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A Bayesian Change Point with Regime Switching Model

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Abstract : This paper extends the threshold model to Markov Switching model in order to relax a linear function between dependent and independent variables. The model allows non-linear function using the idea of Threshold model. We conducted both simulation and real data studies to evaluate the performance of the proposed model and found that the model performs well in both simulation and application studies. The application study revealed the negative impact of unemployment rate on industry production index when the market stays in recession and depression period. Conversely, the positive impact of Unemployment rate to Industry Production index is empirically evident during expansion and boom period.

Keywords : Markov Switching model; Threshold model; Bayesian; unemployment rate and industry production index.
2010 Mathematics Subject Classification : 47H09; 47H10.

1 Introduction

The classical linear regression model has received considerable attention in many financial applications. However, the model is poorly suited for capturing and detecting instability in statistical relationship between the dependent and independent variables. The economic variables behave differently in various stages and hence the explanation of the effects that the independent variables have on the

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dependent variables is also different. There are two types of non-linear model have been applied in many studies and researchers were successful in their attempts to explain the relationship between the economic variables in different regimes; see e.g., Tong ([1, 2]), Granger and Teraasvirta [3], Hansen [4] and Chen, So and Liu [5] for more thorough discussions on Threshold model and Hamilton [6] for discussion on Markov Switching model.

After the Threshold model was purposed by Tong in 1970s, the threshold autoregression (TAR) ([7]) has become popular for analyzing a non-linear time series and has been applied in many nonlinear regression contexts (See, [8]). The regression model is separated into two or more states of economic or regimes based on the unknown or known threshold point (w). The model can switch the regime whenever threshold variable(q_t) across. In contrast to Threshold model, the Markov Switching of Hamilton (1989) is another type of non-linear model that also has become the most popular in non-linear time series literature. Kuan [9] highlighted that Threshold model has some limitations. First, nonlinear optimization algorithms are not easy to reach the global optimization in the parameter space and second, the model was design to estimate certain nonlinear patterns of data. Thus, the Markov switching model is proposed to capture more complex nonlinear patterns during different time periods. The switching mechanism could occur any time and regime switching is governed by an unobserved state variable that follows the first order Markov chain, i.e. S_t is governed by S_{t-1}.

To estimate the parameter set in each regime, the Hamilton filter is used to split the Markov Switching model into two or more regimes. It is an iterative algorithm for calculating the distribution of the state variable S_t by filtering the density of linear function between dependent and independent variables. We expected that the assumption of this linear function might not fit to explain the real economic behaviour. For example, if we conduct two regimes switching model to analyse the economic data, we will specify one regime as economic upturn and the other as economic downturn. The problem is if the linear function between dependent variable and independent variable is hold, the model will consider the linear relation function in both regimes and thereby failing to explain the non-linear relationship between variables in each regime. In the real economic cycle, many economists have suggested that there are four stages of economic cycles namely, expansion, boom, and recession and depression stage. Thus, it is reasonable to split the data into two regimes, the economic upturn and the economic downturn. Thus, it is fruitful to go beyond the conventional Markov Switching model by extending the Threshold model to Markov Switching model. To the best of our knowledge, the combination of Markov switching and threshold model was firstly proposed by Ardia [10]. He extend the Markov Switching model to Threshold asymmetric GARCH model. This paper generalizes the model of Ardia [10] by replacing the GARCH by regression model and proposed a Markov Switching Threshold regression (MS-Treg) to detect and analyze the extreme structural change in the economic data. From now on, let we call the regime specified in the Markov Switching model as state in order to make the explanation reasonable and understandable. In this study, we consider the two states with two regime Markov

Switching Threshold regression model (MS(2)-T(2)-reg). The model, therefore, relax the linear function and proposes a non-linear function in each state. To estimate the parameters in our proposed model, we employ a Bayesian method, based on Markov chain Monte Carlo (MCMC) method, which is more informative, flexible, and efficient than a maximum likelihood based approach Harris [11].

The objective of this paper is to develop a Markov Switching model for allowing the non-linear function between dependent and independent variables in each state. To accomplish this goal, we extend the idea of Threshold model to Markov Switching regression and proposed a Markov Switching Threshold regression model. However, our proposed model has to be proven that it will work in the real data analysis thus we conduct both simulation and real data studies to confirm that our proposed model can perform well and accurately. In addition, we use Kullback Leibler divergence (KLD) (also relative entropy) to measure the distance between the true and candidate models to check the robustness of our model. We took the idea of robustness check following what introduced by Glasserman and Xu [12]. They suggested that the model cannot avoid the imperfect assumption and estimation. Error in these assumption and estimation will bring about the in the model. To overcome these problems, we employ the KLD to measure the distance between the true model and alternative models in order to check the robustness of our purposed model. This robust approach starts from a true model and finds the worst case error in risk measurement that would present through a deviation from the true model.

The remainder of this study proceeds as follows. Section 2 presents a brief review of the non-linear models. In Section 3 we describe the MS-Treg model. The simulation study and the robustness check provided in Section 4. Section 5 presents empirical applications to the impact of Industry Production index on US. Civilian unemployment rate. Section 6 concludes the paper and provides a future research recomendation.

2 Review of Markov Switching and Threshold Regression Model

2.1 Markov Switching Regression Model

Consider the following Gaussian Regime Switching regression model

$$y_t = \beta_{0,S_t} + \beta_{S_t} X_t + \varepsilon_{t,S_t} \tag{2.1}$$

where $\varepsilon_{t,S_t}\,\tilde{}\,i.i.d.\,N(0,\sigma^2_{S_t})$, $y_{i,t}$ is dependent variable, X_t is $(k \times 1)$ vector of independent variables and state variable $S_t = i,\ \ i = 1, ..., k$. The state variable is unobserved and is assumed to evolve following a first order Markov chain with probability of transition from state i to state j thus

$$\Pr(S_t = j\,|S_{t-1} = i) = p_{ij}, \quad \sum_{j=1}^{k} p_{ij} = 1, \quad \text{for } i = 1, ..., k \tag{2.2}$$

The specification in Eq. (2.1) assumes that the probability of a change in state depends on the previous state. We do not know which state the process is in but can only estimate the probabilities of switching or staying in its own state. Thus, for k states, the transition matrix Q can be written as

$$Q = \begin{bmatrix} p_{11} & \cdots & \cdots & p_{1k} \\ \vdots & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ p_{k1} & \cdots & \cdots & p_{kk} \end{bmatrix}. \tag{2.3}$$

2.2 Threshold Regression Model

We consider a two regimes parametric threshold regression model of the form

$$y_t = \alpha_1 I_t[X_t \in L_t] + \alpha_2 I_t[X_t \in U_t] + e_t \tag{2.4}$$

where L_t and U_t are a subset of lower and upper regime, respectively. I is an indicator function such as

$$I_t = 1 \; if \, q_t > w \text{ and } I_t = 0 \text{ if} \, q_t \leq w \tag{2.5}$$

where α and w are the estimated coefficients and threshold parameter, respectively, and $e \sim N(0, \sigma^2)$ is an n 1 vector of independent and identically distributed (iid) errors with normal distribution. The movement of X_t between the regimes is controlled by q_t, If q_t are greater or lower than w, the separated X_t can be estimated as regressions. In Eq. (2.4) the slope with respect to X_t is equal to α_1 when the $q_t \leq w$, and equals α_2 when the $q_t > w$. For example if we consider a single covariate, we can rewrite Eq. (2.6) as

$$y_t = \alpha_1 \, [X_t \leq w] + \alpha_2 \, [X_t > w] + e_t. \tag{2.6}$$

3 Definition and Posterior of Markov Switching-Threshold Regression Model

3.1 Markov-Switching Threshold Regression (MSTreg)

To construct our proposed model, the non-linear function in each state is proposed in the model. The typical setup of our model is

$$y_t = \alpha_{1,S_t} I_t[X_t \in L_t] + \alpha_{2,S_t} I_t[X_t \in U_t] + e_{t,S_t} \tag{3.1}$$

whereas $e_{t,S_t} \sim N(0, \sigma^2_{S_t})$, and α_{S_t} and $\sigma^2_{S_t}$ are regime-dependent parameters. In the model (3.1) there is after a change in the regime an immediate one-time jump in the process mean. Occasionally, it may be more plausible to assume that the mean smoothly approaches a new level after the transition from one state to another. In such a situation the following model with regime-dependent intercept term α_{0,S_t} may be used:

$$y_t = \alpha_{0,S_t} + \alpha_{1,S_t} I_t[X_t \in L_t] + \alpha_{2,S_t} I_t[X_t \in U_t] + e_{t,S_t}. \quad (3.2)$$

A variable y_t is allowed to be a non-linear function of variables in vector of X_t with coefficients that depend on the state in period t. There are a discrete number of states, k. In this study we consider the simple case of two states and two regimes MS-Treg, therefore the MS(2)-T(2)-reg can be formed as

$$\begin{aligned} y_t &= \alpha_{0,S_t=1} + \alpha_{1,S_t=1} I_t[X_t \le w] + \alpha_{2,S_t=1} I_t[X_t > w] + e_{t,S_t=1} \\ y_t &= \alpha_{0,S_t=2} + \alpha_{1,S_t=2} I_t[X_t \le w] + \alpha_{2,S_t=2} I_t[X_t > w] + e_{t,S_t=2} \end{aligned} \quad (3.3)$$

where w is an unknown threshold parameter which is assumed to be the same in both states and state independent. The state S_t is unobserved and follows first order Markov process with transition matrix

$$Q = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}$$

where$p_{11} + p_{12} = p_{21} + p_{22} = 1$. To split the non-linear function into two state, the Hamilton filter is conducted to estimate the distribution of discrete state variable $\Pr(S_t = j\,|\Gamma_t)$. Let Γ_t denote as all information available at time t which include the data and parameters set at time t. Then the Hamilton filter comprises two recursive steps: first is the prediction step, defining $\Pr(S_t = j\,|\Gamma_{t-1}) = Q\cdot\Pr(S_{t-1} = j\,|\Gamma_{t-1})$ and second is the updating step defining $\Pr(S_t = j) = \mathrm{h}\cdot(\Pr(S_t = j\,|\Gamma_{t-1}))$. In this study, the full log likelihood function of our proposed model is

$$\begin{aligned} L = y_t\,|X_t, \Gamma_{t-1}) &= \sum_1^T f(y_t|S_t = j, X_t, \Gamma_{t-1})(S_t = j\,|\Gamma_{t-1}) \\ &= \sum_1^T \left(\frac{1}{\sqrt{2\pi\sigma^2_{S_t=1}}} \times \left[-\frac{(E_{S_t=1})^2}{2(\sigma^2_{S_t=1})} \right] ((S_t = 1\,|\Gamma_{t-1}) \right) \\ &+ \sum_1^T \left(\frac{1}{\sqrt{2\pi\sigma^2_{S_t=2}}} \times \left[-\frac{(E_{S_t=2})^2}{2(\sigma^2_{S_t=2})} \right] ((S_t = 2\,|\Gamma_{t-1}) \right) \end{aligned} \quad (3.4)$$

where $E_{S_t=1} = y_t - \alpha_{0,S_t=1} + \alpha_{1,S_t=1} I_t[X_t] + \alpha_{2,S_t=1} I_t[X_t > w]$, $E_{S_t=2} = y_t - \alpha_{0,S_t=2} + \alpha_{1,S_t=2} I_t[X_t \le w] + \alpha_{2,S_t=2} I_t[X_t > w]$, $\Pr(S_t = 1\,|\Gamma_{t-1})$ and $\Pr(S_t = 2\,|\Gamma_{t-1})$ are derived from the Hamilton Filter algorithm.

3.2 Posterior Estimation

The study conducted a Bayesian estimation for inference. Generally the conventional methods rely on normality assumptions and asymptotic arguments.

However, under the MCMC sampling methods which are more complicated and realistic applications, there is no inherent reliance on asymptotic arguments and assumptions [13]. Thus, this leads the Bayesian approach to outperform the classical approach. Given the observation, $y = y_1, ..., y_T$ and $X = X_1, ..., X_T$, the posterior distribution of $\Gamma = \left\{\alpha, \sigma^2, w, p_{11}, p_{22}\right\}$ is given by

$$P(\Gamma \left| y, x\right.) \,\alpha\, \; L(y, X \left|\Gamma\right.) P(\Gamma) \tag{3.5}$$

where $P(\Gamma)$ is the prior distribution of Γ and $L(y, X \left|\Gamma\right.)$ is the likelihood function in Eq. (3.4). In this study, we adopt a Metropolis Hasting algorithm which can work with the multivariate distributions and is easy for estimating the complicated conditional distribution of our proposed model. Lynch [13] suggested that it is easier to use a random walk MH algorithm and to let the computer to derive the complex conditionals and let the computer do the job.In summary, the Bayesian estimation involves the following steps:

1. To sampler the posterior distribution, Vrontos, Dellaportas, and Politis [14], suggested to reduction of the computation cost by using simultaneous updating of the highly correlated parameters group. Thus, the study separates the parameters into 4 groups consisting of coefficient parameter (α), variance parameterσ^2, threshold parameter (w) and transition matrix parameter groups (p_{11}, p_{22}).

2. To estimate the posterior distribution in the model, we need to specify the prior for unknown parameters. In this study, we choose the priors as follows.

1) The conditional posterior for coefficient group is $p(\alpha_j \left|\alpha_{j-1}, \sigma^2, w, Q, y, X\right.)$ where αparameter are assumed to be normal density,

$$\tilde{\alpha}\tilde{\ } N(\alpha_j \left|\alpha_{j-1}, \sigma^2, w, Q, y, X\right. \;, \Sigma_\alpha)$$

where Σ_α is the variance of $\tilde{\alpha}$.

2) The conditional posterior for variance-covariance group is

$$p(\sigma_j^2 \left|\alpha, \sigma_{j-1}^2, w, Q, y, X\right.)$$

where σ^2 parameters are assumed to be Inverse $Gamma(\frac{v+n}{2}, \frac{v\psi+ns^2}{2})$ where v and ψ are hyper-parameter for shape and rate parameters and $s^2 = e'e/n$

3) The conditional posterior for transition matrix group is

$$p(Q_j \left|\beta, \sigma^2, w, Q_{j-1}, y, X\right.),$$

the diffuse prior for the transition probabilities is Dirichlet. $\tilde{Q}_j\tilde{\ } Dirichlet(q)$ where q is the vector of scale parameters.

4) Finally, we can write the conditional posterior distribution for was

$$p(w \left|\Gamma, Y_c, X_c\right.) = \prod_{t=1}^{n} \left\{ \sum_{j=1}^{2} \left(\frac{1}{\sqrt{2\pi\sigma_j^2}} \times \exp\left[-\frac{\left(E_{(S_t=j)}\right)^2}{2(\sigma_j^2)} \right] \right) \right\}$$

3. Draw a candidate parameter, $\Gamma^1 = \left\{\alpha^1, \sigma^{2,1}, w^1, p_{11}^1, p_{22}^1\right\}$ from a proposal function density . These candidate parameters can be sampled from

$$\Gamma^{j+1} = \Gamma^{j} + \tilde{\Gamma}^{j\sim} N(0, \varsigma)$$

where ς is a standard deviation which is obtained from Maximum Likelihood estimated standard errors of the parameters multiplied by a fraction (0.5) in order to produce a reasonable acceptance rate.

4. Each group of parameters is updated in sequence by a Metropolis-Hastings step and the parameters of each group is simultaneously updated conditional on the remaining blocks. MH-sampler is carried out by cycling repeatedly through draws of each parameter block conditional on the remaining parameter blocks. Then we simulate u from $U(0,1)$ and compare it with ratio R:

$$R = \frac{P(\Gamma_c^{j+1}|y,x)}{P(\Gamma^{j}|y,x)}$$

In each group, if $R > u$, we accept the candidate parameters otherwise, we reject then and retain the previous parameters values in this iteration. Set $j = j+1$ and return to step 3 until enough draws are obtained.

4 Simulation Study

4.1 Simulation Result

In this section, we provide a simulation study to check whether our proposed model works or not. In this simulation study, we set the true parameter values of MS(2)-T(2)-reg as specified in Table 1. T=300 and 500 observations were generated. The model errors are assumed to follow a Normal distribution with $e_1 \sim NN(0,1)$ and $e_2 \sim N(0,2)$. The independent variables X_t are randomly simulated from Normal (w,10) to ensure the structural change in the data.

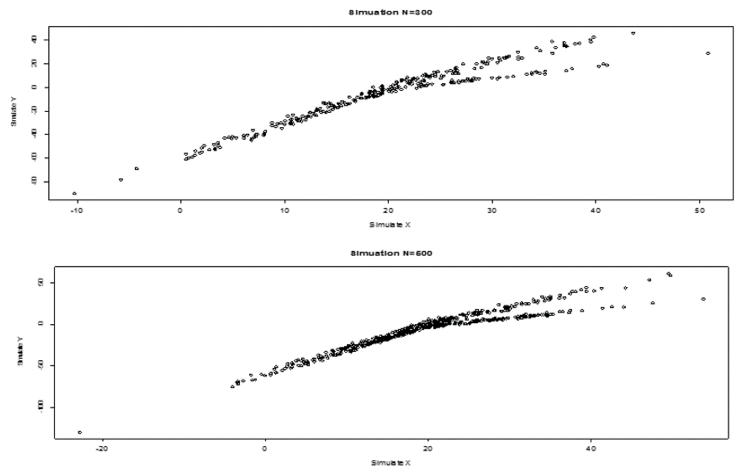


Figure 1: Scatter Plot of simulated Independent data (X) and dependent data (Y).

Table 1: Simulation Results

MS(2)-T(2)-reg				
N	Parameter	True Value	Estimated Value	Acceptance rate
	$\alpha_{0,S_t=1}$	-2	-2.2271(0.1402)	40.53%
	$\alpha_{1,S_t=1}$	3	3.0081(0.0019)	
	$\alpha_{2,S_t=1}$	1	1.0181(0.0001)	
	$\alpha_{0,S_t=2}$	1.5	0.9155(0.3985)	
	$\alpha_{1,S_t=2}$	3	2.9831(0.0075)	
	$\alpha_{2,S_t=2}$	2	2.0513(0.0050)	
	$\sigma^2_{S_t=1}$	1	1.0181(0.0002)	75.25%
	$\sigma^2_{S_t=2}$	2	1.8562(0.0011)	
	w	20	19.8743(0.0295)	35.57%
	p_{11}	0.95	0.9237(0.0173)	19.06%
	p_{22}	0.95	0.9237(0.0173)	
500	$\alpha_{0,S_t=1}$	-2	-2.0952(0.0380)	47.15%
	$\alpha_{1,S_t=1}$	3	3.0121(0.0018)	
	$\alpha_{2,S_t=1}$	1	1.0008(0.0048)	
	$\alpha_{0,S_t=2}$	1.5	1.4084 (0.0724)	
	$\alpha_{1,S_t=2}$	3	3.0149(0.0022)	
	$\alpha_{2,S_t=2}$	2	1.9772(0.0048)	
	$\sigma^2_{S_t=1}$	1	1.0008(0.0002)	73.37%
	$\sigma^2_{S_t=2}$	2	1.9767(0.0001)	
	w	20	19.904(0.0288)	29.68%
	p_{11}	0.95	0.9421 (0.0156)	15.43%
	p_{22}	0.95	0.9374(0.0157)	

Source: Calculation

The model is tested against a number of simulated data sets. The posterior mean parameter estimates are found to converge to the true parameters. The parameter means are close to the true values with the reasonable standard deviations. The acceptance rate for each block is around 30%-50%. Table 1 also demonstrates the estimation of parameter set of the model, the estimated posterior means are close to their true values. Figure 2. compares the estimated regime (line) with true regime (dot line). Our model successfully differentiated between state 1 and 2. Finally, we plot the fitted MS(2)-T(2)-reg lines in Figure 3. We can observe that the two fitted lines show a large positive slope with threshold value around 20 (green dot) for both states and their switching to small positive slope for both states when X exceed the threshold value. Overall, the simulation study suggests that MS(2)-T(2)-reg is quite accurate with the simulation data when T=200 and

500 observations.In this paper, we take 10,000 iterations for an MCMC algorithm where the first 2,000 iterations serve as a burn-in period in order to discard the uncertainty of the algorithm. The acceptance rate also computed while the iteration draw to check how well the algorithm jumps around in the parameter space.

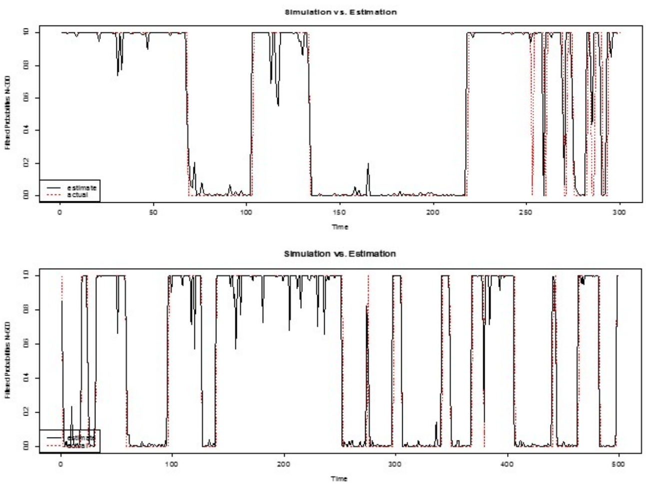


Figure 2: Simulated Filtered Probabilities Plot

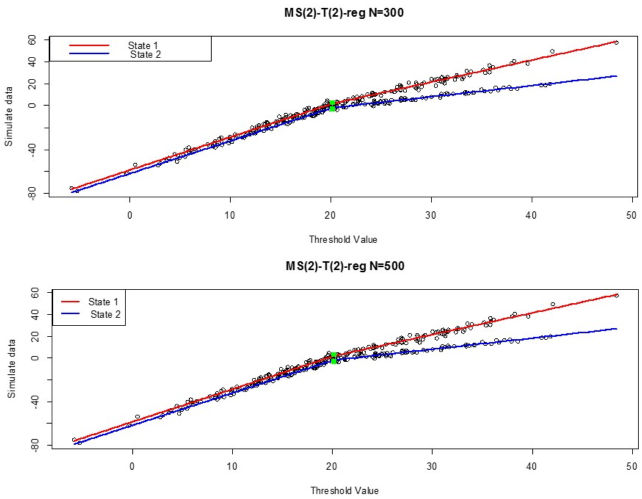


Figure 3: Estimated Markov Switching Threshold regression with break point (green dot)

4.2 Model Accuracy and Comparison with Other Models

In the financial analysis, the models for this purpose inevitably rely on imperfect assumptions and estimations and thereby creating the model risk. The model risk is a loss consequential to using the wrong model specification such as distribution assumption, the structure of the model and estimation Glasserman and Xu [12]. Thus if we adopt a wrong model, we might face the wrong result. To overcome these problem, we conducted the Kullback Leibler divergence (KLD) (also relative entropy) that as introduced in Glasserman and Xu [12] to check the robustness of the model. In this subsection we focus only on the model structure thus we compare our proposed model with three conventional models.

1) Linear regression model (reg)

2) Threshold regression with two regimes (T-reg)

3) Markov switching regression model with four states (MS-reg)

Note that the study employ a Bayesian method was employed to estimate the parameters in our model. If the true posterior is known, thus the proposed model is accurate. However, it will be true only in the simulation study. In practice, we conduct the Kullback-Leiber divergence (KLD) which is a measure of the difference between two probabilities distribution [15]. Thus we take this approach to measure the distance between the true model and alternative model through their posterior distribution. Consider the continuous probability distribution, let $\widehat{f}$ and f be denoted as the density of $\widehat{F}$and F thus we define the relative entropy of $\widehat{f}$ with respect to f to be]

$$R(f,\widehat{f}) = \int_{-\infty}^{\infty} f(x)\log\frac{f(x)}{\widehat{f}(x)}dx$$

where $R(f,\widehat{f}) > 0$ otherwise equals zero only if $\widehat{f}=f$. In this study, we define $\widehat{f}$ as an alternative or approximated posterior distribution and define f as a true posterior function when all parameters are known.

In this section, we aim to check the robustness of MSTreg by measuring the distance between true model posterior function and its approximation, when the parameter and model are correctly specified and when the model is misspecified. The study conducts Monte Carlo simulation to simulate the true model explained in section 4.1. We compare the true MSTreg function (by simulation) with its best approximations, reg, T-reg, and MS-reg (in terms of posterior function).

Figure 1 illustrates two panels of each simulation data, we can observe its best approximation (approximate MSTreg) achieve its minimum and is the closest to true function line (red dashed line) when compare with the other three models. According to the results, we can claim that our proposed model is a robust model. Adopting a wrong model makes the distance between the true model and alternative model larger and it will lead to the low accuracy of the model.Furthermore, we investigate the performance of our model under specific parameter perturbations. The Monte Carlo simulation is conducted here to generate the relative entropy for each error variance (σ^2). We vary σ^2 from 0.5 to 5 (which produces the worst case): the relative entropy between the true simulated posterior (Grey shade) and

MS-treg (dashed red), reg (dashed blue), T-reg (deep blue), and MS-reg (dashed green) are shown in Figure 5.

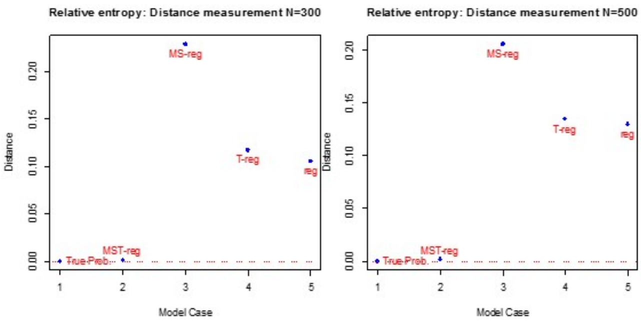
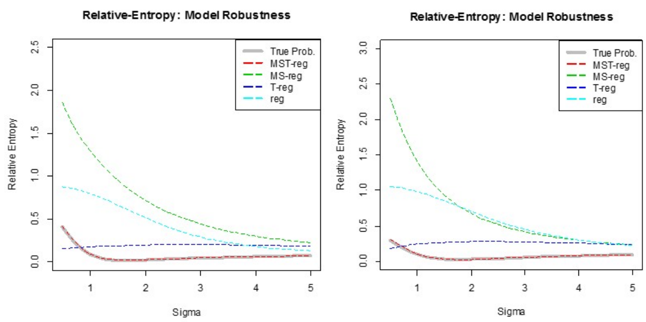


Figure 4: Estimated Markov Switching Threshold regression with break point(red dot)



(a) (b)

Figure 5: The distance between true model and alternative model under perturbations in sigma parameter

The results show that the relative between the true model and other models decreases when σ^2 increases. Increasing the error variance makes the relative entropy smaller for all the models. This indicates that a large error variance will make all model not different. However, our model does not seem to be more affected by the increase in the error variance since the lines between the MST-reg (dased red) and true model (Grey shade) are very close together. We can confirm that our model performs well in this simulation study.

5 Application on the Impact of US. Civilian Unemployment Rate to Industry Production Index

The data set considered, derived from the Thomson Reuter Data stream, Faculty of Economics, Chiang Mai University, consisted of monthly data, form the ending of December 1962 to November 2015, of the Industry Production index (IPI) and Unemployment rate of United States (UNP). The data series examined are transformed in to growth rate. In this study, we setup the model specification as follows:

$$IPI_t = \alpha_{0,S_t=1} + \alpha_{1,S_t=1} I_t[UNP_{t-1} \le w] + \alpha_{2,S_t=1} I_t[UNP_{t-1} > w] + e_{t,S_t=1}$$

$$IPI_t = \alpha_{0,S_t=2} + \alpha_{1,S_t=2} I_t[UNP_{t-1} \le w] + \alpha_{2,S_t=2} I_t[UNP_{t-1} > w] + e_{t,S_t=2}.$$

The results reported in this section are related to the fitting of an MS(2)-T(2)-reg to analyse the impact of US. Civilian Unemployment rate on Industry Production index. We plot a scatter plot of IPI and Unemployment rate in Figure 6.

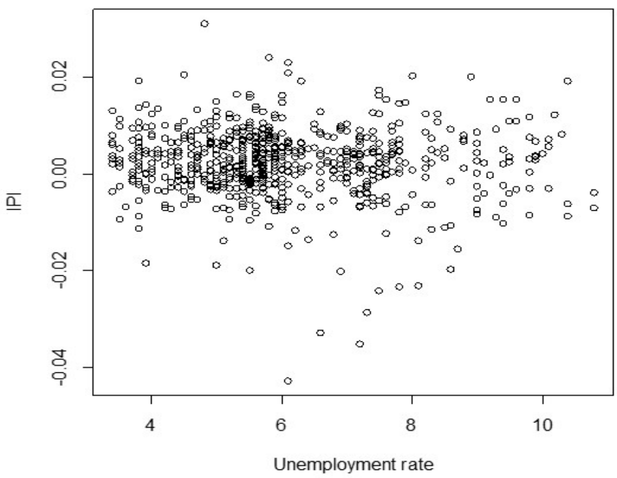


Figure 6: Scatter Plot of IPI and Unemployment rate

The estimated parameters for MS(2)-T(2)-reg, which are obtained from the Bayesian estimation, are shown in Table 2. The results provide two states where each state has two regimes. The intercept term α_0 of state 1 ($S_t = 1$)and state 2 ($S_t = 2$) seem to have an economic interpretation. $\alpha_{0,S_t=1}$ appears to have a lower value when compared with $\alpha_{0,S_t=2}$, hence we can indicate state 1 and state 2 as low economic state and high economic state, respectively. For each state, the same threshold value for both states is assumed in this study.

Table 2: Estimates Results

MS(2)-T(2)-reg				
N	Parameter	True Value	Estimated Value	Acceptance rate
	$\alpha_{0,S_t=1}$	-2	-0.0010(0.0048)	52.53%
	$\alpha_{1,S_t=1}$	3	-0.0033(0.0043)	
	$\alpha_{2,S_t=1}$	1	-0.0001079	
	$\alpha_{0,S_t=2}$	1.5	0.0003(0.0016)	
	$\alpha_{1,S_t=2}$	3	0.0300(0.0201)	
	$\alpha_{2,S_t=2}$	2	0.0083*(0.0039)	
	$\sigma^2_{S_t=1}$	1	0.0003(0.0020)	65.81%
	$\sigma^2_{S_t=2}$	2	0.0150*(0.0075)	
	w	20	6.0864(0.0020)	65.81%
	p_{11}	0.95	0.8761* (0.0432)	53.18%
	p_{22}	0.95	0.8597*(0.0514)	

Source: Calculation

The threshold estimate is 6.0864. The slope with respect to UNP equals α_{1,S_t} for UNP less than w, and equals α_{2,S_t} for values of UNP greater than w. Thus, we can split the data into two regimes for each state. Following Makiw (2003) we interpret regime 1 and 2 of state 1 as recession and depression, respectively. Mankiw [16] suggested that such periods are called recessions if they are mild and depressions if they are more severe. In this state, the estimates show negative coefficients of 0.0033 and 0.0166 for regime 1 and regime 2, respectively. Consider state 2, we interpret regime 1 and 2 as expansion and boom economy, respectively. Contrary to state 1, the estimates show positive coefficients of 0.0030 and 0.0083, respectively. This indicates that the high unemployment rate will produce a negative effect on IPI in the next month for both recession and depression periods but positive on IPI for both expansion and boom economy. Surprisingly, for the expansion period, we find the opposite effect that the more production grows, the higher the unemployment level. We expect that unemployment increases the supply of labour available for firms to employ. Thus, it will push a downward pressure on wages as labour is less scarce and more labour are willing to get a job at a lower wage. This will have a positive effect on industry as their labour costs will fall. Moreover, Table 2 also provides an estimated probabilities of staying in regime 1 (p_{11}) and regime 2 (p_{22}). This results indicate that both regimes are persistent. The mean regime at time t is shown in Figure 7. The results illustrate a filtered probabilities along our sample sizes. We can observe that the economic environment is identified to stay in high economic state more than the other. The model can capture the turbulent economic crisis during 1982-1984 which coincided with the peak of the depression when the nationwide unemployment rate was 10.8%, highest since the Great Depression in 1930; and during 2007-2009, which coincided with the Hamburger Crisis in US.

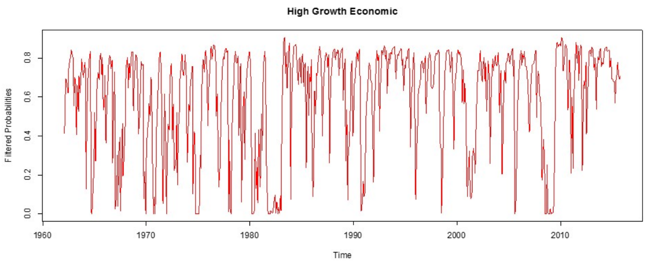


Figure 7: Filtered Probabilities in High Growth Economic

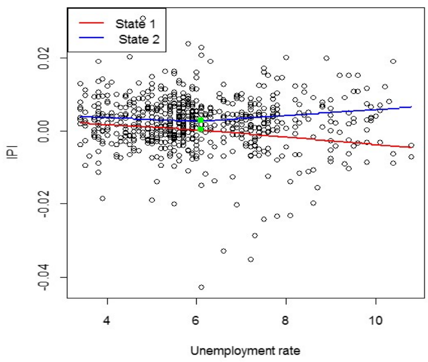


Figure 8: Scatter Plot of IPI and Unemployment rate with Estimated MS(2)-T(2)-reg model

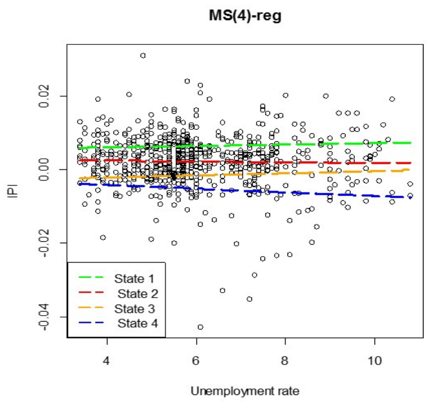


Figure 9: Scatter Plot of IPI and Unemployment rate with Estimated MS(4)-reg model

Furthermore, in this section, we plot the scatter plot of IPI and Unemployment rate with estimated Markov Switching Threshold regression with two states and two regimes (MS(2)-T(2)-reg) and a Markov Switching regression with four regimes (MS(4)-reg model). Though the Hamilton filter is conducted as a tool for split the Markov Switching model into two or more regimes, we expect the linear function assumption between independent and dependent variables is not convenient in the real economic data. To confirm our expectation, we compare the potential between MS(2)-T(2)-reg and MS(4)-reg model by plotting the model regression fitted line against scatter plot between IPI and UNP. Figure 8 and 9 illustrate MS(2)-T(2)-reg and MS(4)-reg fitted lines against scatter plot, respectively. Consider our proposed model, Figure 7 shows two different fitted lines consisting of state 1 line (red line) and state 2 line (blue line). We can see that the fitted regression in state 2 shows a small positive slope with threshold value around 6 (green dot), with switching to large positive slope above this value. Conversely, state 2 shows a small negative slope and switches to large negative slope when UNP exceeds the threshold value. Comparing to MS(4)-reg, we can observe that both model fitted lines perform well to explain the relationship between UNP and IPI. However, we compared the models in terms of a root mean square error $RMSE = \sqrt{1/(N-q)\sum_{i=1}^{N}\varepsilon_i^2}$. We found that the RMSE of MS(2)-T(2)-reg = 0.6268% and MS(4)-reg =0.7163%. Thus, our proposed model is slightly better than the conventional MS(4)-reg for this real data analysis.

6 Conclusion and Future Work

The study extends the Threshold model to a Markov Switching regression model and introduced a Markov Switching threshold regression model. Both simulation and real data studies are conducted to evaluate the performance of our model. The Bayesian method is adopted as an estimation tool for estimating our model. Robustness check through KLD or relative entropy confirms our model is robust. We measure the distance between true model and alternative models in terms of their distributions and found that our proposed model is closest to the true distribution. In addition, the study applied the proposed model to study the impact of unemployment rate to the industry production index and found that the model provides a particularly good description for this real data study. The model can capture and identify the structural change in this data with two states and two regimes. Finally, we conducted RMSE to measure the performance of our model against Markov Switching regression with four regimes and found that our model show a lower value of RMSE. For future study, as we do not consider the prior sensitivity here, we suggest that prior sensitivity check should be taken into account when the model is employed in real data analysis. Moreover, the model can be extend to Markov Switching Smooth Threshold regression in order to allow for higher degree of flexibility in model parameters through smooth transition.

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