Thai Journal of Mathematics
Volume 16 (2016) Number 1 : 43–51
http://thaijmath.in.cmu.ac.th
ISSN 1686-0209

A Generalization on $\mathcal{I}-$Asymptotically Lacunary Statistical Equivalent Sequences

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Abstract : This paper presents the concept of generalized asymptotically $\mathcal{I}$-lacunary statistical equivalent of order α to multiple L and $\mathcal{I}$ is an ideal of the subset of positive integers. In addition to this definition inclusion theorems are also presented. The study leaves some interesting open problems.

Keywords : Lacunary sequence; Ideal convergence; Asymptotically equivalent sequences; Statistical convergence of order α.
2010 Mathematics Subject Classification : 40G15; 40A35.

1 Introduction

Before continuing with this paper we present some definitions and preliminaries. The concept of $\mathcal{I}-$convergence was introduced by Kostyrko, Mačaj and Wilczyński in a metric space [1]. Later it was further studied by Dems [2], Savas ([3],[4], [5],[6][7],[8], [9] [10]) and Mursaleen et al. [11, 12]. $\mathcal{I}-$convergence is a generalization form of statistical convergence, which was introduced by Fast (see [13]) and that is based on the notion of an ideal of the subset of positive integers $\mathbb{N}$.

The following definitions and notions will be needed.

Definition 1.1. *A family $\mathcal{I} \subset 2^{\mathbb{N}}$ is said to be an ideal of $\mathbb{N}$ if the following conditions hold:*
(a) $A, B \in I$ implies $A \cup B \in \mathcal{I}$,

(b) $A \in I,\ B \subset A$ *implies* $B \in \mathcal{I}$,

An ideal is called non-trivial if $U^*(F)2115 \notin \mathcal{I}$ and a non-trivial ideal is called admissible if $\{n\} \in \mathcal{I}$ for each $n \in \mathbb{N}$.

Definition 1.2. A family of sets $F \subset 2^{\mathbb{N}}$ is a *filter* in $\mathbb{N}$ if and only if

(i) $\emptyset \notin \mathcal{F}$
(ii) For each $A, B \in F$ we have $A \cap B \in \mathcal{F}$
(iii) For each $A \in F$ and each $B \supseteq A$ we have $B \in \mathcal{F}$.

Proposition 1.3. *$\mathcal{I}$ is a non-trivial ideal in $\mathbb{N}$ if and only if*

$$F = F(\mathcal{I}) = \{M = \mathbb{N}\backslash A : A \in \mathcal{I}\}$$

is a filter in $\mathbb{N}$, (see [1]).

Definition 1.4. A real sequence $x = (x_k)$ is said to be $\mathcal{I}-$ *convergent to* $L \in \mathbb{R}$ if and only if for each $\varepsilon > 0$ the set

$$A_\varepsilon = \{k \in \mathbb{N} : |x_k - L| \geq \varepsilon\}$$

belongs to $\mathcal{I}$. The number L is called the $\mathcal{I}-$limit of the sequence x, (see, [1]).

Remark 1.5. *If we take $\mathcal{I} = \mathcal{I}_f = \{A \subseteq \mathbf{N} : A$ is a finite subset $\}$. Then $\mathcal{I}_f$ is a non-trivial admissible ideal of $\mathbf{N}$ and the corresponding convergence coincide with the usual convergence.*

In 1993, Marouf presented definitions for asymptotically equivalent sequences and asymptotic regular matrices.

Definition 1.6 ([14])**.** Two nonnegative sequences $x = (x_k)$ and $y = (y_k)$ are said to be *asymptotically equivalent* if

$$\lim_k \frac{x_k}{y_k} = 1$$

and it is denoted by $x \sim y$.

Definition 1.7 (Fridy, [15])**.** The sequence $x = (x_k)$ has *statistic limit* L, denoted by $st - \lim x = L$ provided that for every $\epsilon > 0$,

$$\lim_n \frac{1}{n}\{\text{the number of } k \leq n : |x_k - L| \geq \epsilon\} = 0.$$

In 2003, Patterson defined asymptotically statistical equivalent sequences by combining definitions 1.6 and 1.7 as follows:

Definition 1.8 (Patterson, [16])**.** Two nonnegative sequences $x=(x_k)$ and $y=(y_k)$ are said to be *asymptotically statistical equivalent of multiple* L provided that for every $\epsilon>0$,

$$\lim_n \frac{1}{n}\{\text{the number of } k<n : \left|\frac{x_k}{y_k}-L\right| \geq \epsilon\}=0$$

(denoted by $x \overset{S_L}{\sim} y$), and simply asymptotically statistical equivalent if $L=1$.

A lacunary sequence is an increasing integer sequence $\theta=(k_r)$ such that $k_0=0$ and $h_r=k_r-k_{r-1}\to\infty$ as $r\to\infty$. The intervals determined by θ will be denoted $I_r=(k_{r-1},k_r]$ and the ratio $\frac{k_r}{k_{r-1}}$ will be denoted by q_r.

In 2006, Patterson and Savaş presented definitions for asymptotically lacunary statistical equivalent sequences, (see [17]).

Definition 1.9 ([17])**.** Let $\theta=(k_r)$ be a lacunary sequence; the two nonnegative sequences $x=(x_k)$ and $y=(y_k)$ are said to be *asymptotically lacunary statistical equivalent of multiple* L provided that for every $\varepsilon>0$

$$\lim_r \frac{1}{h_r}\left|\left\{k\in I_r : \left|\frac{x_k}{y_k}-L\right|\geq\varepsilon\right\}\right|=0$$

where the vertical bars indicate the number elements in the enclose set.

Definition 1.10. Let $\theta=(k_r)$ be a lacunary sequence; the two number sequences $x=(x_k)$ and $y=(y_k)$ are said to be *strong asymptotically lacunary equivalent of multiple* L provided that

$$\lim_r \frac{1}{h_r}\sum_{k\in I_r}\left|\frac{x_k}{y_k}-L\right|=0$$

In 2008, Savas and Patterson [18] gave an extension on asymptotically lacunary statistical equivalent sequences and they investigated some relations between strongly asymptotically lacunary equivalent sequences and strongly Cesàro asymptotically equivalent sequences.

Definition 1.11 ([18])**.** Let $\theta=(k_r)$ be a lacunary sequence and $p=(p_k)$ is a sequence of positive real numbers. Two number sequences $x=(x_k)$ and $y=(y_k)$ are said to be *strongly asymptotically lacunary equivalent of multiple* L provided that

$$\lim_r \frac{1}{h_r}\sum_{k\in I_r}\left|\frac{x_k}{y_k}-L\right|^{p_k}=0$$

(denoted by $x \overset{N_\theta^{L(p)}}{\sim} y$) and simply strongly asymptotically lacunary equivalent if $L=1$.

Throughout $\mathcal{I}$ will stand for a proper admissible ideal of $\mathbb{N}$ and by sequence we always mean sequences of real numbers.

Recently, Savas[5] defined $\mathcal{I}-$asymptotically lacunary statistical equivalent sequences by using the definitions $\mathcal{I}-$convergence and asymptotically lacunary statistical equivalent sequences together.

Definition 1.12 ([5])**.** Let $\theta = (k_r)$ be a lacunary sequence; the two nonnegative sequences $x = (x_k)$ and $y = (y_k)$ are said to be *$\mathcal{I}-$asymptotically lacunary statistical equivalent of multiple* L provided that for every $\varepsilon > 0$ and $\delta > 0$,

$$\left\{r \in \mathbb{N} : \frac{1}{h_r}\left|\left\{k \in I_r : \left|\frac{x_k}{y_k} - L\right| \geq \varepsilon\right\}\right| \geq \delta\right\} \in \mathcal{I}.$$

In this case we write $x \stackrel{S_\theta^L(\mathcal{I})}{\sim} y$.

Quite recently Savas[6] has given the following definitions.

Definition 1.13. Let θ be a lacunary sequence; the two nonnegative sequences $x = (x_k)$ and $y = (y_k)$ are said to be *asymptotically $\mathcal{I}$-lacunary statistical equivalent of order* α , where $0 < \alpha < 1$, to multiple L provided that for any $\epsilon > 0$ and $\delta > 0$

$$\{r \in \mathbb{N} : \frac{1}{h_r^\alpha}|\{k \in I_r : |\frac{x_k}{y_k} - L| \geq \epsilon\}| \geq \delta\} \in \mathcal{I},$$

(denoted by $x \stackrel{S_\theta^L(I)^\alpha}{\sim} y$) and simply asymptotically $\mathcal{I}$-lacunary statistical equivalent of order α if $L = 1$. Furthermore, let $S_\theta^L(\mathcal{I})^\alpha$ denote the set of x and y such that $x \stackrel{S_\theta^L(\mathcal{I})^\alpha}{\sim} y$.

More investigations in this direction and more applications of asymptotically equivalent sequence can be found in [16, 17, 18].

In this paper we shall generalize the above definition by using sequence of positive real numbers and also some inclusion theorems are proved.

2 Main Results

In this section we shall give some new definitions and also examine some inclusion relations.

Definition 2.1. Let $\theta = (k_r)$ be a lacunary sequence and $p = (p_k)$ is a sequence of positive real numbers. Two number sequences $x = (x_k)$ and $y = (y_k)$ are said to be strongly asymptotically $\mathcal{I}-$ lacunary equivalent of order $\alpha 51$, where $0 < \alpha < 1$, to multiple L for the sequence p provided that

$$\left\{r \in \mathbb{N} : \frac{1}{h_r^\alpha}\sum_{k \in I_r}\left|\frac{x_k}{y_k} - L\right|^{p_k} \geq \varepsilon\right\} \in \mathcal{I}.$$

In this situation we write $x \overset{N_\theta^{L(p)}(\mathcal{I})^\alpha}{\sim} y$.

If we take $p_k = p$ for all $k \in \mathbb{N}$ we write $x \overset{N_\theta^{Lp}(\mathcal{I})^\alpha}{\sim} y$ instead of $x \overset{N_\theta^{L(p)}(\mathcal{I})}{\sim} y$. For $\alpha = 1$ the above definition coincides with strongly $\mathcal{I}-$asymptotically lacunary equivalent of multiple L, [5].

Definition 2.2. Let $p = (p_k)$ be a sequence of positive real numbers. Two number sequences $x = (x_k)$ and $y = (y_k)$ are said to be *strongly asymptotically* $\mathcal{I}-$ *Cesáro equivalent of order* α , where $0 < \alpha < 1$, to multiple L provided that

$$\left\{ n \in \mathbb{N} : \frac{1}{n^\alpha} \sum_{k=1}^{n} \left| \frac{x_k}{y_k} - L \right|^{p_k} \geq \varepsilon \right\} \in \mathcal{I}$$

(denoted by $x \overset{\sigma^{(p)}(\mathcal{I})^\alpha}{\sim} y$) and simply strongly Cesáro $\mathcal{I}-$asymptotically equivalent if $L = 1$.

We now prove some inclusion theorems

Theorem 2.3. *Let $\theta = (k_r)$ be a lacunary sequence. Then, If $x \overset{N_\theta^{Lp}(\mathcal{I})^\alpha}{\sim} y$ then $x \overset{S_\theta^{L}(\mathcal{I})^\alpha}{\sim} y$*

Proof. Let $x \overset{N_\theta^{Lp}(\mathcal{I})^\alpha}{\sim} y$ and $\varepsilon > 0$ be given. Then,

$$\begin{aligned} \sum_{k \in I_r} \left| \tfrac{x_k}{y_k} - L \right|^p &\geq \sum_{k \in I_r \ \& \ \left| \frac{x_k}{y_k} - L \right| \geq \varepsilon} \left| \tfrac{x_k}{y_k} - L \right|^p \\ &\geq \varepsilon^p \left| \left\{ k \in I_r : \left| \tfrac{x_k}{y_k} - L \right| \geq \varepsilon \right\} \right| \end{aligned}$$

and so

$$\tfrac{1}{\varepsilon^p h_r^\alpha} \sum_{k \in I_r} \left| \tfrac{x_k}{y_k} - L \right|^p \geq \tfrac{1}{h_r^\alpha} \left| \left\{ k \in I_r : \left| \tfrac{x_k}{y_k} - L \right| \geq \varepsilon \right\} \right|$$

Then for any $\delta > 0$,

$$\left\{ r \in \mathbb{N} : \frac{1}{h_r^\alpha} \left| \left\{ k \in I_r : \left| \frac{x_k}{y_k} - L \right| \geq \varepsilon \right\} \right| \geq \delta \right\} \subseteq \left\{ r \in \mathbb{N} : \frac{1}{h_r^\alpha} \sum_{k \in I_r} \left| \frac{x_k}{y_k} - L \right|^p \geq \varepsilon^p \delta \right\} \in I.$$

Therefore $x \overset{S_\theta^{L}(\mathcal{I})^\alpha}{\sim} y$. □

Remark 2.4. *The following two conditions remain true for $0 < \alpha < 1$ is not clear and we leave them as open problems.*

(2) $x \in l_\infty$ and $x \overset{S_\theta^{L}(\mathcal{I})^\alpha}{\sim} y \Rightarrow x \overset{N_\theta^{Lp}(\mathcal{I})^\alpha}{\sim} y$,

(3) $S_\theta^L(\mathcal{I})^\alpha \cap l_\infty = N_\theta^{Lp}(\mathcal{I})^\alpha \cap l_\infty$.

We now investigate the relationship between $x \overset{N_\theta^{L(p)}(\mathcal{I})^\alpha}{\sim} y$ and $x \overset{S_\theta^L(\mathcal{I})^\alpha}{\sim} y$.

Theorem 2.5. *Let $\theta = (k_r)$ be a lacunary sequence, $\inf_k p_k = h$ and $\sup_k p = H$. Then,*

$$x \overset{N_\theta^{L(p)}(\mathcal{I})^\alpha}{\sim} y \text{ implies } x \overset{S_\theta^L(\mathcal{I})^\alpha}{\sim} y.$$

Proof. Assume that $x \overset{N_\theta^{L(p)}(\mathcal{I})^\alpha}{\sim} y$ and $\varepsilon > 0$. Then,

$$\begin{aligned}
\frac{1}{h_r^\alpha}\sum_{k\in I_r}\left|\frac{x_k}{y_k}-L\right|^{p_k} &= \frac{1}{h_r^\alpha}\sum_{k\in I_r \ \& \ \left|\frac{x_k}{y_k}-L\right|\geq\varepsilon}\left|\frac{x_k}{y_k}-L\right|^{p_k} + \frac{1}{h_r^\alpha}\sum_{k\in I_r \ \& \ \left|\frac{x_k}{y_k}-L\right|<\varepsilon}\left|\frac{x_k}{y_k}-L\right|^{p_k} \\
&\geq \frac{1}{h_r^\alpha}\sum_{k\in I_r \ \& \ \left|\frac{x_k}{y_k}-L\right|\geq\varepsilon}\left|\frac{x_k}{y_k}-L\right|^{p_k} \\
&\geq \frac{1}{h_r^\alpha}\sum_{k\in I_r \ \& \ \left|\frac{x_k}{y_k}-L\right|\geq\varepsilon}(\varepsilon)^{p_k} \\
&\geq \frac{1}{h_r^\alpha}\sum_{k\in I_r \ \& \ \left|\frac{x_k}{y_k}-L\right|\geq\varepsilon}\min\left\{(\varepsilon)^h,(\varepsilon)^H\right\} \\
&\geq \frac{1}{h_r^\alpha}\min\left\{(\varepsilon)^h,(\varepsilon)^H\right\}\left|\left\{k\in I_r : \left|\frac{x_k}{y_k}-L\right|\geq\varepsilon\right\}\right|
\end{aligned}$$

and

$$\left\{r\in\mathbb{N} : \frac{1}{h_r^\alpha}\left|\left\{k\in I_r : \left|\frac{x_k}{y_k}-L\right|\geq\varepsilon\right\}\right|\geq\delta\right\}\subseteq\left\{r\in\mathbb{N} : \frac{1}{h_r^\alpha}\sum_{k\in I_r}\left|\frac{x_k}{y_k}-L\right|^{p_k}\geq\delta\min\left\{(\varepsilon)^h,(\varepsilon)^H\right\}\right\}\in I.$$

Thus we have $x \overset{S_\theta^L(\mathcal{I})}{\sim} y$. □

Theorem 2.6. *Let $\mathcal{I}$ be an ideal and $\theta = \{k_r\}$ is a lacunary sequence, then*

$$x \overset{\sigma^{(p)}(\mathcal{I})}{\sim} y \text{ implies } x \overset{N_\theta^{L(p)}(\mathcal{I})}{\sim} y.$$

if $\liminf\limits_{r} q_r^\alpha > 1$.

Proof. If $\liminf_r q_r^\alpha > 1$ then there exists $\delta > 0$ such that $q_r^\alpha \geq 1+\delta$ for all $r \geq 1$. Since $h_r = k_r - k_{r-1}$, we have $\frac{k_r}{h_r} \leq \frac{1+\delta}{\delta}$ and $\frac{k_{r-1}}{h_r} \leq \frac{1}{\delta}$. Let $\varepsilon > 0$ and define the set,

$$S = \left\{k_r \in \mathbb{N} : \frac{1}{k_r}\sum_{k=1}^{k_r}\left|\frac{x_k}{y_k}-L\right|^{p_k} < \varepsilon\right\}.$$

We can easily say that $S \in F(\mathcal{I})$ which is the filter of the ideal $\mathcal{I}$.

$$\begin{aligned}\frac{1}{h_r^\alpha}\sum_{k\in I_r}\left|\frac{x_k}{y_k}-L\right|^{p_k} &= \frac{1}{h_r^\alpha}\sum_{k=1}^{k_r}\left|\frac{x_k}{y_k}-L\right|^{p_k}-\frac{1}{h_r}\sum_{k=1}^{k_{r-1}}\left|\frac{x_k}{y_k}-L\right|^{p_k}\\ &= \frac{k_r}{h_r^\alpha}\frac{1}{k_r}\sum_{k=1}^{k_r}\left|\frac{x_k}{y_k}-L\right|^{p_k}-\frac{k_{r-1}}{h_r^\alpha}\frac{1}{k_{r-1}}\sum_{k=1}^{k_{r-1}}\left|\frac{x_k}{y_k}-L\right|^{p_k}\\ &\leq \left(\frac{1+\delta}{\delta}\right)\varepsilon-\frac{1}{\delta}\varepsilon'\end{aligned}$$

for each $k_r \in S$. Choose $\eta = \left(\frac{1+\delta}{\delta}\right)\varepsilon - \frac{1}{\delta}\varepsilon'$. Therefore,

$$\left\{r\in\mathbb{N}:\frac{1}{h_r^\alpha}\sum_{k\in I_r}\left|\frac{x_k}{y_k}-L\right|^{p_k}<\eta\right\}\in F(\mathcal{I})$$

and it completes the proof. □

Remark 2.7. *The converse of this result is not clear for $\alpha < 1$ and we leave it as an open problem.*

For the next result we assume that the lacunary sequence θ satisfies the condition that for any set $C \in F(I)$, $\bigcup\{n : k_{r-1} < n < k_r, r \in C\} \in F(I)$.

Theorem 2.8. *For a lacunary sequence θ satisfying the above condition,*

$$x \overset{N_\theta^{L(p)}(\mathcal{I})}{\sim} y \text{ implies } x \overset{\sigma^{(p)}(\mathcal{I})}{\sim} y$$

if $\sup_r \sum_{i=0}^{r-1}\frac{h_{i+1}^\alpha}{(k_{r-1})^\alpha} = B(say) < \infty$.

Proof. If $\limsup q_r < \infty$ then there exists $B > 0$ such that $q_r < B$ for all $r \geq 1$. Let $x \overset{N_\theta^{L(p)}(\mathcal{I})}{\sim} y$ and define the sets T and R such that,

$$T=\left\{r\in\mathbb{N}:\frac{1}{h_r^\alpha}\sum_{k\in I_r}\left|\frac{x_k}{y_k}-L\right|^{p_k}<\varepsilon_1\right\}$$

and

$$R=\left\{n\in\mathbb{N}:\frac{1}{n^\alpha}\sum_{k=1}^{n}\left|\frac{x_k}{y_k}-L\right|^{p_k}<\varepsilon_2\right\}.$$

Let

$$A_j=\frac{1}{h_j^\alpha}\sum_{k\in I_j}\left|\frac{x_k}{y_k}-L\right|^{p_k}<\varepsilon_1$$

for all $j \in T$. It is obvious that $T \in F(\mathcal{I})$. Choose n is any integer with $k_{r-1} < n < k_r$ where $r \in T$.

$$\begin{aligned}
&\frac{1}{n^\alpha}\sum_{k=1}^{n}\left|\frac{x_k}{y_k}-L\right|^{p_k}\\
\le\ &\frac{1}{k_{r-1}^\alpha}\sum_{k=1}^{k_r}\left|\frac{x_k}{y_k}-L\right|^{p_k}\\
=\ &\frac{1}{k_{r-1}^\alpha}\left(\sum_{k\in I_1}\left|\frac{x_k}{y_k}-L\right|^{p_k}+\sum_{k\in I_2}\left|\frac{x_k}{y_k}-L\right|^{p_k}+...+\sum_{k\in I_r}\left|\frac{x_k}{y_k}-L\right|^{p_k}\right)\\
=\ &\frac{k_1^\alpha}{k_{r-1}^\alpha}\left(\frac{1}{h_1^\alpha}\sum_{k\in I_1}\left|\frac{x_k}{y_k}-L\right|^{p_k}\right)+\frac{(k_2-k_1)^\alpha}{k_{r-1}^\alpha}\left(\frac{1}{h_2^\alpha}\sum_{k\in I_2}\left|\frac{x_k}{y_k}-L\right|^{p_k}\right)+...\\
&+\frac{(k_r-k_{r-1})^\alpha}{k_{r-1}^\alpha}\left(\frac{1}{h_r^\alpha}\sum_{k\in I_r}\left|\frac{x_k}{y_k}-L\right|^{p_k}\right)\\
=\ &\frac{k_1^\alpha}{k_{r-1}^\alpha}A_1+\frac{(k_2-k_1)^\alpha}{k_{r-1}^\alpha}A_2+\cdots+\frac{(k_r-k_{r-1})^\alpha}{k_{r-1}^\alpha}A_r\\
\le\ &\sup_{j\in C}A_j.\sup_r\sum_{i=0}^{r-1}\frac{(k_{i+1}-k_i)^\alpha}{k_{r-1}^\alpha}\\
<\ &\varepsilon_1 B
\end{aligned}$$

Choose $\varepsilon_2 = \frac{\varepsilon_1}{B}$ and in view of the fact that $\bigcup\{n : k_{r-1} < n < k_r, r \in T\} \subset R$ where $T \in F(I)$ it follows from our assumption on θ that the set R also belongs to $F(I)$ and this completes the proof of the theorem. □

Acknowledgements : I would like to express my gratitude to the referees of the paper for his useful comments and suggestions towards the quality improvement of the paper.

References

[1] P. Kostyrko, Šalát, T. and Wilezyński, , W., $\mathcal{I}-$Convergence, Real Analysis Exchange. 26 (2) (2000/2001) 669-686.

[2] K. Dems, On I-Cauchy sequences, Real Anal. Exchance. 30 (2004-2005) 123-128.

[3] P. Das, E. Savaş, On I-statistical and I-lacunary statistical convergence of order *alpha* Bull. iranian Soc. 40 (2) (2014) 459-472.

[4] E. Savaş, R.F. Patterson, An extension asymptotically lacunary statistical equivalent sequences, Aligarh Bull. Math. 27 (2) (2008) 109-113.

[5] E. Savaş, On $\mathcal{I}-$ asymptotically lacunary statistical equivalent sequences, Advances in Difference Equations.2013, 2013:111. DOI: 10.1186/1687-1847-2013-111.

[6] E. Savaş, On asymptotically $\mathcal{I}$-lacunary statistical equivalent sequences of order α, the 2014 International Conference on Pure Mathematics-Applied Mathematics Venice, Italy, March 15-17, 2014.

[7] E. Savaş, $\triangle^m$ -strongly summable sequences spaces in 2-normed spaces defined by Ideal convergence and an orlicz function, App. Math. Comp. 217 (2010) 271-276.

[8] E. Savaş, A-sequence spaces in 2-normed space defined by ideal convergence and an Orlicz function, Abst. Appl. Analy., Vol. 2011 (2011), Article ID 741382.

[9] E. Savas, On some new sequence spaces in 2-normed spaces using Ideal convergence and an Orlicz function, J. Ineq. Appl. Article Number: 482392 DOI: 10.1155/2010/482392 Published: 2010.

[10] E. Savaş, P. Das, A generalized statistical convergence via ideals, Appl. Math. Letters. 24 (2011) 826 - 830.

[11] Mursaleen M. and Sharma Sunil K., *Spaces of ideal convergent sequences*, The Scientific World Journal, Volume 2014, Article ID 134534, 1-5.

[12] Mursaleen M. , Alotaibi A. , *On I-convergence in random 2-normed spaces*, Mathematica Slovaca, 61 (6) (2011) 933-940.

[13] H. Fast, Sur la Convergence Statistique, Coll. Math. 2 (1951) 241-244.

[14] M. S. Marouf, Asymptotic equivalence and summability, Internat. J.Math & Math. Sci. 16 (4) (1993) 755-162.

[15] J. A. Fridy, On statistical convergence, Analysis. 5 (1985) 301-313.

[16] R.F. Patterson, On asymptotically statistical equivalent sequences, Demonstratio Mathematica. XXXVI (1) (2003), 1-7.

[17] R.F. Patterson, E. Savaş, On asymptotically lacunary statistical equivalent sequences, Thai Journal of Mathematics. 4 (2) (2006) 267-272.

[18] E. Savaş, R.F. Patterson, An extension Asymptotically lacunary statistical equivalent sequences, Aligarh Bull. Math. 27 (2) (2008) 109-113.

(Received 19 September 2014)
(Accepted 25 January 2015)