 3.1 converge to a solution of the strongly nonlinear general nonconvex variational inequalities (2.5).*

Proof. Let $x^* \in C_r$ be a solution of Wiener-Hopf equation (3.2) and from Lemma 3.1, we have

$$x^* = (1-\alpha_n)x^* + \alpha_n(g(u) - \rho(Tu - Au)).$$

From the algorithm 3.1, we have

$$\begin{aligned}
\|x_{n+1} - x^*\| &= \|(1-\alpha_n)x_n + \alpha_n(g(u_n) - \rho Tu_n + \rho A(u_n)) - x^*\| \\
&= \|(1-\alpha_n)x_n + \alpha_n(g(u_n) - \rho Tu_n + \rho A(u_n)) - ((1-\alpha_n)x^* \\
&\quad +\alpha_n(g(u) - \rho(Tu - Au)))\| \\
&\leq (1-\alpha_n)\|x_n - x^*\| \\
&\quad +\alpha_n\|g(u_n) - \rho Tu_n + \rho A(u_n) - g(u) + \rho Tu - \rho Au\| \\
&\leq (1-\alpha_n)\|x_n - x^*\| \\
&\quad +\alpha_n\|g(u_n) - g(u) - \rho Tu_n + \rho Tu\| + \alpha_n\rho\|Au_n - Au\| \\
&\leq (1-\alpha_n)\|x_n - x^*\| \\
&\quad +\alpha_n\|(u_n - u) - \rho(Tu_n - Tu) - (u_n - u) + (g(u_n) - g(u))\| \\
&\quad +\alpha_n\rho\|Au_n - Au\| \\
&\leq (1-\alpha_n)\|x_n - x^*\| \\
&\quad +\alpha_n\|(u_n - u) - \rho(Tu_n - Tu)\| + \alpha_n\|(u_n - u) - (g(u_n) - g(u))\| \\
&\quad +\alpha_n\rho\|Au_n - Au\| \\
&\leq (1-\alpha_n)\|x_n - x^*\| + \alpha_n\|(u_n - u) - \rho(T_1u_n - T_1u)\| + \alpha_n\rho\|T_2u_n - T_2u\| \\
&\quad +\alpha_n\|(u_n - u) - (g(u_n) - g(u))\| + \alpha_n\rho\|Au_n - Au\|. \qquad (3.6)
\end{aligned}$$

From T_1 are both μ_1-Lipschitz continuous and γ-strongly monotone mapping and from Lemma 2.6, we obtain

$$\begin{aligned}
\|(u_n - u) - \rho(T_1u_n - T_1u)\|^2 &\leq \|u_n - u\|^2 - 2\rho\langle u_n - u, T_1u_n - T_1u\rangle + \rho^2\|T_1u_n - T_1u\|^2 \\
&\leq \|u_n - u\|^2 - 2\rho\gamma\|u_n - u\|^2 + \rho^2\mu_1^2\|u_n - u\|^2 \\
&= (1 - 2\rho\gamma + \rho^2\mu_1^2)\|u_n - u\|^2.
\end{aligned}$$

It follows that

$$\|(u_n - u) - \rho(T_1u_n - T_1u)\| \leq \sqrt{1 - 2\rho\gamma + \rho^2\mu_1^2}\|u_n - u\|. \qquad (3.7)$$

On the other hand, from T_2 is μ_2-Lipschitz continuous, we have

$$\|T_2u_n - T_2u\| \leq \mu_2\|u_n - u\|. \qquad (3.8)$$

From g are both σ-Lipschitz continuous and η-strongly monotone mapping and from Lemma 2.6, we get

$$\begin{aligned}\|(u_n-u) &- \rho(g(u_n)-g(u))\|^2 \\ &\leq \|u_n-u\|^2-2\langle u_n-u, g(u_n)-g(u)\rangle+\|g(u_n)-g(u)\|^2 \\ &\leq \|u_n-u\|^2-2\eta\|u_n-u\|^2+\sigma^2\|u_n-u\|^2 \\ &= (1-2\eta+\sigma^2)\|u_n-u\|^2.\end{aligned}$$

It follows that

$$\|(u_n-u)-\rho(g(u_n)-g(u))\| \leq \sqrt{1-2\eta+\sigma^2}\|u_n-u\|. \tag{3.9}$$

Since A is β-Lipschitz continuous, we have

$$\|Au_n-Au\| \leq \beta\|u_n-u\|. \tag{3.10}$$

Thus, by (3.6), (3.7), (3.8), (3.9) and (3.10), we have

$$\|x_{n+1}-x^*\| \leq (1-\alpha_n)\|x_n-x^*\|+\alpha_n(\sqrt{1-2\rho\gamma+\rho^2\mu_1^2}+\rho\mu_2+\sqrt{1-2\eta+\sigma^2}+\rho\beta)\|u_n-u\|. \tag{3.11}$$

From algorithm 3.1, (3.9) and definition of P_C, we have

$$\begin{aligned}\|u_n-u\| &= \|(u_n-u)-(g(u_n)-g(u))+(P_Cx_n-P_Cx^*)\| \\ &\leq \|(u_n-u)-(g(u_n)-g(u))\|+\|P_Cx_n-P_Cx^*\| \\ &\leq \|(u_n-u)-(g(u_n)-g(u))\|+t_s\|x_n-x^*\| \\ &\leq (\sqrt{1-2\eta+\sigma^2})\|u_n-u\|+t_s\|x_n-x^*\|.\end{aligned}$$

Hence,

$$\|u_n-u\| \leq \frac{t_s}{(1-k)}\|x_n-x^*\|. \tag{3.12}$$

It follows that

$$\begin{aligned}\|x_{n+1}-x^*\| &\leq (1-\alpha_n)\|x_n-x^*\|+\alpha_n(\sqrt{1-2\rho\gamma+\rho^2\mu_1^2}+\rho\mu_2+k \\ &\quad +\rho\beta)\frac{t_s}{(1-k)}\|x_n-x^*\| \\ &= (1-\alpha_n)\|x_n-x^*\|+\alpha_n\theta\|x_n-x^*\|, \end{aligned} \tag{3.13}$$

where $\theta := (\sqrt{1-2\rho\gamma+\rho^2\mu_1^2}+\rho\mu_2+k+\rho\beta)\frac{t_s}{(1-k)}$, it follows that

$$\begin{aligned}\|x_{n+1}-x^*\| &\leq (1-\alpha_n)\|x_n-x^*\|+\alpha_n\theta\|x_n-x^*\| \\ &= (1-(1-\theta)\alpha_n)\|x_n-x^*\| \\ &\leq \prod_{i=0}^{n}(1-(1-\theta)\alpha_i)\|x_0-x^*\|. \end{aligned} \tag{3.14}$$

Since $\Sigma_{n=0}^{\infty}\alpha_n = \infty$ and conditions (3.5), we obtain

$$\lim_{n\to\infty}\prod_{i=0}^{n}(1-(1-\theta)\alpha_i) = 0. \tag{3.15}$$

It follows from (3.14) and (3.15), we have

$$\lim_{n\to\infty}\|x_n - x^*\| = 0. \tag{3.16}$$

Which is $x^* \in C_r$ satisfying the strongly nonlinear general nonconvex variational inequalities (2.5). This completes the proof. □

Corollary 3.4 ([12]). *Let P_K be the Lipschitz continuous operator with constant $\delta = \frac{r}{r-r'}$. Let T, g be strongly monotone with constant $\alpha > 0, \eta > 0$, respectively, and Lipschitz continuous with constant $\beta > 0, \sigma > 0$, respectively. Let the operator A be Lipschitz continuous with constant $\gamma > 0$. If there exists a constant ρ such that*

$$\begin{aligned} & |\rho - \frac{(\alpha\delta - \gamma(1-(1+\delta)k))}{\delta(\beta^2-\gamma^2)}| \\ < \ & \frac{\sqrt{(\alpha\delta - \gamma(1-(1+\delta)k))^2 - (\beta^2-\gamma^2)(\delta^2-(1-(1+\delta)k)^2)}}{\delta(\beta^2-\gamma^2)}, \qquad (3.17) \\ \delta\rho\alpha \ > \ & 1, k<1, k = \sqrt{1-2\eta+\sigma^2}, \\ \delta\alpha \ > \ & \gamma(1-(1+\delta)k) + \sqrt{(\beta^2-\gamma^2)(\delta^2-(1-(1+\delta)k)^2)}, \end{aligned}$$

and $\alpha_n \in [0,1], \forall n \geq 0; \Sigma_{n=0}^{\infty}\alpha_n = \infty$, then the approximate solution x_n obtained from Algorithm 3.1 converges to a solution $z \in H$ satisfying the nonconvex Wiener-Hopf equation (3.2).

Proof. From Theorem 3.3, let $T_2 \equiv 0$ it follows that $T = T_1$, then we have the result of [12]. □

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