Thai **J**ournal of **M**athematics : (2016) 183–200
Special Issue on Applied Mathematics : Bayesian Econometrics

http://thaijmath.in.cmu.ac.th
ISSN 1686-0209

Multi-Asset Portfolio Returns: A Markov Switching Copula-Based Approach

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Abstract : The motivation for undertaking this paper stems from doubt that whether investors should keep the same strategy on the portfolio over periods of market regime shift. This paper investigates portfolio risk structure for multi-asset allocation issue using a Markov Switching copula-based approach. With this method we focus on returns in the different regime to improve the performance of portfolios. We conduct a Markov Switching with high dimension copula in order to measure a dependency of the variables, thus the model is flexible and can capture the economic behaviour change over time. The conditional Value at Risk is taken into account in the economic change and we employ Bayesian estimation method to estimate parameters of the model.

Keywords : GARCH; Markov Switching Multivariate Copula; Value-at-Risk; Expected Shortfall.
2010 Mathematics Subject Classification : 47H09; 47H10.

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1 Introduction

The Chinese stock market crash has occurred since June 2015. Notably, not only was Shanghai main share index down 8.49 percent of its value on 24 August, the markets in Japan, Europe and America also suffered the meltdown. Furthermore, the Bloomberg Commodity Index has hit a low for more than 15 years. There appears to be some correlation between stock markets and commodity futures. Should investors include commodities in their portfolios to reduce risk or

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increase returns? There exists a large body of literature documenting this issue. Daskalaki and Skiadopoulos [1] , found that commodities offer in-sample diversification benefits only in the case where higher order moments are taken into account and these benefits are not preserved in the out-of-sample framework. Bessler and Wolff [2] investigated individual commodities and commodity groups separately as well as alternative commodity indices. They found that aggregate commodity indices, industrial and precious metals as well as energy improved the performance of a stock-bond portfolio for most asset allocation strategies but hardly traced positive portfolio effects for agricultural and livestock commodities.

So far, many studies have worked on stock and commodity portfolio returns using conventional model, such as minimum-variance portfolio optimization strategy and sample-based mean-variance optimization model. The dependence between financial asset returns is explained by those conventional models, which only can explain dependence between random variables in the linear regression. In an investment environment, there are no outliers. An incorrect model for portfolio optimization can lead to significant loss of investment. Embrechts, Lindskog and McNeil [3] noticed that linear correlation can often be quite misleading and should not be taken as the canonical dependence measure. In order to capture heavy tail information regarding the financial market, we use the copula-based GARCH model to get value at risk (VaR) and Expected ShortfallES). The copula-based GARCH model can be used to analyze asymmetric or tail dependence structure (see Patton [4] and Wu, Chung and Chang [5]). There are already several papers that show its advantages. For example, Autchariyapanitkul, Chanaim and Sriboonchitta [6] and Ayusuk and Sriboonchitta [7] investigated multivariate t-copula and Vine copula based on GARCH model to explain portfolio risk structure for high-dimensional asset allocation issue. But most still worked on strong assumption of no economic change. We need to relax this assumption since many papers presented the different structure of dependency for a long time. So the dependency may be represented as two regimes, i.e., high dependence regime and low dependence regime[8]. Thus we need Markov Switching technique. Markov Switching models have become popular for modeling non-linearities and regime shifts. Why is it interesting to focus on a dynamic asset allocation context? Because high and low regime can affect asset pricing and focusing on the different regime can remove some short-term impacts in market price dynamics and distortion of performance of portfolios. Ntantamis and Zhou [9] investigated the relation between different market states (bull and bear markets) to examine whether being in a different market phases for a given commodity can provide information about whether the corresponding commodity stocks or stock market indices are in a comparatively market states. Moreover, most investigators used MLE as an estimator. In this paper we employ a Bayesian estimation since the likelihood function is difficult to estimate in the discrete margins case [10]. Moreover, if estimation of the copula parameters is undertaken jointly with the parameters of the marginal models, the maximum likelihood estimator is difficult to reach the global maximum and is not easy to be converged.

This study contributes to the literature in several aspects. First, the high dimensional copula is extended to Markov Switching and conduct a Markov Switching with high dimensional copula in order to measure a dependency of the variables, thus the model is flexible and can capture the economic behaviour change over time. Second, the conditional Value-at-Risk is taken into account in the economic change, thus it will be the more accurate risk measure than the conventional method, which is measured under the one dimension.

Our empirical results confirm that rice futures found useful in investors portfolios. Furthermore, we consider the stock and commodity returns in high dependence regime and low dependence regime. We found that rubber futures add more value than rice and oil futures in stock and commodity portfolios.

The remainder of this study is organized as follows. In section 2 we present the multivariate copula and Markov Switching model. Section 3 describes our dataset of commodity futures and stock indices. In section 4 we discuss our empirical results. Section 5 concludes.

2 Methodology

2.1 Basic Concepts of Copula

Copula is a multivariate probability distribution that is used to describe the dependence between random variables. Sklar's Theorem [11] states that any multivariate joint distribution can be written in terms of univariate marginal distribution functions and a copula which describes the dependence structure between the variables. Consider the multivariate case with n random variables, given n variables $x_1, ..., x_n$ with marginal distribution $F_1(x_1), ..., F_n(x_n)$, Sklars theorem [11] introduced a linkage between distributions of $x_1, ..., x_n$ and bind their marginals using copula function. That is $H\left(x_1, ..., x_n\right) = C\left(F_1(x_1), ..., F_n(x_n)\right) = C\left(u_1, ..., u_n\right)$ where $u_1, ..., u_n$ are uniform in the [0,1] interval. If marginals $F_1(x_1), ..., F_n(x_n)$ are continuous distribution functions, then there is a unique copula function C but if$F_1(x_1), ..., F_n(x_n)$ are discrete then C is not unique. For multivariate case, the copula density c is obtained by

$$c\left(F_1(x_1), ..., F_n(x_n)\right) = \frac{h\left({F_1}^{(-1)}\left(u_1\right), ..., {F_n}^{(-1)}\left(u_n\right)\right)}{\prod_{i=1}^{n} f_i\left({F_i}^{(-1)}\left(u_i\right)\right)} \tag{2.1}$$

where

h= the density function associated to H

f_i=the density function of each marginal distribution

c= the copula density.

There are two famous classes of copula functions, namely Elliptical and Archimedean. However, this study will focus on the Elliptical class. Elliptical copula function is a variance-covariance structure similar to the multivariate normal family, but is essentially richer because its marginal tails are allowed to decrease to zero exponentially, according to power, or at many other rates and also has symmetrical tail dependence. The dependence structure, related to this function, is the Pearsons correlation which has the value of its parameter in the [-1,1] interval. The copula functions in Elliptical class are the Gaussian and the Student-t copulas.

2.1.1 Gaussian Copula

The Gaussian, or Normal copula is a linear correlation with symmetric function because the upper and the lower tail dependences are equal, and so it has no tail dependence in this function. In the multivariate case, let $\Phi()$ be standard normal cumulative distribution, thus Gaussian copula density can be written as

$$f_{(n)}(R) = \frac{1}{\left|R^{1/2}\right|} exp\left\{\frac{-1}{2}\gamma\left(R^{-1} - I\right)\gamma'\right\}\left(\prod_{i=1}^{n} exp\left\{\frac{-1}{2}\gamma_i^2\right\}\right)^{-1} \tag{2.2}$$

2.1.2 Student-t Copula

The Student-t copula has a linear correlation coefficient and has symmetrical tail dependence. However, it can capture some tail dependence. Thus the multivariate Student-t copula density can be written as

$$f_{(t)}(X) = \frac{\frac{\Gamma[(v+n)/2]\left|R^{1/2}\right|}{\sqrt{v^n \pi^n \Gamma(v/2)}}}{\left\{1 + \frac{R^{-1}(x-\mu)'(x-\mu)}{v}\right\}^{\frac{v+n}{2}}} \prod_{i=1}^{n}\left\{1 + \frac{(x-\mu)'(x-\mu)}{v}\right\}^{\frac{v+n}{2}} \tag{2.3}$$

Where, v is degree of freedom parameter and Γ is gamma function.

2.2 ARMA(p, q) GARCH Models for Univariate Distributions

To model the marginal distribution of each random variable, we employ a univariate ARMA(p, q)-GARCH(m, n) specification that can be described as

$$y_t = \phi_0 + \sum_{i=1}^{p} \phi_i y_{t-i} + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j} + \varepsilon_t \tag{2.4}$$

$$\varepsilon_t = h_t \eta_t \tag{2.5}$$

$$h_t^2 = \alpha_0 + \sum_{i=1}^{m} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{n} \beta_j h_{t-j}^2 \tag{2.6}$$

where (2.4) and (2.6) are the conditional mean and variance equation, respectively. ε_t is the residual term which consists of the standard variance, h_t, and the standardized residual, η_t, which is proposed to have a Gaussian distribution, a Student-t distribution, a generalized error distribution (GED), a skewed GED and a skewed-t distribution. The best-fit ARMA(p, q)-GARCH(m, n) will give the standardized residuals to be transformed into a uniform distribution in (0,1).

2.3 Value at Risk with Copula

Value at Risk (VaR) and conditioned Value at Risk or Expected Shortfall (ES) has been widely used to measure risk since the 1990s. The VaR of portfolio can be written as

$$VaR_\alpha = inf\left\{l \in R : P\left(L > l\right) \leq 1 - \alpha\right\} \tag{2.7}$$

where, α is a confidence level with a value [0,1] which presents the probability of Loss L to exceed l but not larger than $(1-\alpha)$. While an alternative method, ES, is the extension of the VaR approach to remedy two conceptual problems of VaR ([12]). Firstly, VaR measures only percentiles of profit-loss distribution with difficulty to control for non-normal distribution. Secondly, VaR is not sub-additive. ES can be written as

$$ES_\alpha = E\left(L | L > VaR_\alpha\right). \tag{2.8}$$

To find the optimal portfolios, Rockafellar and Uryasev [13] introduced the portfolio optimization by calculating VaR and extend VaR to optimized ES. The approach focused on the minimizing of ES to obtain the optimal weight of a large number of instruments. In other words, we can write the problem as in the following The objective function is to

$$\text{Minimize} \quad ES_\alpha = E\left(L | L > inf\left\{l \in R : P\left(L > l\right) \leq 1 - \alpha\right\}\right) \tag{2.9}$$

Subject to

$$R_p = \sum_{i=1}^{n} \left(w_i \bullet r_i\right)$$

$$\sum_{i=1}^{n} \left(w_i\right) = 1$$

$$0 \leq w_i \leq 1, \quad i = 1, 2, ..., n$$

where R_p is an expected return of the portfolios, w_i is a vector of weight portfolio, and r_i is the return of each instrument.

2.4 Regime Switching Copula

In general, financial time series exhibit different behaviour and lead to different dependencies over time; for this reason, the dependence structure of the variables may be determined by a hidden Markov chain with two regimes or more. Therefore, it is reasonable to extend the copula to Markov Switching [14] and obtain Markov Switching copula. Thus the model becomes more flexible since it allows the dependence copula parameter $(R_{c,t}^{S_t})$ to be governed by an unobserved variable at time t (S_t). Let S_t be the state variable, which is assumed to have two states (k=2), namely high dependence regime and low dependence regime. The joint distribution of $x_1, ..., x_n$ conditional on S_t, is defined as

$$\left(x_{1,t}, ..., x_{n,t} | S_t = i\right) \sim C_t^{S_t}\left(u_{1t}, ..., u_{nT} | \theta_{c,t}^{S_t}, R_{c,t}^{S_t}\right), \quad i = 1, 2. \tag{2.10}$$

The unobservable regime (S_t) is governed by the first order Markov chain, which is characterized by the following transition probabilities (P):

$$P_{ij} = Pr\left(S_{t+1} = j | S_t = i\right) \quad \text{and} \quad \sum_{j=1}^{k} P_{ij} = 1 \quad \text{for} \quad i = 1, 2 \tag{2.11}$$

where P_{ij} is the probability of switching from regime i to regime j, and these transition probabilities can be formed in a transition matrix P, as follows:

$$P = \begin{bmatrix} p_{11} & p_{12} = 1 - P_{11} \\ p_{21} = 1 - P_{22} & p_{22} \end{bmatrix} \tag{2.12}$$

The Gaussian copula density function from Eq.(2.2) can be rewritten in the likelihood function form as

$$L_{(n)}\left(u_1, .., u_n | \theta_1, .., \theta_n, R\right) = \frac{1}{\left|R^{1/2}\right|} \prod_{i=1}^{T} \left(exp\left\{ \frac{-1}{2} \gamma \left(R^{-1} - I\right) \gamma' \right\} \prod_{j=1}^{n} f_i\left(x_{ij}; \theta_j\right) \right) \tag{2.13}$$

where $f_i\left(x_{ij}; \theta_j\right)$ is the density function obtained from the ARMA-GARCH step and we assume this function to be fix. Similarly, the Student-t copula density function from Eq.(2.3) can be rewritten in the likelihood function form as

$$L_{(t)}\left(u_1, .., u_n | \theta_1, .., \theta_n, R, v\right) = \prod_{i=1}^{T} \left(\frac{\frac{\Gamma[(v+n)/2]\left|R^{1/2}\right|}{\sqrt{v^n \pi^n \Gamma(v/2)}}}{\left\{1 + \frac{R^{-1}(x-\mu)'(x-\mu)}{v}\right\}^{\frac{v+n}{2}}} \prod_{j=1}^{n} f_i\left(x_{ij}; \theta_j, v\right) \right). \tag{2.14}$$

In this study, the method of Kim's filtering algorithm [15] is conducted to filter the state variable S_t and let $L_{(t)}$ and $L_{(n)}$ be $L(T)$ and $L(N)$ respectively, thus we can write the two regime Markov Switching copula log likelihood as

$$\log L_N\left(\theta_{N,S_t}, R_{N,S_t}, P\right) = \sum_{S_t=1}^{2} \log L(N) Pr\left[S_t | \theta_{N,S_{t-1}}, R_{N,S_{t-1}}, P\right] \quad \text{Gaussian} \tag{2.15}$$

$$\log L_T\left(\theta_{T,S_t}, R_{T,S_t}, P\right) = \sum_{S_t=1}^{2} \log L(T) Pr\left[S_t | \theta_{T,S_{t-1}}, R_{T,S_{t-1}}, P\right] \qquad \text{Student} - t. \tag{2.16}$$

To evaluate the log-likelihood in Eq. (2.15) and Eq. (2.16), we need to calculate the weight $Pr\left[S_t|\theta_{n,S_{t-1}}, R_{n,S_{t-1}}\right]$ and $Pr\left[S_t|\theta_{t,S_{t-1}}, R_{t,S_{t-1}}, v_{S_{t-1}}\right]$ for S_t=1,2 because the estimation of the Markov Switching copula needs inferences on the probabilities of S_t,

$$Pr\left(S_t = 1|w_t\right) = \frac{logL\left(\theta_{S_t=1}, R_{S_t=1}, P\right) Pr\left(S_t = 1|w_{t-1}\right)}{\sum_{S_t=1}^{2} logL(\theta_{S_t}, R_{S_t}, P) Pr\left[S_t = S_t|w_{t-1}\right]} \tag{2.17}$$

$$Pr\left(S_t = 2|w_t\right) = 1 - Pr\left(S_t = 1|w_t\right) \tag{2.18}$$

where w is all the information set of the model.

2.5 Prior Distributions

In the Bayesian approach we need to specify the prior distribution for all parameters sets in the model consisting of transition matrix parameters and dependence parameters to obtain the posterior distribution. We define the distribution of our parameters following Smith [10] and Smith, Gan and Kohn [16]. The uniform prior $Unif(-1,1)$ is given for the dependence parameters $R_{c,t}^{S_t}$ while the Dirichlet distribution with the hyper-parameters (α_1, α_2) is assumed to be our prior since the transition matrix parameter is the probability [0,1] and suitable for make the persistence of the probability of staying in their own regime. For v , we use a uniform prior on [2, 50]. Since the marginal models are application specific, so are the priors on the marginal parameters, we adopt non-informative priors in our empirical work. Thus, the log posterior distribution of Markov Switching copula becomes

$$Pr\left(\Theta, P|u_1, ..., u_n\right) = \sum_{k=1}^{2} \log L(N) Pr\left[S_t|\Theta_{t-1}\right] + \log\left(Pr\left(\Theta, S\left(t\right)\right)\right) \quad \text{Gaussian} \tag{2.19}$$

$$Pr\left(\Psi, P|u_1, ..., u_n\right) = \sum_{k=1}^{2} \log L(T) Pr\left[S_t|\Psi_{t-1}\right] + \log\left(Pr\left(\Psi, S\left(t\right)\right)\right) \quad \text{Student} - t \tag{2.20}$$

where $\log\left(Pr\left(\Theta, S\left(t\right)\right)\right)$ and $\log\left(Pr\left(\Psi, S\left(t\right)\right)\right)$ are the log prior distribution for Gaussian and Student-t copulas respectively.

To sample all of these parameters based conditional posterior distribution, we employ the Markov chain Monte Carlo, Metropolis Hasting algorithm. To draws these parameters, first of all, the target distribution function is set as a truncated normal [-1,1] for dependence parameters and truncated normal [0,1] for transition matrix. We run the Metropolis Hasting sampler for 10,000 iterations where the first 2,000 iterations serve as a burn-in period. For Metropolis Hasting algorithm, we apply it to find all parameter sets together where the acceptance ratio is

$$r = \frac{Pr\left(\theta^{*}|u_1,...,u_n\right)Pr\left(\theta_{i-1}|\theta^{*}\right)}{Pr\left(\theta_{i-1}|u_1,...,u_n\right)Pr\left(\theta^{*}|\theta_{i-1}\right)} \tag{2.21}$$

where θ is $\Theta = \left\{\theta_{n,S_{t-1}}, R_{n,S_{t-1}}, P\right\}$ or $\Psi = \left\{\theta_{t,S_{t-1}}, R_{t,S_{t-1}}, v_{S_{t-1}}, P\right\}$.
If $r \geq 1 \Rightarrow \theta = \theta^{*}$.
if $r < 1 \Rightarrow$ draw Uniform [0,1].
if $U \leq 1 \Rightarrow \theta_i = \theta^{*}$ else $\theta_i = \theta_{i-1}$.

3 Dataset and Estimation

In this study, we use the data set comprising the Stock Exchange of Thailand index (SET), Hang Seng Index (HSI), Brent oil spot price (OIL), rubber commodity price (Rubber), and rice commodity price (RICE). For the period July, 2008 to April, 2015 , totally 1766 observations. The data are collected from Thomson and Reuter DataStream, Chiang Mai, University. All the series have been transformed into the difference of the logarithm. We would like to focus on Thailand market and to mix stock market and commodity market. We choose oil, rubber and rice as representation of commodity market. There are several reasons. First, the rice price has a significant effect on quantity of rubber production in Thailand with an estimated elasticity of -2.6 (see [17]). Second, Li and Yang [18] using A Copula-based GARCH model approach found that the rubber price is affected by the price of oil. Thailand has become the largest rubber exporter in the world. Thai rubber rank second in value of agricultural export after rice.

Table 1 provides the summary statistics for each rate of returns. As previously found in other studies, these return rates demonstrate excess kurtosis and negative skewness except HSI. In addition, from the results of Jarque-Bera test, we may state that they do not exhibit Gaussian distribution.

In the estimation of copula with Markov switching, the method consists of three steps. The first step is the estimation of the ARMA-GARCH to obtain the standardized residual for each stock and transform it into uniform[0,1]; the second step involves maximizing the Markov Switching copula log-likelihood in order to get the starting value of dependence parameters. Finally, the Bayesian estimation is conducted to estimate the posterior mean of the parameter sets in the model. Note that Gaussian and Student-t copulas are two families that we employ to join the marginal distribution in this study.

Then, the obtained final mean posterior parameter of dependence between all variables will be extended to compute the VaR and the ES in two different regimes, using the following method. First, the Monte Carlo simulations are used to simulate the joint-dependent distribution uniform from the fitted Markov Switching copula model.

We simulate 10,000 replications of the portfolio returns for each regime and, then we multiply the inverse of the marginal distribution with the random variable

Table 1: Data Descriptive Statistics

	SET	HSI	OIL	RUBBER	RICE
Mean	0.0002	0.00006	-0.00021	-0.00012	0.00001
Median	0.0002	0.00002	0	0	0
Maximum	0.03409	0.05821	0.05518	0.02879	0.0266
Minimum	-0.05037	-0.05902	-0.04429	-0.03803	-0.06982
Std. Dev.	0.0063	0.00702	0.00917	0.00729	0.00427
Skewness	-0.67331	0.12004	-0.0258	-0.41097	-2.79167
Kurtosis	9.86278	13.734	7.09583	5.8445	50.19978
Jarque-Bera	3599.050	8483.831	1234.618	645.086	166224.198
Probability	0	0	0	0	0
Sum	0.35651	0.10509	-0.36391	-0.20691	0.02093
Sum Sq. Dev.	0.06995	0.08694	0.14853	0.09373	0.03223
Observations	1766	1766	1766	1766	1766

to obtain ε_{it}^{k}. To find the return of each variable ($r_{it}^{(k)}$), we perform the estimation using the following formula:

$$r_{it}^{(k)} = \widetilde{u_{it}} + \sqrt{h_{it}} \cdot \varepsilon_{it}^{(k)}$$

where $\widetilde{u_{it}}$ is the simulated mean form ARMA equation. To compute the portfolio return in each regime, we specify an equally weighted portfolio return, that is, $X_{pt} = 0.2SET_t + 0.2HSI_t + 0.2BRENT_t + 0.2Rubber_t + 0.2Rice_t$. In this computation, we compute all the risk measures at 1%, 5%, and 10% levels. Then, The study conducts two backtesting of Kupiec [19] measure the accuracy of the obtained VaR and the ES estimates (See [12]).

4 Empirical Result

4.1 ARMA-GARCH Results

We used ARMA-GARCH process to appropriately analyze the volatility and estimate the marginal. We selected the optimal lag and marginal distribution assumption for ARMA(p, q)-GARCH(1,1,) by using AIC and found that the returns on SET, HSI , OIL, RUBBER and RICE satisfied ARMA(1,1), ARMA(3,4), ARMA(5,5), ARMA(1,1), and ARMA(2,1) with GARCH(1,1) respectively. In addition, we compared various margins assumption and the lowest Akaike Information criterion (AIC) is preferred. We found that the margins of SET, HSI and OIL are GED and the margins of RUBBER and RICE are normal distributed. The parameters of each are all significant as shown in Table 2. The estimated ARCH effects equal 0.097, 0.066, 0.054, 0.076 and 0.059. These results indicate that a shock to the growth rate of return has short-run persistence in all cases.

Table 2: Estimates of ARMA-GARCH parameters for raw returns

	SET	HSI	OIL	RUBBER	RICE
C	0.000217	0.00004	0.00001	0.000017	-0.00001
	(0.000054)	(0.000042)	(0.000)	(0.00008)	(0.00002)
AR(1)	4.889	0.4057	-0.1448	0.5811	0.685
	(0.04503)	(0.1338)	(0.00001)	(0.1238)	(0.1523)
AR(2)		0.5062	-0.4316		0.08698
		(0.1585)	(0.00001)		(0.0296)
AR(3)		-0.3447	0.01003		
		(0.04722)	(0.00001)		
AR(4)			0.09061		
			(0.00001)		
AR(5)			-0.5544		
			(0.00001)		
MA(1)	-0.4821	-0.3981	0.09564	-0.4294	-0.7283
	-0.05096	-0.1321	(0.00002)	(0.1380)	(0.1519)
MA(2)		-0.5066	0.4451		
		(0.1580)	(0.00002)		
MA(3)		0.3334	-0.06321		
		(0.05054)	(0.00002)		
MA(4)		-0.00263	-0.0685		
		(0.01808)	(0.00002)		
MA(5)			0.5581		
			(0.00002)		
α_0	0	0	0	0.000002	0
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
ARCH(1)	0.09768	0.06619	0.05497	0.07657	0.05904
	(0.01709)	(0.01189)	(0.0106)	(0.01038)	(0.00692)
GARCH(1)	0.894	0.9287	0.9446	0.8849	0.927
	(0.01692)	(0.01212)	(0.0102)	(0.01616)	(0.00683)
SHAPE	1.197	1.212	1.325		
	(0.05665)	(0.06707)	(0.06127)		
LogL	6784.036	6741.797	6230.19	6364.204	7274.951
normalized	3.84147	3.8176	3.527854	3.603739	4.119452
BERK-test	0.8249	0.9882	0.5459	0.9989	0.9987
ARCH-LM	0.4467	0.5332	0.1002	0.2778	0.9975

Source: Calculation

The values of the GARCH coefficient are 0.894, 0.928, 0.944, 0.884, and 0.927 that illustrate each growth rate of return has a long-run persistence of volatility. Testing for marginal distribution that satisfies the two preconditions: uniformity and serial independence is a critical step in constructing multivariate models using copula. We used the Berkowitz test to confirm the marginal has uniform distribution and ARCH-LM Test to ensure residuals are i.i.d random variables and no autocorrelation.

4.2 Model Selection

In this section, we compare two copula functions, namely Gaussian and Student-t copulas. The Deviance Information criterion (DIC) is employed to compare the performance of our purposed models. Table 3 provides an evidence that MS-copula with Student-t function presents a lower DIC than Gaussian copula. Thus, we adopt MS-copula with Student-t function to be inference in our study. Moreover, the acceptance rate is considered here about how often was a proposal rejected by the Metropolis Hastings acceptance criterion. In the general, acceptance rates between 20% and 40% are optimal since these will confirm the good mixing between the proposal function and the target distribution. In the present study the acceptance is 40.27% for marginal parameters in our Markov Switching Student-t copula model.

Table 3: My caption

	Acceptance	DIC
Gaussian	0.4456	-2423.259
Student-t	0.4027	-2761.691

Source: Calculation

4.3 Markov Switching Student-t Copula

Table 4 shows the solutions of multivariate Student-t copula parameters with regime switching. We can use these values to construct efficient portfolio and find optimal plans for best expected returns with minimum loss which will be reported in the last section. Table 4 reports the estimated parameters of the Markov Switching Student-t copula. The results show that the value of the matrix dependence parameter in regime 1 is higher than regime 2. Thus, we can interpret regime 1 to be the high dependence regime, while regime 2 is the low dependence regime. Moreover, recently, the studies of Karimalis and Nimokis [20], found an evidence that the dependence among assets during market upturns is less than that during market downturns. Thus, this confirms the high dependence regime as the market downturn regime and the low dependence regime as the market upturn regime. Next, we take into consideration the estimated dependence parameters for 2 regimes, we observe that all of pair copula parameters present a positive dependence in both regimes except for RICE-OIL pair in regime 2. The presence of a positive dependence among these commodity prices gives us some economic inference that these prices are moving in the same direction and that the scope for the diversification of these commodity prices to reduce risk is more limited. In addition, the transition probability matrix of these commodity prices are also reported in Table 4. The $Pr(S_t = 1)$ is 91.09% and $Pr(S_t =)$ is 94.5% while the probabilities of regime switching between these two regimes are less than 10%.

Table 4: Empirical copula parameters

Regime 1					
	SET	HSI	OIL	RUBBER	RICE
SET	1	0.6804 (0.0013)	0.4722 (0.005)	0.3267 (0.0023)	0.1359 (0.0023)
HSI	0.6804 (0.0013)	1	0.5349 (0.0033)	0.422 (0.0023)	0.0821 (0.0023)
OIL	0.4722 (0.005)	0.5349 (0.0033)	1	0.309 (0.0033)	0.0166 (0.004)
RUBBE	0.3267 (0.0023)	0.422 (0.0023)	0.309 (0.0033)	1	0.0437 (0.0035)
RICE	0.1359 (0.0023)	0.0821 (0.0023)	0.0166 (0.004)	0.0437 (0.0035)	1

Regime2					
	SET	HSI	OIL	RUBBER	RICE
SET	1	0.4519 (0.0017)	0.0539 (0.0015)	0.2986 (0.0014)	0.1301 (0.0015)
HSI	0.4519 (0.0017)	1	0.0566 (0.0018)	0.3103 (0.0017)	0.0929 (0.0013)
OIL	0.0539 (0.0015)	0.0566 (0.0018)	1	0.0501 (0.0014)	-0.0124 (0.0018)
RUBBE	0.2986 (0.0014)	0.3103 (0.0017)	0.0501 (0.0014)	1	0.1623 (0.0017)
RICE	0.1301 (0.0015)	0.0929 (0.0013)	-0.0124 (0.0018)	0.1623 (0.0017)	1
	Regime1	Duration	Regime2	Duration	
Regime1	0.9109	11.224	0.0891	1.0978	
Regime2	0.055	1.0583	0.945	18.1669	

Source: Calculation

The results indicate that both regimes are persistent because of the high values obtained from the probabilities. Moreover, the duration of stay is short for both the regimes, with the duration equal to 11.24 days for the high dependence regime and 18.16 days for the low dependence regime. This result, apparently, indicates that the dependence between these returns has high fluctuation.

4.4 Regime Probabilities

As, we mentioned before, regime 1 can be interpreted as high dependence regime while regime 2 is interpreted as low dependence regime.

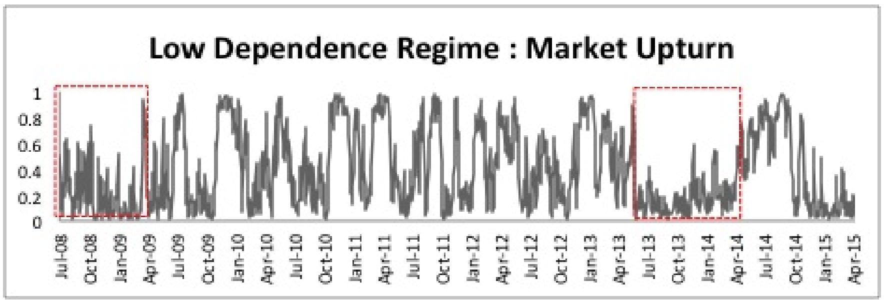

Figure 1: Filtered Probabilities of Market upturn regime.

Figure 1 plots the posterior mean regime at each time point for low dependence regime or market upturn. In this section, we analyse the evolution of the regime probabilities at each time period and find two interesting periods. First, we can observe that the 2 main sub periods (box plot line) consist of the July 2008 to April 2009 and June 2013 to April 2014 mostly take place in market downturn. These periods correspond to the US. Financial crisis in 2008 and the European Crisis in 2013-2014. We found that these two periods created a large negative effect on the world economy. The demand in commodity market shrunk and thereby lowering price of the commodities. The model seems to capture the financial behaviour well since it could detect the two great crises in our samples.

4.5 Value at Risk and Expected Shortfall Estimation

Table 5: Risk Measurement

Regime 1		
	VaR%	ES%
1%	-4.51	-5.57
5%	-2.91	-3.95
10%	-2.09	-3.21
Regime 2		
	VaR%	ES %
1%	-4.62	-5.58
5%	-3.02	-4.02
10%	-2.11	-3.28

Source: Calculation

Further estimation results on the expected VaR and ES are reported in Table 5. We calculated the expected values of 1%, 5%, and 10% VaR and ES on an equally weighted portfolio based on the Markov Switching Student-t copula.

Table 6: Result of Kupiec and Christoffersen Tests for VaR and ES

			Regime1	Regime2
Copula		α	Kupiec	
Student-t	VaR	1%	-14.9126	-14.95024
		5%	-9.2124	-4.0825
		10%	-5.6669	-1.5461
	ES	1%	-0.1081	-0.5694
		5%	-1.7473	-4.3055
		10%	-6.1858	-7.5136

Source: Calculation

Table 7: Optimal Portfolios weight

Regime 1							
Port	SET	HSI	OIL	RUBBER	RICE	Ret%	Risk%
1	0.4255	0.1608	0.1608	0.3220	0.0151	0	4.57
2	0.4659	0.1464	0.0725	0.3104	0.0046	0.01	4.58
3	0.4304	0.2246	0.044	0.3005	0	0.02	4.62
4	0.4834	0.2308	0.0060	0.2796	0	0.02	4.71
5	0.5642	0.1997	0	0.2360	0	0.03	4.84
6	0.6417	0.1748	0	0.1833	0	0.04	5
7	0.7242	0.1421	0	0.1335	0	0.04	5.18
8	0.8018	0.1171	0	0.0809	0	0.05	5.41
9	0.8603	0.1220	0	0.0175	0	0.06	5.67
10	1	0	0	0	0	0.06	5.96
Regime 2							
Port	SET	HSI	OIL	RUBBER	RICE	Ret%	Risk%
1	0.3587	0.2227	0.0812	0.3309	0.0063	0	4.6
2	0.3762	0.2185	0.0745	0.3306	0	0	4.6
3	0.4304	0.2246	0.044	0.3005	0	0.01	4.61
4	0.4834	0.2308	0.0060	0.2796	0	0.01	4.67
5	0.5642	0.1997	0	0.2360	0	0.01	4.78
6	0.6417	0.1748	0	0.1833	0	0.02	4.92
7	0.7242	0.1421	0	0.1335	0	0.02	5.12
8	0.8018	0.1171	0	0.0809	0	0.03	5.39
9	0.8603	0.1220	0	0.0175	0	0.03	5.7
10	1	0	0	0	0	0.03	6.13

Source: Calculation

For regime 1 or market downturn, the estimated VaR values are 4.51%, 2.91%, and 2.09%, respectively, while the estimated ES values are, respectively, 5.57%, 3.95%, and 3.21%. In the case of VaR, we can indicate that it might be 1%, 5%, and 10% sure that this portfolio will fall more than 4.51%, 2.91%, and 2.09%. If we take ES into account, it might be 1%, 5%, and 10% sure that this portfolio will fall more than 5.57%, 3.95%, and 3.21%. For regime 2 or market upturn, the result from VaR shows that it might be 1%, 5%, and 10% sure that this portfolios will fall more than 4.62%, 3.02%, and 2.11% while ES shows that it might be 1%, 5%, and 10% sure that this portfolio will fall more than 5.58%, 4.02%, and 3.28%. We observe that the probability of loss in regime 2 is higher than regime 1. This result confirms that the investor will face higher risk during the market upturn.

The study conducts two backtesting of Kupiec [11] measure the accuracy of the obtained VaR and the ES estimates (See [12]). The backtest at 99%, 95%, and 90% confidence levels are shown in Table 6. We can observe that our portfolio, at 1%, 5%, and 10% levels, are not statistically significant at 10% level.Thus, it is not possible to reject the null hypothesis that the expected proportion of violation is equal to the VaR confidence level (α). Therefore, the Markov Switching Student-t copula was concluded as the appropriate model to estimate the VaR and the ES in both two regimes.

Figure 2 illustrates the efficiency frontier for two regimes embracing the 10 portfolios in the Table 7. In this section, we also provide the optimal weight investment for these stock and commodities price in the market upturn and market downturn. The results can be interpreted separately for the two regimes. For example, in regime 1 or market downturn, these investors who are risk lover and want to gain high returns can allocate their investment in SET 86.04%, HSI 12.21%, and Rubber 1.75% in order to get the highest return at 0.06% and risk at5.67%. In contrast the investors who are risk averse and afraid of risk, they can invest in SET 46.60%, HSI 14.64%, OIL 7.26% and Rubber 31.04% rice 0.46% to face with the lowest risk (4.57%). Similar investors response are advised for to regime 2 or market upturn. In addition, we observed that SET index presents the best choice of investing when compare with other stock and commodity prices while rice presents the worse choice.

5 Conclusion and Future Works

This paper offers portfolio risk structure for multi-asset allocation issue using a Markov Switching copula-based approach. We intendedly deal with two different regimes to improve the performance of portfolios. We focus on Thai market and use the data set comprising the stock index of SET, HIS, and commodity price of OIL, Rubber and RICE for the period 2008:07-2015:04. There are three main findings. The first is that we found evidence that MS-copula with Student-t function present lower DIC than Gaussian copula. Thus, we adopt MS-copula with Student-t function to be inference in our study. The second finding is that the results of multivariate Student-t copula parameters with regime switching confirm the high dependence regime as the market downturn regime and the low dependence regime as the market upturn regime. This model also could capture the

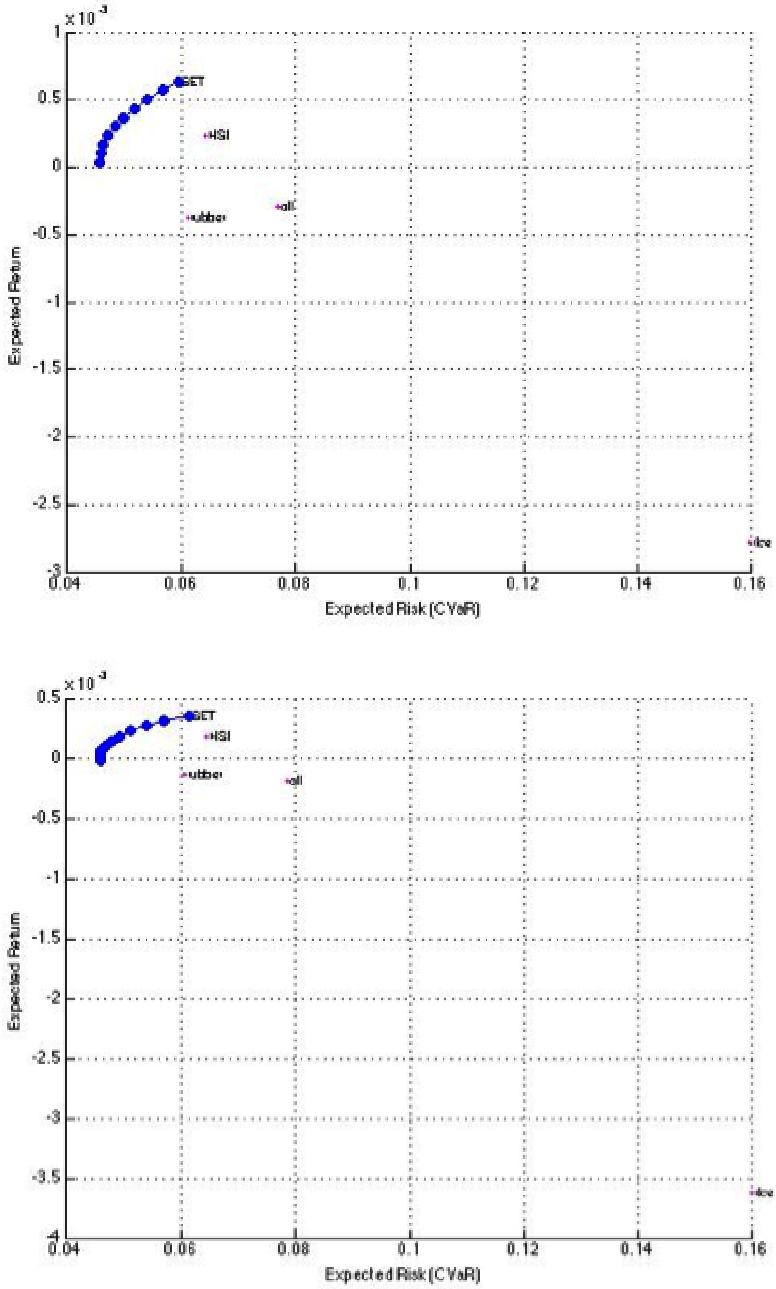

Figure 2: Efficient frontier for two regimes.

financial behavior well since it could detect the two great crises in our samples. Finally, the estimation of Expected Shortfall (ES) confirms that the investors will face higher risk with markets upturn. We also obtained the optimal weight for the portfolios which varies with the ES in the market upturn and market downturn. Further researches on this work can be pursued from different angles. Since there are two main classes of copula functions, namely Elliptical and Archimedean, our study only focuses on the Elliptical class which has symmetrical tail dependence. It would be interesting to see whether Archimedean copulas benefit from these advantages in multi-asset allocation issue using a Markov Switching approach when the data set has asymmetrical tail dependence. Additionally, in our paper, we assume that the dependence of copula parameters does not change over time. It would be interesting to extend dynamic portfolio risk for multi-asset allocation issue using a Markov Switching with time-varying copulas.

Acknowledgements : We are grateful for financial support from Puay Ungpakorn Centre of Excellence in Econometrics, Faculty of Economics, Chiang Mai University. We thanks Nguyen, H. T. for valuable comment to improve this paper.

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(Received 6 August 2016)
(Accepted 14 October 2016)