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Non-Normality and the Fuzzy Theory for Variable Parameters $\overline{X}$ Control Charts

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Abstract : This paper applies the fuzzy theorem to non crispy data in creating the $\overline{X}$ control charts. The triangular fuzzy membership function with α - cut is introduced to eliminate less important data. The non normality data are generated to evaluate the performance of the variable parameters $\overline{X}$ control charts (VP), the fuzzy variable parameters $\overline{X}$ control charts (FVP), and the fuzzy variable parameters $\overline{X}$ control charts by weighted variance method (FVP-WV).The performances of the variable parameters $\overline{X}$ control charts are Average Adjusted Time to Signal (AATS), Average Number of Observations to Signal (ANOS), and Average Time to Signal (ATS), the fuzzy variable parameters $\overline{X}$ control charts by weighted variance method (FVP-WV) shows the best in every aspect.

Keywords : VP control charts; fuzzy $\overline{X}$ control charts; α - cut; non-normality distribution.
2010 Mathematics Subject Classification : 90C70.

1 Introduction

The $\overline{X}$ control charts are widely used to maintain and establish statistical control of a process. The $\overline{X}$ control charts were introduced by W.Shewhart in the 1920s. The $\overline{X}$ control charts have two types : fixed parameter and variable

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parameter.Many studied about adaptive $\overline{X}$ control charts,the work of Raynolds et al.[1] introduced the idea of varying the $\overline{X}$ control chats sampling interval. Prabhu et al[2] and Costa [3] proposed the size of the samples was the second design parameter to be considered variable. Finally all design parameter were considered variable by costa[4]. These studies are under the assumption that the data come from a normal distribution.The two topics in non-normal distribution data was presented by Yan-Kwang Chen[5] in the variable sampling interval (VSI) $\overline{X}$ control charts using Burrs distribution and Yu-Chang Lin and Chao-Yu Chou [6] proposed On the design of variable sample size and sampling intervals $\overline{X}$ charts under non-normality using Burrs distribution and Non-normality and the variable parameters control charts using Gamma distribution and t distribution. The previous papers used the standard normal distribution to transform non-normality data to normal before constructing the control charts. The performance indicators of the control charts are calculated using the Markov chain approach.A. Pongpullponsak, W.Suracherkeiti and C.Panthong[7], studied the variable parameter $\overline{X}$ control charts using shewhart method(SH), the weighted variance method (WV) and the scaled weighted variance method (SWV) for skewed distribution are Burrs Weibull and lognormal. The weighted variance method (WV) and the scaled weighted variance method (SWV) create control chart from skewed data directly without any transformation. The economic model was the tools in investigating the performance of these control charts. The problem in control charts making causes from uncertain data e.g. human errors, measuring devices, and environmental conditions. Many studied were done to combine statistical methods and fuzzy set theory. Fuzzy sets theory was first introduced by Zadeh [8]. Zadeh outlined generalized theory of uncertainty (GTU) which presented a change of perspective and direction in thinking about the system and uncertainties. Gullbay et al.[9] suggested the α-cut fuzzy control charts for linguistic data.Kahraman,C [10] proposed an alternative approach to fuzzy control charts:Direct fuzzy approach (DFA) , They developed fuzzy approaches to control charts based on fuzzy transformation methods, used the trapezoidal membership function. Zarandi et al.[11] presented a new hybrid method based on a combination of fuzzified sensitivity criteria and fuzzy adaptive sampling rules to determine the sample size and sample interval of the control charts in order to determine the sample size and sample interval of the control charts. in order to control and improve process efficiency at its best. It was discovered by Senturk and Erginel [12] that control charts could be used to solve the problem of uncertain data by using fuzzy theory. The topic of the research studied was fuzzy $\widetilde{\overline{X}} - \widetilde{R}$ and $\widetilde{\overline{X}} - \widetilde{S}$ control charts using α - cut. The methods used in the transformation of fuzzy sets into scalars are fuzzy mode, fuzzy median and α - level fuzzy midrange. Which one you choose to use depends on the difficulty of the computation or preference as in Wang[13] the α - level fuzzy midrange was selected. The aim of this study is to introduce the framework of FVP-WV which are Gamma distributions, using α - cut with the methods of α - level fuzzy midrange. First of all, we transform VP-WV to FVP-WV. To obtain FVP-WV charts, triangular fuzzy numbers (a,b,c) are used. Secondly α - cut FVP charts are developed by using α - cut approach. Thirdly α -level fuzzy midrange for FVP-WV are calculated by using α - level fuzzy midrange transformation techniques. Finally, we can use ANOS , AATS and ATS to determine the efficiency of the charts.

2 Research Methodology

Pongpullponsak and Panthong [7] studied the fuzzy theory for variable parameters $\overline{X}$ control charts by weighted variance for suitable non-normal data.

2.1 Fuzzy variable parameters $\overline{X}$ control charts by weighted variance method : FVP - WV

The FVP-WV applied membership by a triangular fuzzy number (a,b,c)[13]. Therefore, the control limits are as follows :

$U\tilde{C}L_i = \overline{\overline{X_i}} + \frac{k_i W_{U_i}\overline{R_i}}{3} = (\overline{\overline{X}}_{a,i} + \frac{k_i W_{U_{a,i}}\overline{R}_{a,i}}{3}, \overline{\overline{X}}_{b,i} + \frac{k_i W_{U_{b,i}}\overline{R}_{b,i}}{3}, \overline{\overline{X}}_{c,i} + \frac{k_i W_{U_{c,i}}\overline{R}_{c,i}}{3}),$

$U\tilde{C}L_i = \overline{\overline{X_i}} + \frac{w_i W_{U_i}\overline{R_i}}{3} = (\overline{\overline{X}}_{a,i} + \frac{w_i W_{U_{a,i}}\overline{R}_{a,i}}{3}, \overline{\overline{X}}_{b,i} + \frac{w_i W_{U_{b,i}}\overline{R}_{b,i}}{3}, \overline{\overline{X}}_{c,i} + \frac{w_i W_{U_{c,i}}\overline{R}_{c,i}}{3}),$

$C\tilde{L}_i = (\overline{\overline{X}}_{a,i}, \overline{\overline{X}}_{b,i}, \overline{\overline{X}}_{c,i}),$

$L\tilde{W}L_i = \overline{\overline{X_i}} - \frac{w_i W_{L_i}\overline{R_i}}{3} = (\overline{\overline{X}}_{a,i} - \frac{w_i W_{L_{a,i}}\overline{R}_{a,i}}{3}, \overline{\overline{X}}_{b,i} - \frac{w_i W_{L_{b,i}}\overline{R}_{b,i}}{3}, \overline{\overline{X}}_{c,i} - \frac{w_i W_{L_{c,i}}\overline{R}_{c,i}}{3}),$

$L\tilde{C}L_i = \overline{\overline{X_i}} - \frac{k_i W_{L_i}\overline{R_i}}{3} = (\overline{\overline{X}}_{a,i} - \frac{k_i W_{L_{a,i}}\overline{R}_{a,i}}{3}, \overline{\overline{X}}_{b,i} - \frac{k_i W_{L_{b,i}}\overline{R}_{b,i}}{3}, \overline{\overline{X}}_{c,i} - \frac{k_i W_{L_{c,i}}\overline{R}_{c,i}}{3}),$

where $W_{U,i}$, $W_{L,i}$ is a constant of WV method ; i = 1,2.

2.2 An α - cut fuzzy weighted variance method control charts

An α - cut consists of any elements whose membership is greater than or equal to α. Applying α - cut of fuzzy sets, the values of $\overline{\overline{X}}_{a,i}$, $\overline{\overline{X}}_{c,i}$,$\overline{R}_{a,i}$, $\overline{R}_{c,i}$ are determined as follows:

$\overline{\overline{X}}^{\alpha}_{a,i} = \overline{\overline{X}}_{a,i} + \alpha(\overline{\overline{X}}_{b,i} - \overline{\overline{X}}_{a,i}),$

$\overline{\overline{X}}^{\alpha}_{c,i} = \overline{\overline{X}}_{c,i} - \alpha(\overline{\overline{X}}_{c,i} - \overline{\overline{X}}_{b,i}),$

$\overline{R}^{\alpha}_{a,i} = \overline{R}_{a,i} + \alpha(\overline{R}_{b,i} - \overline{R}_{a,i}),$

$\overline{R}^{\alpha}_{c,i} = \overline{R}_{c,i} - \alpha(\overline{R}_{c,i} - \overline{R}_{b,i}).$

Therefore, the α-cut fuzzy mean control limits by weighted variance method are

$U\tilde{C}L_i{}^{\alpha} = (\overline{\overline{X}}^{\alpha}_{a,i} + \frac{k_i W_{U_{a,i}}\overline{R}^{\alpha}_{a,i}}{3}, \overline{\overline{X}}_{b,i} + \frac{k_i W_{U_{b,i}}\overline{R}_{b,i}}{3}, \overline{\overline{X}}^{\alpha}_{c,i} + \frac{k_i W_{U_{c,i}}\overline{R}^{\alpha}_{c,i}}{3}),$

$U\tilde{W}L_i{}^{\alpha} = (\overline{\overline{X}}^{\alpha}_{a,i} + \frac{w_i W_{U_{a,i}}\overline{R}^{\alpha}_{a,i}}{3}, \overline{\overline{X}}_{b,i} + \frac{w_i W_{U_{b,i}}\overline{R}_{b,i}}{3}, \overline{\overline{X}}^{\alpha}_{c,i} + \frac{w_i W_{U_{c,i}}\overline{R}^{\alpha}_{c,i}}{3}),$

$C\tilde{L}^{\alpha}_i = (\overline{\overline{x}}^{\alpha}_{a,i}, \overline{\overline{x}}_{b,i}, \overline{\overline{x}}^{\alpha}_{c,i}),$

$L\tilde{W}L_i{}^{\alpha} = (\overline{\overline{X}}^{\alpha}_{a,i} - \frac{w_i W_{L_{a,i}}\overline{R}^{\alpha}_{a,i}}{3}, \overline{\overline{X}}_{b,i} - \frac{w_i W_{L_{b,i}}\overline{R}_{b,i}}{3}, \overline{\overline{X}}^{\alpha}_{c,i} - \frac{w_i W_{L_{c,i}}\overline{R}^{\alpha}_{c,i}}{3}),$

$L\tilde{C}L_i{}^{\alpha} = (\overline{\overline{X}}^{\alpha}_{a,i} - \frac{k_i W_{L_{a,i}}\overline{R}^{\alpha}_{a,i}}{3}, \overline{\overline{X}}_{b,i} - \frac{k_i W_{L_{b,i}}\overline{R}_{b,i}}{3}, \overline{\overline{X}}^{\alpha}_{c,i} - \frac{k_i W_{L_{c,i}}\overline{R}^{\alpha}_{c,i}}{3}),$

where i = 1,2.

2.3 An α - level fuzzy midrange for α - cut fuzzy mean by weighted variance method control charts

An α - level fuzzy midrange for α - cut fuzzy mean control limits by weighted variance method control chart is developed to construct the control charts, which is a more efficient, suitable statistical tool in order to control process more efficiently. Therefore, the α - level fuzzy midrange for α - cut fuzzy mean control limits by weighted variance method control limits are,

$$U\tilde{C}L^{\alpha}_{mr-i} = \left(\frac{\overline{\overline{X}}^{\alpha}_{a,i}+\overline{\overline{X}}^{\alpha}_{c,i}}{2}\right) + \frac{k_i W_{U,i}\overline{R}^{\alpha}_{a,i}+\overline{R}^{\alpha}_{c,i}}{6},$$

$$U\tilde{W}L^{\alpha}_{mr-i} = \left(\frac{\overline{\overline{X}}^{\alpha}_{a,i}+\overline{\overline{X}}^{\alpha}_{c,i}}{2}\right) + \frac{w_i W_{U,i}\overline{R}^{\alpha}_{a,i}+\overline{R}^{\alpha}_{c,i}}{6},$$

$$\widetilde{C}L^{\alpha}_{mr-i} = \left(\frac{\overline{\overline{X}}^{\alpha}_{a,i}+\overline{\overline{X}}^{\alpha}_{c,i}}{2}\right),$$

$$L\tilde{W}L^{\alpha}_{mr-i} = \left(\frac{\overline{\overline{X}}^{\alpha}_{a,i}+\overline{\overline{X}}^{\alpha}_{c,i}}{2}\right) - \frac{w_i W_{L,i}\overline{R}^{\alpha}_{a,i}+\overline{R}^{\alpha}_{c,i}}{6},$$

$$L\tilde{C}L^{\alpha}_{mr-i} = \left(\frac{\overline{\overline{X}}^{\alpha}_{a,i}+\overline{\overline{X}}^{\alpha}_{c,i}}{2}\right) - \frac{k_i W_{L,i}\overline{R}^{\alpha}_{a,i}+\overline{R}^{\alpha}_{c,i}}{6}.$$

2.4 Fuzzy transformation techniques

Fuzzy transformation techniques have four types : fuzzy mode, fuzzy median, fuzzy average and α - level fuzzy midrange. In this study, the α - level fuzzy midrange transformation technique is used for FVP control charts.

The α - level fuzzy midrange f^{α}_{mr} is defined as the midpoint of the α - level cuts. Let A^{α} is α - level cuts, nonfuzzy sets that consist of any elements whose membership is greater than or equal to α. If a^{α} and b^{α} are end points of A^{α} then $f^{\alpha}_{mr} = \frac{1}{2}(a^{\alpha} + c^{\alpha})$.

In fact the fuzzy mode is a special case of α - level fuzzy midrange when α=1.

The definition of α - level fuzzy midrange of sample for fuzzy $\tilde{\overline{X}}$ control charts is

$$S^{\alpha}_{mr-\overline{X},j} = \frac{(\overline{X}_{a_j} + \overline{X}_{c_j}) + \alpha\left[(\overline{X}_{b_j} - \overline{X}_{a_j}) - (\overline{X}_{c_j} - \overline{X}_{b_j})\right]}{2}.$$

Then, the condition of process control for each sample can be defined as
$Process\ control =$

$$\begin{cases} warning\,control\,; UCL^{\alpha}_{mr-\overline{X}} \geq S^{\alpha}_{mr-\overline{X}.j} > UWL^{\alpha}_{mr-\overline{X}} \\ and\, LWL^{\alpha}_{mr-\overline{X}} < S^{\alpha}_{mr-\overline{X}.j} \leq LWL^{\alpha}_{mr-\overline{X}} \\ in\,control \quad ; LWL^{\alpha}_{mr-\overline{X}} \leq S^{\alpha}_{mr-\overline{X}.j} \leq UWL^{\alpha}_{mr-\overline{X}} \\ out\,of\,control \quad ; S^{\alpha}_{mr-\overline{X}.j} < LCL^{\alpha}_{mr-\overline{X}}\, and\, S^{\alpha}_{mr-\overline{X}.j} > UCL^{\alpha}_{mr-\overline{X}}. \end{cases}$$

2.5 Distributed data

Gamma Distribution to compare the performance of the control charts, the parameters of the Gamma distribution are varied as presented in the work of Lin and Chou [6]. The gamma distribution has the probability density function

$$f(x) = \frac{1}{\beta^{\alpha}\Gamma(\alpha)} x^{\alpha-1} e^{-\frac{x}{\beta}} \quad , \; x > 0,$$

where α and β are the shape and scale parameters, respectively. The mean of a gamma distribution is $\alpha\beta$ and its variance is $\alpha\beta^2$, (α,β)at (4,1), (2,1), (1,1) are used in this study.

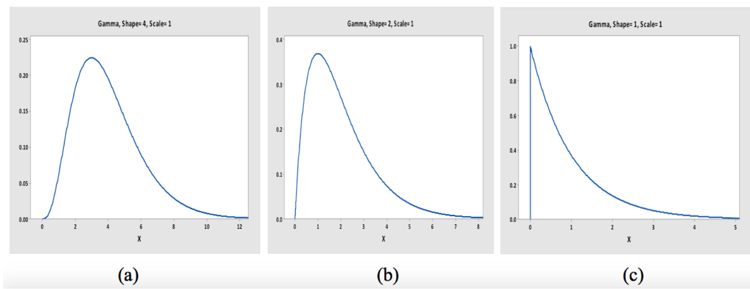

Fig. 1. The probability density function for various gamma distributions: (a) Gamma (4, 1) and (b) Gamma (2, 1) and (c) Gamma (1, 1).

2.6 Calculations of ANOS , ATS and AATS

Costa [3] purpos the operation of VP-WV, the design parameter should be chosen such that $n_1 > n_2$, $h_1 < h_2$, $w_1 < w_2$ *and* $k_1 < k_2$.

The parameters calculation based the probability as follows,

$$p_0 = P(|M| < w_i \mid |M| < k_i) \, , \, i = 1 \, , \, 2,$$

where M is non-normal random variable .

Calculations of ANOS , ATS and AATS The calculated of these value are applied The Markov chain approach according to the condition[4]. State1 ; The process is in-control and the sample point fall in the central regions. State2 ; The process is in-control and the sample point fall in the warning regions. State3 ; The process is out-of-control and the sample point fall in the action regions (absorbing state). The widely used performance indicators for adaptive control charts [4] are: 1. ANOSthe average number of observations to signal, which is defined as the expected number of individual observations from the start of the process to the time when the chart indicates an out-of-control signal. 2. ATSthe average time to signal, which is defined as the expected value of the time from the start of the process to the time when the chart indicates an out-of-control signal. 3. AATSthe adjusted average time to signal, which is defined as the expected value of the time from the occurrence of an assignable cause to the time when the chart indicates an out-of control signal.

Let be the state transition probability matrix : $Q = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}$

where p_ij are probability which sample in stage i and sample point fall in state j. Let P be the transition probability matrix when the process is in control :

$$P = \begin{bmatrix} P_1 & 1 - P_1 \\ P_2 & 1 - P_2, \end{bmatrix}$$

where p_i as the conditional probability that the sample size is n_i and the sample point fall in the central regions, i = 1 , 2. ANOS , ATS and AATS can be calculated by

$$ATS = r(I - Q)^{-1}h$$
$$ANOS = r(I - Q)^{-1}n$$
$$AATS = \pi[(I - Q)^{-1} - \tfrac{1}{2}]h.$$

3 Research Results and Discussion

The purpose of this study is to compare the efficiency of VP FVP and FVP-WV control charts using under non-normality is gamma distributions with $\alpha = 4, 2, 1$ and $\beta = 1$. The procedure is repeated 10,000 times. This study shows values of parameters, AATS, ATS and ANOS when mean shifts occurred in the process $\delta = 0.0, 0.25, 0.50, 0.75, 1.0, 2.0$ and 3.0.

Table 1 In-control ATS value for various adaptive control charts under various distributions.

Charts	VP		FVP		FVP	
(n_1, n_2)	(2,22)	(4,9)	(5,8)	(3,6)	(2,7)	(4,5)
(h_1, h_2)	(1.16,0.10)	(1.23,0.10)	(1.22,1.14)	(1.43,0.11)	(1.52,1.19)	(1.66,0.12)
(k_1, k_2)	(6.00,2.37)	(6.00,2.47)	(3.09,3.00)	(3.36,2.10)	(3.39,3.11)	(3.56,2.31)
(w_1, w_2)	(1.44,1.39)	(1.28,1.25)	(1.85,1.27)	(1.68,1.23)	(1.57,1.29)	(1.50,1.28)
Distribution	ATS					
Normal	370.40	370.40	370.40	370.40	370.40	370.40
Gamma(4,1)	409.55	363.97	500.43	375.93	511.22	399.54
Gamma(2,1)	454.68	355.61	553.21	340.25	557.30	365.81
Gamma(1,1)	514.31	341.09	558.90	340.22	561.82	356.29

Table 1 showed changes in ATS values of FVP and FVP-WV methods making a new control limit by using fuzzy theory. Weighted standard deviation tended to be in the same direction with VP method. The more increased skewness dispersed away from normal distribution, ATS values decreased. This meant it showed high false alarm rate in the process. However, FVP and FVP-WV methods had more coefficient than FVP-WV method with ATS values at least.

Table 2-1 Values of ATS, AATS and ANOS for various adaptive control charts under Gamma (4,1) distributions.

Charts	VP		FVP		FVP-WV	
(n_1, n_1)	(2,22)	(4,9)	(5,8)	(3,6)	(2,7)	(4,5)
(h_1, h_1)	(1.16,0.10)	(1.23,0.10)	(2.42,2.14)	(3.43,3.00)	(2.73,2.01)	(3.66,2.12)
$(k_{1,1}, k_{1,2})$	(10.55,2.53)	(9.15,2.75)	(2.44,2.23)	(2.95,2.54)	(4.33,3.22)	(3.17,2.88)
$(k_{2,1}, k_{2,2})$	(2.73,2.20)	(3.47,2.19)	(2.40,2.04)	(2.68,1.23)	(3.57,1.82)	(2.50,2.11)
$(w_{1,1}, w_{1,2})$	(1.54,1.42)	(1.23,1.28)	(1.56,1.12)	(2.00,1.96)	(2.61,1.00)	(2.00,1.55)
$(w_{2,1}, w_{2,2})$	(1.29,1.35)	(1.22,1.21)	(1.21,1.01)	(1.78,1.77)	(2.45,0.98)	(1.96,1.15)
Shift	ATS					
$\delta = 0$	370.40	370.40	370.40	370.40	370.40	370.40
	AATS					
0.25	62.05	133.01	60.96	68.92	60.11	60.02
0.50	8.18	17.61	8.09	20.43	8.28	8.38
0.75	3.63	3.25	3.22	3.21	3.01	3.01
1	2.35	1.29	2.46	2.44	2.29	2.30
2	1.18	0.74	1.00	0.92	1.00	1.00
3	0.76	0.70	0.35	0.35	0.35	0.35
Shift	ANOS					
$\delta = 0$	1851.99	1851.99	1851.99	1851.99	1851.99	1851.99
0.25	343.93	712.44	333.64	453.80	329.08	431.13
0.50	66.40	136.50	59.96	132.71	52.55	130.92
0.75	32.31	40.79	29.66	30.24	28.91	32.83
1	26.43	20.21	23.87	18.11	20.43	20.21
2	24.45	12.56	19.70	12.00	19.70	12.00
3	23.84	12.20	20.94	12.00	20.04	12.00

From table 2-1, when production process changed less mean average values, AATS values of three control charts were similar. When production process changed more mean average values, FVP-WV method showed the least AATS values. FVP-WV method showed least ANOS values at slightly different changes of mean averages, followed by FVP method. However, changing process of mean averages by FVP-WV and FVP methods showed almost no differences in AATS values.

Table 2-2 Values of ATS ,AATS and ANOS for various adaptive control charts under Gamma (2,1) distributions.

Charts	VP		FVP		FVP-WV	
(n_1, n_1)	(2,22)	(4,9)	(5,8)	(3,6)	(2,7)	(4,5)
(h_1, h_1)	(1.16,0.10)	(1.23,0.10)	(3.50,2.04)	(10.01,4.30)	(12.45,5.66)	(9.92,4.21)
$(k_{1,1}, k_{1,2})$	(12.58,2.59)	(10.55,2.87)	(4.67,3.55)	(2.95,2.54)	(3.40,3.25)	(4.11,3.03)
$(k_{2,1}, k_{2,2})$	(1.99,2.13)	(2.73,2.07)	(4.22,3.67)	(4.00,3.91)	(3.07,2.65)	(3.62,3.20)
$(w_{1,1}, w_{1,2})$	(1.57,1.43)	(1.33,1.29)	(2.75,2.00	(3.47,2.66)	(2.98,2.00)	(2.45,2.09)
$(w_{2,1}, w_{2,2})$	(1.21,1.34)	(1.18,1.20)	(2.74,1.90)	(2.73,2.33)	(2.70,1.77)	(2.44,2.12)
Shift	ATS					
$\delta = 0$	370.40	370.40	370.40	370.40	370.40	370.40
	AATS					
0.25	75.85	106.17	74.88	98.02	70.26	96.89
0.50	9.18	21.64	9.03	17.34	9.03	16.00
0.75	3.94	3.65	3.94	3.05	3.57	3.21
1	2.56	1.32	2.33	1.09	2.32	1.06
2	1.23	0.73	1.11	0.73	1.00	0.70
3	0.75	0.70	0.72	0.70	0.70	0.65
Shift	ANOS					
$\delta = 0$	1851.99	1851.99	1851.99	1851.99	1851.99	1851.99
0.25	390.41	834.40	382.24	799.96	377.58	607.96
0.50	72.21	163.04	67.09	157.88	66.78	148.74
0.75	33.35	46.82	30.76	45.99	27.54	40.73
1	26.75	21.89	22.54	21.44	21.11	18.08
2	24.53	12.58	22.11	11.38	20.00	10.57
3	23.81	12.20	20.95	11.00	15.69	12.52

From to table 2-2, when the process changed mean averages at every level, AATS and ANOS values of three control charts showed similar values. However, FVP-WV method showed the least value, followed by FVP method and VP method in sequence.

Table 2-3 Values of ATS, AATS and ANOS for various adaptive control charts under Gamma (1,1) distributions.

Charts	VP		FVP		FVP-WV	
(n_1, n_1)	(2,22)	(4,9)	(5,8)	(3,6)	(2,7)	(4,5)
(h_1, h_1)	(1.16,0.10)	(1.23,0.10)	(3.32,2.57)	(15.56,8.38)	(11.30,5.33)	(6.12,4.50)
$(k_{1,1}, k_{1,2})$	(15.52,2.69)	(12.58,3.03)	(3.67,3.00)	(3.05,3.00)	(3.36,3.09)	(3.01,2.09)
$(k_{2,1}, k_{2,2})$	(1.41,2.04)	(1.99,1.90)	(2.78,2.11)	(3.00,2.88)	(3.04,2.94)	(3.00,2.02)
$(w_{1,1}, w_{1,2})$	(1.59,1.44)	(1.34,1.29)	(2.30,1.55)	(2.29,1.52)	(2.30,2.21)	(2.00,1.67)
$(w_{2,1}, w_{2,2})$	(1.10,1.31)	(1.13,1.17)	(1.45,1.40)	(1.65,1.50)	(1.50,1.45)	(1.59,1.33)
Shift	ATS					
$\delta = 0$	370.40	370.40	370.40	370.40	370.40	370.40
	AATS					
0.25	99.17	204.65	93.06	179.11	87.34	155.73
0.50	10.44	28.20	10.40	26.89	9.83	26.36
0.75	4.36	4.30	4.00	4.12	3.50	4.18
1	2.86	1.36	2.34	1.00	2.01	0.79
2	1.30	0.71	1.12	0.68	1.05	0.43
3	0.70	0.70	0.64	0.70	0.54	0.50
Shift	ANOS					
$\delta = 0$	1851.99	1851.99	1851.99	1851.99	1851.99	1851.99
0.25	464.23	1025.14	413.87	907.04	411.86	900.85
0.50	81.01	205.56	59.55	168.41	58.51	160.92
0.75	34.94	57.00	33.67	55.59	33.04	51.77
1	27.18	24.87	25.14	22.98	22.68	20.94
2	24.63	12.63	20.13	11.04	20.12	10.09
3	23.75	12.20	20.13	11.00	17.59	9.93

From table 2-3, according to changing process showed distributions which dispersed away from normal distribution when production process changed less mean averages, AATS values of three control charts showed very different values. When production process changed more mean averages, FVP-WV showed the least AATS values. ANOS values at changing mean averages had very different values, FVP-WV and FVP methods showed similar values but less than ANOS values of VP method. However, when changing process of mean averages at level 3, FVP-WV and FVP methods showed a smaller difference of AATS values.

4 Conclusions

In this study, we compared coefficients of variation of parameters on control charts between FVP method and FVP-WV method which made up by using Fuzzy theory and VP method. The non-normal distribution using Gamma distribution was dispersed away from normal distribution. It was found that FVP and FVP-WV methods considered out of control very well, specifically mean averages with more changes in process. FVP-WV method showed AATS ANOS values less than other methods because FVP-WV also weighted deviations. However, using Fuzzy theory to produce the new control limits showed that control charts by FVP and FVP-WV methods had less false alarm and more coefficient than VP method. In further study, we may study from other non-normal distributions such as Weibull distribution, Lognormal distribution and Burr's distribution. Moreover, using control charts by SWV method and simulation model to compare coefficients was suggested in further study.

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