Thai Journal of Mathematics : (2016) 22-36
Special Issue (ACFPTO2016) on : Advances in fixed point theory towards real world optimization problems

http://thaijmath.in.cmu.ac.th
Online ISSN 1686-0209

Chaos Synchronization of Two Different Chaotic Systems via Nonsingular Terminal Sliding Mode Techniques

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Abstract : This research studies the problem of finite-time chaos synchronization between two different chaotic systems with uncertain parameters and external disturbances. A nonsingular terminal sliding mode and super-twisting sliding mode control methods are proposed to solve this problem. Based on the Lyapunov stability theory, the proposed control schemes can ensure the finite-time synchronization between the master and the slave chaotic systems under parameter uncertainties and external disturbances. Numerical simulations are presented to demonstrate the applicability and effectiveness of the proposed control techniques.

Keywords : chaos synchronization; finite-time control; nonsingular terminal sliding mode; super-twisting sliding mode control.
2010 Mathematics Subject Classification : 93B12; 93D05; 93D09

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1 Introduction

Synchronization of chaotic systems has become a great deal of interest among many researches due to its potential applications in secure communication, power convertors, biological systems, information processing and chemical reactions [1]. In practice, system uncertainties and external disturbances are ubiquitous in reality. In addition, owing to un-modeled dynamics, structural variations of the system and measurement and environment noises, the chaotic systems should be considered with uncertainties and external disturbances. Thus, synchronization of chaotic systems with uncertainties and external disturbances is effectively important in applications. A number of control techniques have been proposed to synchronize of chaotic systems such as adaptive control [2], [11], passive control [3], sliding mode control [4], [5], [13], [14], [15], [16], backstepping control [6],[7],[17],[18], active control [8],[9], [19], fuzzy control [10], [20], observer-based control [12] and so on. However, most of the aforementioned works have studied asymptotical synchronization of chaotic systems. In other word, they have guaranteed that the slave system state can reach the master system state over an infinite time horizon. In real world applications, it is more practicable to realize synchronization in a finite time.

To obtain fast convergence speed in a control system, the finite-time control technique is a powerful strategy. Finite-time control methods can force the controlled systems to their targets in finite time. Up to now, finite-time controllers have been designed to stabilize a number of nonlinear systems. Terminal sliding mode control (TSMC) has been developed by introducing the fractional power term into the sliding surface. This technique offer the convergence of system states in finite time [21]. Thus, it could ensure finite-time convergence and strong robustness when the terminal sliding mode (TSM) is reached. However, using this technique, there often exists a singularity when the conventional TSMC is applied in actual cases. To overcome this difficulty, the adopted nonsingular terminal sliding mode (NTSM) concept has been proposed in [22], [23] to ensure finite-time stability and good control precision. Recently, Wang et al. [24] has proposed a novel terminal sliding mode controller and applied it into chaotic systems. However, the discontinuity property of the switching surface made it inconvenient for actual applications.

Higher order sliding mode control (HOSMC) is an extension of the traditional sliding mode control. This control method can preserve the advantages of SMC. It also gives higher accuracy and chattering attenuation. The main characteristic of HOSM is based on the action of a discontinuous control in the higher-order time derivative [25], [26], [27], [28], so the chattering can be attenuated because the control signal is continuous. Furthermore, HOSM can bring better accuracy than conventional SMC while the robustness of the control system is similar to SMC. Super twisting (ST) control algorithm is a well-known second-order sliding mode control method [28], [29]. It has been presented in [30] for the attitude tracking of a four rotors UAV.

This research studies chaos synchronization of different two chaotic systems with uncertain parameters and external disturbances. The Lyapunov stability theory is used to guarantee the stable synchronization. We propose a nonsingular terminal sliding mode and super-twisting sliding mode controllers to make the states of the slave system have same amplitude with the states of the master system in finite time.

The rest of the paper is organized as follows. In Section 2, preliminary concepts and problem statement are stated. Section 3 presents a nonsingular terminal sliding mode design. The finite-time stability is also analyzed. In Section 4, a new finite-time STW controller is designed. In Section 5, simulation results are given. Conclusions are presented in Section 6.

2 System Description and Problem Statement

The problem discussed in this study concerns with the the master-slave configuration in the presence of system uncertainties and external disturbances. The master system is described as follows:

$$\begin{aligned} \dot{y}_1(t) &= y_2(t), \\ \dot{y}_2(t) &= g(y,t) + \Delta g(y,t), \end{aligned} \tag{2.1}$$

where $y_1(t), \quad y_2(t) \in R$ are the states of the master system, $y = [y_1 \quad y_2]^T \in R^2$, $g(y,t) \in R$ is the nonlinear function of the master system, $\Delta g(y,t) \in R$ is the uncertain term of the master system.
The slave system is described by

$$\begin{aligned} \dot{x}_1(t) &= x_2(t), \\ \dot{x}_2(t) &= f(x,t) + \Delta f(x,t) + v(t) + b(x,t)u, \end{aligned} \tag{2.2}$$

where $x_1(t), \quad x_2(t) \in R$ are the states of the slave system, $x = [x_1 \quad x_2]^T \in R^2$, $f(x,t) \in R$ is the nonlinear function term of the slave system, $\Delta f(x,t) \in R$ is the uncertain term of the slave system, $v(t)$ is the disturbance input of the slave system, $b(x,t) \in R$ is the nonzero control coefficient of the slave system and $u(t) \in R$ is the control input.
We define the synchronization error as

$$e_1 = x_1 - y_1 \quad \text{and} \quad e_2 = x_2 - y_2. \tag{2.3}$$

From the master system (2.1) and the slave system (2.2), we get the error dynamic system

$$\begin{aligned} \dot{e}_1(t) &= e_2(t), \\ \dot{e}_2(t) &= f(x,t) - g(y,t) + d(x,y,t) + b(x,t)u(t), \end{aligned} \tag{2.4}$$

where $d(x,y,t) = \Delta f(x,t) - \Delta g(y,t) + v(t)$ is the error perturbation term including system uncertain term and disturbance.

Assumption 2.1. *The error perturbation term $d(x,y,t)$ and it first time derivative $\dot{d}(x,y,t)$ are bounded i.e.,*

$$|d(x,y,t)| \leq D_1 \quad and \quad |\dot{d}(x,y,t)| \leq D_2, \tag{2.5}$$

where D_1 and D_2 are positive constants.

We consider the master and slave chaotic systems described by (2.1) and (2.2), respectively. The aim is to find a controller $u(t)$ so that the error state e_1 and e_2 in (2.4) converge to zero in finite time. In other word, $\lim\limits_{t\to T} \|e(t)\| = 0$, where T is a positive constant and $\|\cdot\|$ denote the Euclidean norm.
Next, the following Lemmas that will be used in the later section are provided.

Lemma 2.1. *([31]) consider the system*

$$\dot{x} = f(x), \quad f(0) = 0, \quad x \in R^n \tag{2.6}$$

where $f : D \to R^n$ is continuous on an open neighborhood $D \subset R^n$. Assume that there is a continuous differential positive-definite function $V(x) : D \to R$, real numbers $\beta > 0$ and $0 < \gamma < 1$, such that

$$\dot{V}(x) + \beta V^{\gamma}(x) \leq 0, \quad \forall x \in D. \tag{2.7}$$

Then, the origin of system (2.6) is a locally finite-time stable equilibrium, and the setting time, depending on the initial state $x(0) = x_0$, satisfies

$$T \leq \frac{V^{1-\gamma}(x_0)}{\beta(1-\gamma)}. \tag{2.8}$$

In addition, if $D = R^n$ and $V(x)$ is also radially unbounded, then the origin is a globally finite-time stable equilibrium of system (2.6).

Lemma 2.2. *([32]) For any numbers $\lambda_1 > 0, \lambda_2 > 0, 0 < \varpi < 1$, an extended Lyapunov condition of finite-time stability can be given in the form of fast terminal sliding mode as*

$$\dot{V}(x) + \lambda_1 V(x) + \lambda_2 V^{\varpi}(x) \leq 0. \tag{2.9}$$

The setting time can be estimated by

$$T \leq \frac{1}{\lambda_1(1-\varpi)} \ell n \left(\frac{\lambda_1 V^{1-\varpi}(x_0) + \lambda_2}{\lambda_2} \right). \tag{2.10}$$

3 Finite-time nonsingular terminal sliding mode controller

In this section, a new nonsingular terminal sliding surface and finite-time terminal sliding mode controller are designed. The finite-time stability of synchronization system under the action of the proposed control law is analyzed.
We define a nonsingular terminal sliding surface as:

$$s = \dot{e}_1 + \beta_1 e_1 + \beta_2 e^{-\lambda t}|e_1|^{-2\alpha+1} sign(e_1) \tag{3.1}$$

where $\beta_1, \beta_2 > 0$, $0 < \alpha < 1$ and $\lambda > 0$.
When sliding mode occurs, the following is satisfied:

$$s = \dot{e}_1 + \beta_1 e_1 + \beta_2 e^{-\lambda t}|e_1|^{1-2\alpha} sign(e_1) = 0, \tag{3.2}$$

which can be obtained as

$$\dot{e}_1 = -\beta_1 e_1 - \beta_2 e^{-\lambda t}|e_1|^{1-2\alpha} sign(e_1). \tag{3.3}$$

A NTSM controller is designed as

$$u(t) = -(qs + \eta sign(s) + f - g + \beta_1 e_2 + \beta_2 \phi), \tag{3.4}$$

where q and η are positive constants and

$$\phi = \beta_2 \left(e^{-\lambda t}|e_1|^{-2\alpha}\dot{e}_1 - 2\alpha e^{-\lambda t}|e_1|^{-2\alpha}\dot{e}_1 - \lambda e^{\lambda t}|e_1|^{-2\alpha} e_1 \right). \tag{3.5}$$

Theorem 3.1. *For the systems (2.4), if the control law is designed as (3.5) and the gain η satisfies $\eta > |d(x, y, t)|$, the synchronization errors e_1 and e_2 will converge to the terminal sliding surface $s = 0$ in finite time.*

Proof. Consider the following Lyapunov function:

$$V_1 = \frac{1}{2}s^2 \tag{3.6}$$

Finding the first derivative of the sliding surface (3.1) with respect to time, we have

$$\dot{s} = \dot{e}_2 + \beta_1 \dot{e}_1 + \beta_2 \phi, \tag{3.7}$$

where ϕ is expressed by (3.5).
Differentiating V_1 with respect to time, and substituting (3.4) and (2.4) in to the differential result, we obtain

$$\begin{aligned}
\dot{V}_1 &= s(\dot{e}_2 + \beta_1 \dot{e}_1 + \beta_2 \phi) \\
&= s(f - g + d + u + \beta_1 \dot{e}_2 + \beta_2 \phi) \\
&= s\Big(f - g + d + \big(-(qs + \eta sign(s) + f - g + \beta_1 e_2 + \beta_2 A)\big) + \beta_1 \dot{e}_2 + \beta_2 \phi\Big) \\
&\le -qs^2 - \eta|s| + D_1|s| \\
&= -qs^2 - \epsilon_0|s| \le 0,
\end{aligned} \tag{3.8}$$

when $\epsilon_0 = \eta - D_1 > 0$. Using (3.6), we have $s = \sqrt{2}V_1^{\frac{1}{2}}$. Thus, (3.8), becomes

$$\dot{V}_1 \leq -2qV_1 - \sqrt{2}\epsilon_0 V_1^{\frac{1}{2}} \tag{3.9}$$

By Lemma 2.2, the synchronization error converges to the terminal sliding surface $s = 0$ in finite time T_R defined as

$$T_R \leq \frac{1}{q}\ell n\left(\frac{2qV_1^{\frac{1}{2}}(x_0) + \sqrt{2}\epsilon_0}{\sqrt{2}\epsilon_0}\right) \tag{3.10}$$

This completes the proof. □

Theorem 3.2. *Consider the sliding surface (3.1). If the sliding mode occurs* $(s = 0)$*, then both states of the synchronization errors* e_1 *and* e_2 *converge to zero in finite time*

$$T_s \leq \frac{\ell n\,(1 + (V_2^{\alpha}(0)/a))}{2\alpha\beta_1 - \lambda}, \tag{3.11}$$

where $a = \dfrac{2^{1-\alpha}\alpha\beta_2}{2\alpha\beta_1 - \lambda} > 0$*, with* α, β *and* λ *satisfying* $2\alpha\beta_1 > \lambda$.

Proof. Consider the Lyapunov function:

$$V_2 = \frac{1}{2}e_1^2. \tag{3.12}$$

Substituting (3.3) into the first time derivative of V_2 in (3.12), one obtains

$$\begin{aligned}
\dot{V}_2 &= e_1\dot{e}_1 \\
&= e_1(-\beta_1 e_1 - \beta_2 e^{-\lambda t}|e_1|^{-2\alpha+1} sign(e_1)) \\
&= -\beta_1 e_1^2 - \beta_2 e^{-\lambda t}|e_1|^{-2\alpha+1}|e_1| \\
&= -\beta_1 e_1^2 - \beta_2 e^{-\lambda t}|e_1|^{-2\alpha+2} \\
&= -2\beta_1 V_2 - 2^{1-\alpha}\beta_2 e^{-\lambda t} V_2^{1-\alpha} \leq 0.
\end{aligned} \tag{3.13}$$

Therefore, using the Lyapunov stability, it is obvious that the origin is globally asymptotically stable.
Next, it is required to show that the system state converge to zero in finite time. Multiplying both sides of (3.13) by $\alpha V_2^{\alpha-1}$, we have

$$\alpha V_2^{\alpha-1}\frac{dV_2}{dt} \leq -2\beta_1\alpha V_2^{\alpha} - 2^{1-\alpha}\beta_2\alpha e^{-\lambda t}$$

and

$$\alpha V_2^{\alpha-1}\frac{dV_2}{dt} + 2\beta_1\alpha V_2^{\alpha} \leq -2^{1-\alpha}\beta_2\alpha e^{-\lambda t}. \tag{3.14}$$

Next, multiplying both sides of (3.14) by $e^{2\alpha\beta_1 t}$ yields

$$e^{2\alpha\beta_1 t}\left(\frac{dV_2^\alpha}{dt}+2\beta_1\alpha V_2^\alpha\right)\leq -2^{1-\alpha}\beta_2\alpha e^{(2\alpha\beta_1-\lambda)t}$$

and

$$\frac{d}{dt}\left(e^{2\alpha\beta_1 t}V_2^\alpha\right)\leq -2^{1-\alpha}\alpha\beta_2 e^{(2\alpha\beta_1-\lambda)t}. \tag{3.15}$$

Integrating both sides of (3.15) from 0 to T_s and using $V_2(T_s)=0$, we obtain

$$-e^{2\alpha\beta_1(0)}V_2^\alpha(0) \quad \leq \quad \frac{-2^{1-\alpha}\alpha\beta_2}{2\alpha\beta_1-\lambda}\left[e^{(2\alpha\beta_1-\lambda)T_s}-1\right],$$

which can be written as

$$e^{(2\alpha\beta_1-\lambda)T_s} \quad \leq \quad 1+\frac{V_2^\alpha(0)}{a}, \tag{3.16}$$

where

$$a=\frac{2^{1-\alpha}\alpha\beta_2}{2\alpha\beta_1-\lambda}>0. \tag{3.17}$$

Taking the natural logarithm of both sides of (3.16), one has

$$(2\alpha\beta_1-\lambda)T_s\leq \ell n\left(1+\frac{V_2^\alpha(0)}{a}\right). \tag{3.18}$$

From(3.13), we obtain T_s as

$$T_s\leq\frac{\ell n\left(1+(V_2^\alpha(0)/a)\right)}{2\alpha\beta_1-\lambda} \tag{3.19}$$

This completes the proof. □

4 Finite-time super-twisting nonsingular terminal sliding mode controller

The super-twisting control law is a powerful second-order sliding mode control algorithm. It generates a continuous control signal that drives the sliding variable and its derivative to zero in finite time. In this section, a super-twisting based-nonsingular terminal sliding mode (ST-NTSM) controller is designed.
We use the sliding variable defined in (3.1) and introduce a new reaching law as:

$$\dot{s}=-k_1|s|^{\frac{\gamma+1}{2}}sign(s)-k_2\int_0^t|s|^\gamma sign(s)d\tau, \tag{4.1}$$

where k_1 and k_2 are positive constants and $0 < \gamma < 1$.
Considering the error dynamic system (2.4), the ST-NTSM controller is designed as

$$u = -\left(k_1|s|^{\frac{\gamma+1}{2}}sign(s) + k_2\int_0^t |s|^\gamma sign(s)d\tau + f - g + \beta_1 e_2 + \beta_2\phi\right), \qquad (4.2)$$

where ϕ is expressed by (3.5)
Finding first time derivative of s defined in (3.1) and substituting (4.2) into the result, one has

$$\dot{s} = -k_1|s|^{\frac{\gamma+1}{2}}sign(s) - k_2\int_0^t |s|^\gamma sign(s)d\tau + d. \qquad (4.3)$$

Let us defined

$$\begin{aligned} z_1 &= s \\ z_2 &= -k_2\int_0^t |s|^\gamma sign(s)d\tau + d. \end{aligned} \qquad (4.4)$$

Finding $\dot{z}_1$ and $\dot{z}_2$ from (4.4), one can obtain

$$\begin{aligned} \dot{z}_1 &= -k_1|z_1|^{\frac{\gamma+1}{2}}sign(z_1) + z_2 \\ \dot{z}_2 &= -k_2|z_1|^\gamma sign(z_1) + \dot{d}. \end{aligned} \qquad (4.5)$$

Next, for the system (4.5) under Assumption 2.1, the proof of finite-time stability is given.

Theorem 4.1. *Under Assumption 2.1, the states z_1 and z_2 in (4.5) converge in finite time to the region*

$$\|\zeta\| \le \left(\frac{LD_2}{\lambda_{min}\{Q\}}\right)^{\frac{\gamma+1}{2\gamma}}, \qquad (4.6)$$

where $|\dot{d}| \le D_2$, $\zeta = \begin{bmatrix}|z_1|^{\frac{\gamma+1}{2}}sign(z_1) & z_2\end{bmatrix}^T$, $L = \|[k_1 - 2]\|$, *and*

$$Q = \frac{k_1}{2}\begin{bmatrix}2k_2 + k_1^2(\gamma+1) & -k_1(\gamma+1) \\ -k_1(\gamma+1) & (\gamma+1)\end{bmatrix}.$$

In (4.6), $\lambda_{min}\{Q\}$ denotes the minimum eigenvalue of the matrix Q.

Proof. We select the following Lyapunov function

$$V_3 = \frac{2k_2}{\gamma+1}|z_1|^{\gamma+1} + \frac{1}{2}z_2^2 + \frac{1}{2}(k_1|z_1|^{\frac{\gamma+1}{2}}sign(z_1) - z_2)^2, \qquad (4.7)$$

which can be written as

$$\begin{aligned} V_3 &= \left(\frac{2k_2}{\gamma+1}+\frac{1}{2}k_1^2\right)|s|^{\gamma+1}+z^2-k_1 z|s|^{\frac{\gamma+1}{2}}sign(s) \\ &= \frac{1}{2}\begin{bmatrix}|z_1|^{\frac{\gamma+1}{2}}sign & z_2\end{bmatrix}\begin{bmatrix}\frac{4k_2}{\gamma+1}+k_1^2 & -k_1 \\ -k_2 & 2\end{bmatrix}\begin{bmatrix}|z_1|^{\frac{\gamma+1}{2}}sign(z_1) \\ z_2\end{bmatrix} \end{aligned} \tag{4.8}$$

Letting

$$P=\frac{1}{2}\begin{bmatrix}\frac{4k_2}{\gamma+1}+k_1^2 & -k_1 \\ -k_1 & 2\end{bmatrix}, \tag{4.9}$$

The Lyapunov function V_3 can be obtained as

$$V_3=\zeta^T P\zeta. \tag{4.10}$$

From (4.9), we know that matrix P is symmetricand positive definite, and

$$\lambda_{min}\{P\}\|\zeta\|^2 \leqslant V_3 \leqslant \lambda_{max}\{P\}\|\zeta\|^2, \tag{4.11}$$

where $\lambda_{min}\{P\}$ and $\lambda_{max}\{P\}$ denote the minimum eigenvalue and maximum eigenvalues of the matrix P, respectively.

The first time derivative of Lyapunov function (4.8) along the solutions of system (4.5) is

$$\begin{aligned} \dot{V}_3 &= -2k_1k_2|z_1|^{\frac{3\gamma+1}{2}}+\frac{k_1^2(\gamma+1)}{2}z_2|z_1|^{\gamma}sign(z_1) \\ &\quad +k_1\left(k_2-\frac{k_1^2(\gamma+1)}{2}\right)|z_1|^{\frac{3\gamma+1}{2}}-\frac{k_1(\gamma+1)}{2}z_2^2|z_1|^{\frac{\gamma-1}{2}} \\ &\quad +\frac{k_1^2(\gamma+1)}{2}z_2|z_1|^{\gamma}sign(z_1)-(k_1|z_1|^{\frac{\gamma+1}{2}}sign(z_1)-2z_2)\dot{d}. \end{aligned} \tag{4.12}$$

$\dot{V}_3$ in (4.12) can be rearranged as

$$\begin{aligned} \dot{V}_3 &= -\frac{k_1}{2}|z_1|^{\frac{\gamma-1}{2}}\begin{bmatrix}|z_1|^{\frac{\gamma+1}{2}}sign(z_1) & z_2\end{bmatrix}\begin{bmatrix}2k_2+k_1^2(\gamma+1) & -k_1(\gamma+1) \\ -k_1(\gamma+1) & (\gamma+1)\end{bmatrix} \\ &\quad \times\begin{bmatrix}|z_1|^{\frac{\gamma+1}{2}}sign(z_1) & z_2\end{bmatrix}^T+\begin{bmatrix}k_1 & -2\end{bmatrix}\begin{bmatrix}|z_1|^{\frac{\gamma+1}{2}}sign(z_1) \\ z_2\end{bmatrix}\dot{d} \end{aligned} \tag{4.13}$$

Letting

$$Q=\frac{k_1}{2}\begin{bmatrix}2k_2+k_1^2(\gamma+1) & -k_1(\gamma+1) \\ -k_1(\gamma+1) & (\gamma+1)\end{bmatrix}, \tag{4.14}$$

and

$$L=\|[k_1-2]\|=\sqrt{k_1^2+4}. \tag{4.15}$$

$\dot{V}_3$ becomes

$$\dot{V}_3 \leq -|z_1|^{\frac{\gamma-1}{2}} \zeta^T Q \zeta + L\zeta^T \dot{d}.$$

Using the fact that

$$\|\zeta\|^2 = |z_1|^{\gamma+1} + z_2^2, \tag{4.16}$$

and $0 < \gamma < 1$, one obtains

$$|z_1|^{\frac{\gamma-1}{2}} \geqslant \|\zeta\|^{\frac{\gamma-1}{\gamma+1}}. \tag{4.17}$$

Therefore, using (4.17), we have

$$\begin{aligned} \dot{V}_3 &\leqslant -|z_1|^{\frac{\gamma-1}{2}} \lambda_{min}\{Q\}\|\zeta\|^2 + LD_2\|\zeta\| \\ &\leqslant -\lambda_{min}\{Q\}\|\zeta\|^{\frac{3\gamma+1}{\gamma+1}} + LD_2\|\zeta\| \\ &= -(\lambda_{min}\{Q\}\|\zeta\|^{\frac{2\gamma}{\gamma+1}} - LD_2)\|\zeta\| \\ &\leqslant -\left(\lambda_{min}(Q)\|\zeta\|^{\frac{2\gamma}{\gamma+1}} - LD_2\right) \frac{V_3^{1/2}}{\sqrt{\lambda_{max}(P)}}. \end{aligned} \tag{4.18}$$

If $\lambda_{min}(Q)\|\zeta\|^{\frac{2\gamma}{\gamma+1}} - LD_2 > 0$, (4.18) can be written as $\dot{V}_3 \leq \dfrac{\Omega V_3^{\frac{1}{2}}}{\sqrt{\lambda_{max}(P)}}$ where $\Omega = \lambda_{min}(Q)\|\zeta\|^{\frac{2\gamma}{\gamma+1}} - LD_2 > 0$.
Thus, $\dot{V}_3 \leq 0$ is always kept, when $\lambda_{min}(Q)\|\zeta\|^{\frac{2\gamma}{\gamma+1}} > LD_2$. It follows that $\|\zeta\|$ is reduced and converges to the region $\|\zeta\| \leq \left(\dfrac{LD_2}{\lambda_{min}(Q)}\right)^{\frac{\gamma+1}{2\gamma}}$ in finite time. This completes the proof.

□

5 Numerical simulations

In this section, through a typical numerical example, we study the chaos synchronization based on previous theory result obtained.
We consider the master system (2.1) and the slave system (2.2), where
$g(y,t) = y_1 - 0.2y_2 - y_1^3 - 0.32cos(1.2t),\ f(x,t) = 2x_1 - 1.4x_2 - 0.8x_1^2,$
$\Delta g(y,t) = -0.02y_1,\ \Delta f(x,t) = 0.01x_2,\ v(t) = -0.1sin^2(2t).$
The simulation is carried out with step size 0.001 second. The initial condition is set as $x(0) = [-2\ 3]^T$, $y(0) = [0.8091\ 0.5155]^T$ and control parameters are chosen in Table 1.

Table 1: Control parameters

NTSM	$\beta_1 = \beta_2 = 1, \eta = 2, q = 1, \alpha = 0.3, \lambda = 3$
ST-NTSM	$\beta_1 = \beta_2 = 1.2,\ k_1 = 4, k_2 = 3, \alpha = 0.3, \gamma = 0.1, \lambda = 3$

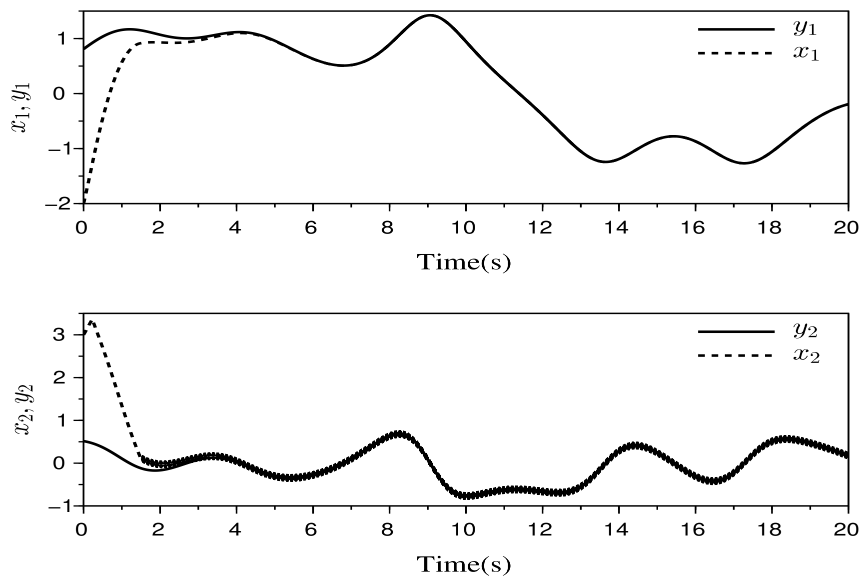

Figure 1: Synchronization of two different chaotic systems for NTSM.

We compare the results between the synchronization error obtained by the proposed NTSM control and ST-NTSM control schemes. As shown in Figure 1 and Figure 2, for both control methods, states of the slave system completely track the states of the master system in about 4 seconds. The control responses from the proposed ST-NTSM control and NTSM control are shown in Figure 3. One can easily see that that the ST-NTSM gives smoother control signal and higher precision synchronization than the NTSM control method. Figure 4 shows the responses of the sliding surfaces. Clearly, the sliding surface from the ST-NTSM method is also smoother than NTSM method. In view of these simulation results, the ST-NTSM control method offers better results of synchronization.

6 Conclusion

In this paper, the NTSM control and ST-NTSM control techniques have been developed to synchronize two different two second-order chaotic systems. NTSM control avoids the singularity problem but this method cannot reduce the chattering phenomenon. The ST-NTSM controller solves the singularity problem and provides better synchronization results and higher accuracy. Using the Lyapunov theory, we have proved the finite-time convergence of synchronous errors. The simulation results are given to show the effectiveness of the developed control methods.

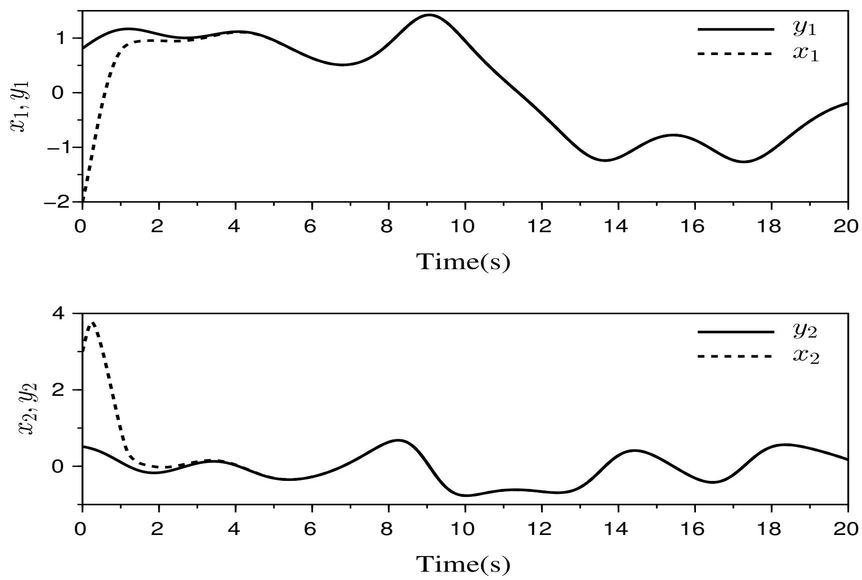

Figure 2: Synchronization of two different chaotic systems for ST-NTSM.

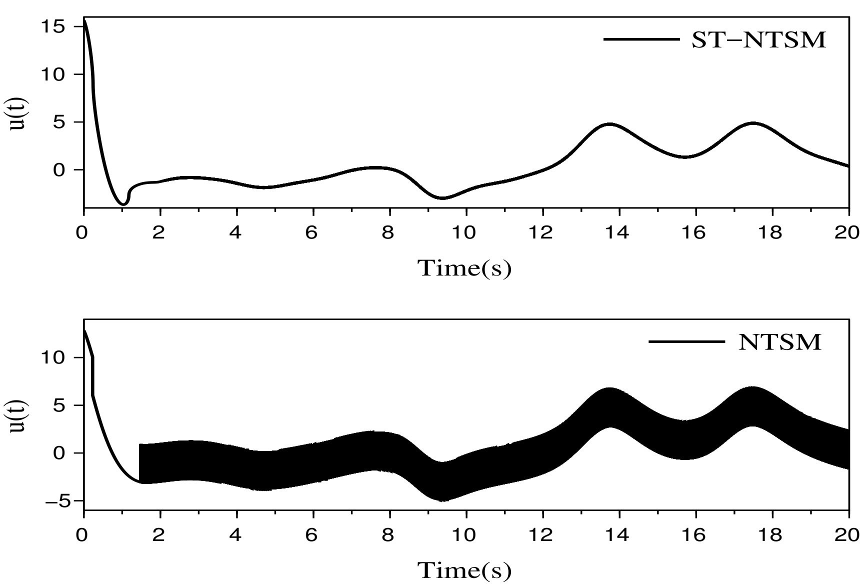

Figure 3: Control responses for ST-NTSM and NTSM.

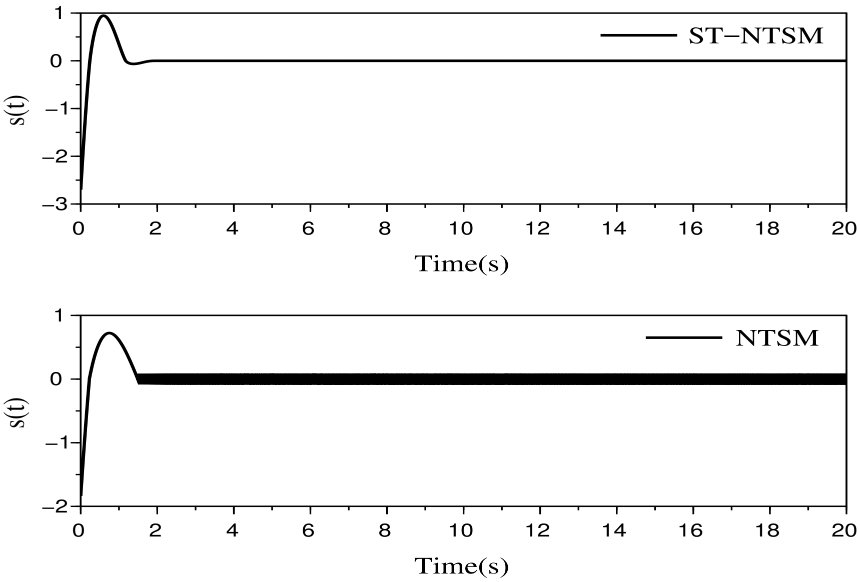

Figure 4: Sliding variables for ST-NTSM and NTSM.

7 Acknowledgement

The research was funded by King Mongkut's University of Technology North Bangkok. Contract no. KMUTNB-GEN-59-17.

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