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Decompositions of Generalized Continuity in Grill Topological Spaces

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Abstract : The paper deals with some new types of generalized open set in grill topological space and its properties. Some generalized continuities will also be considered and its decompositions will be discussed.

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1 Introduction

The concept of Grill [1] and its study [2, 3, 4] in topological space are not new in literature. But the study of grill topological space [5] as like similar to the ideal topological space [6] had been started from 2007. Mathematicians Al-Omari and Noiri [7, 8] and Hatir and Jafari [9] have developed the study of grill topological spaces with continuities and generalized continuities. According to Choquet [1], a grill $\mathcal{G}$ on a topological space X is a non-null collection of nonempty subsets of X satisfying two conditions: (i) $A \in \mathcal{G}$ and $A \subset B \subset X \Rightarrow B \in \mathcal{G}$ and (ii) A, $B \subset X$ and $A \cup B \in \mathcal{G} \Longrightarrow A \in \mathcal{G}$ or $B \in \mathcal{G}$.

In [5] Roy and Mukherjee have introduced a new topology on a topological space X, constructed by the use of a grill on X, and is described as follows:

Let $\mathcal{G}$ be a grill on a topological space (X, τ). Consider the operator $\Phi : \wp(X) \to \wp(X)$ (here $\wp(X)$ stands for the power set of X), given by

$\Phi(A) = \{x \in X : U \cap A \in \mathcal{G}$ for all open neighbourhoods Ucontaining x in $X\}$. Then the map $\Psi : \wp(X) \to \wp(X)$, where $\Psi(A) = A \cup \Phi(A)$ for $A \in \wp(X)$, is a Kuratowski closure operator and hence induces a topology $\tau_{\mathcal{G}}$ on X, strictly finer than τ, in general. An open base $\mathcal{B}$ for the topology $\tau_{\mathcal{G}}$ on X is given by $\mathcal{B}= \{U \setminus A : U \in \tau$ and $A \notin \mathcal{G}\}$. If $\mathcal{G}$ is a grill on the topological space (X, τ) then we will denote $(X, \tau, \mathcal{G})$ as a grill topological space.

The paper which had been written by E. Ekici [10] has motivated me for studying this paper. In this paper we shall introduce some generalized open sets like semi$^{\mathcal{G}}$-open, R-$\mathcal{G}$-set, $\mathcal{A}_{\mathcal{G}}$-set etc. We shall also introduce different types of continuity and characterization of the same. We also decompose these continuities.

Throughout this paper $Cl(K)$ and $Int(K)$ denote the closure and interior of K in (X, τ), respectively for a subset K of a topological space (X, τ). $Cl_{\mathcal{G}}(K)$ and $Int_{\mathcal{G}}(K)$ denote the closure and interior of K in $(X, \tau_{\mathcal{G}})$, respectively for a subset K of a topological space $(X, \tau_{\mathcal{G}})$.

2 R-$\mathcal{G}$ -open sets and $\mathcal{A}_G$-sets

Definition 2.1. *A subset K of a grill topological space $(X, \tau, \mathcal{G})$ is said to be*
(1) R-$\mathcal{G}$-open if $K = Int(Cl_{\mathcal{G}}(K))$;
(2) R-$\mathcal{G}$-closed if its complement is R-$\mathcal{G}$-open.

Lemma 2.2. *Let K be a subset of a grill topological space $(X, \tau, \mathcal{G})$. If N is an open set, then $N \cap Cl_{\mathcal{G}}(K) \subseteq Cl_{\mathcal{G}}(N \cap K)$.*

Proof. Let $x \in N \cap Cl_{\mathcal{G}}(K)$. Then $x \in N$ and $x \in Cl_{\mathcal{G}}(K)$, implies that $x \in N$ and for all $U_x \in \tau_{\mathcal{G}}$, $U_x \cap K \neq \emptyset$. Again $N \in \tau_{\mathcal{G}}$ and $N \cap U_x$ is the open set of $(X, \tau_{\mathcal{G}})$ containing x, so, $N \cap U_x \cap K \neq \emptyset$. This implies that $x \in Cl_{\mathcal{G}}(N \cap K)$. □

Definition 2.3. *A subset K of a grill topological space $(X, \tau, \mathcal{G})$ is said to be*
(1) semi$^{\mathcal{G}}$-open if $K \subseteq Cl(Int_{\mathcal{G}}(K))$;
(2) semi$^{\mathcal{G}}$-closed if its complement is semi$^{\mathcal{G}}$-open .

Theorem 2.4. *For a grill topological space $(X, \tau, \mathcal{G})$ and a subset K of X, the following properties are equivalent:*
(1) K is R-$\mathcal{G}$-closed set;
(2) K is semi$^{\mathcal{G}}$-open and closed.

Proof. (1)⇒ (2): Let K be a R-$\mathcal{G}$-closed set in X. Then we have $K = Cl(Int_{\mathcal{G}}(K))$. It follows that K is semi$^{\mathcal{G}}$-open and closed.

(2)⇒ (1): Suppose that K is a semi$^{\mathcal{G}}$-open set and closed set in X. It follows that $K \subseteq Cl(Int_{\mathcal{G}}(K))$. Since K is closed, then we have $Cl(Int_{\mathcal{G}}(K)) \subseteq Cl(K) = K \subseteq Cl(Int_{\mathcal{G}}(K))$.

Thus $K = Cl(Int_{\mathcal{G}}(K))$ and hence K is R-$\mathcal{G}$-closed. □

Theorem 2.5. *For a grill topological space $(X, \tau, \mathcal{G})$ and a subset K of X, K is an R-$\mathcal{G}$-open set if and only if K is semi$^{\mathcal{G}}$-closed and open.*

Proof. It follows from above Theorem. □

Theorem 2.6. *A subset K of a grill topological space $(X, \tau, \mathcal{G})$ is semi$^{\mathcal{G}}$-open if and only if there exists $N \in \tau_{\mathcal{G}}$ such that $N \subseteq K \subseteq Cl(N)$.*

Proof. Let K be semi$^{\mathcal{G}}$-open, then $K \subseteq Cl(Int_{\mathcal{G}}(K))$. Take $N = Int_{\mathcal{G}}(K)$. Then we have $N \subseteq K \subseteq Cl(N)$.

Conversely, let $N \subseteq K \subseteq Cl(N)$ for some $N \in \tau_{\mathcal{G}}$. We have $N \subseteq Int_{\mathcal{G}}(K)$ and hence $Cl(N) \subseteq Cl(Int_{\mathcal{G}}(K)$. Thus we obtain $K \subseteq Cl(Int_{\mathcal{G}}(K))$. □

Theorem 2.7. *For a grill topological space $(X, \tau, \mathcal{G})$ and a subset K of X, the following properties are equivalent:*

(1) K is a R-$\mathcal{G}$-closed set;

(2) there exists a $\tau_{\mathcal{G}}$-open set L such that $K = Cl(L)$.

Proof. (2)⇒ (1): Suppose that there exists a $\tau_{\mathcal{G}}$-open set L such that $K = Cl(L)$. Since $L = Int_{\mathcal{G}}(L)$, then we have $Cl(L) = Cl(Int_{\mathcal{G}}(L))$. It follows that $Cl(Int_{\mathcal{G}}(Cl(L))) = Cl(Int_{\mathcal{G}}(Cl(Int_{\mathcal{G}}(L)))) = Cl(Int_{\mathcal{G}}(L)) = Cl(L)$. This implies $K = Cl(L) = Cl(Int_{\mathcal{G}}(Cl(L))) = Cl(Int_{\mathcal{G}}(K))$. Thus, $K = Cl(Int_{\mathcal{G}}(K))$ and hence K is an R-$\mathcal{G}$-open set in X.

(1)⇒ (2): Suppose that K is a R-$\mathcal{G}$-closed set in X. We have $K = Cl(Int_{\mathcal{G}}(K))$. We take $L = Int_{\mathcal{G}}(K)$. It follows that L is a $\tau_{\mathcal{G}}$-open set and $K = Cl(L)$. □

Theorem 2.8. *For a grill topological space $(X, \tau, \mathcal{G})$ and a subset K of X, K is semi$^{\mathcal{G}}$-open if $K = L \cap M$ where L is an R-$\mathcal{G}$-closed set and $Int(M)$ is a $\tau_{\mathcal{G}}$-dense set.*

Proof. Suppose $K = L \cap M$ where L is an R-$\mathcal{G}$-closed set and $Int(M)$ is a $\tau_{\mathcal{G}}$-dense set. By Theorem 2.7, there exists a $\tau_{\mathcal{G}}$-open set N such that $L = Cl(N)$. We take $O = N \cap Int(M)$. It follows that O is $\tau_{\mathcal{G}}$-open and $O \subseteq K$. Moreover, we have $Cl(O) = Cl(N \cap Int(M))$ and $Cl(N \cap Int(M)) \subseteq Cl(N)$. Since $Int(M)$ is $\tau_{\mathcal{G}}$-dense, then we have $N = N \cap Cl_{\mathcal{G}}(Int(M)) \subseteq Cl_{\mathcal{G}}(N \cap Int(M)) \subseteq Cl(N \cap Int(M))$. It follows that $Cl(N) \subseteq Cl(N \cap Int(M))$. Furthermore, we have $Cl(O) = Cl(N \cap Int(M)) \subseteq Cl(N) = L \subseteq Cl(N \cap Int(M)) = Cl(O)$. Thus, $O \subseteq K \subseteq L = Cl(O)$. Hence by Theorem 2.6, K is a semi$^{\mathcal{G}}$-open set in X. □

Definition 2.9. *The semi$^{\mathcal{G}}$-closure of a subset K of a grill topological space $(X, \tau, \mathcal{G})$, denoted by $s_{\mathcal{G}}Cl(K)$, is defined by the intersection of all semi$^{\mathcal{G}}$-closed sets of X containing K.*

Theorem 2.10. *Let $(X, \tau, \mathcal{G})$ be a grill topological space and $A \subseteq X$. Then $x \in s_{\mathcal{G}}Cl(A)$ if and only if every semi$^{\mathcal{G}}$-open set U_x containing x, $U_x \cap A \neq \emptyset$.*

Proof. Let $x \in s_{\mathcal{G}}Cl(A)$. Suppose that $U_x \cap A = \emptyset$, U_x is an semi$^{\mathcal{G}}$-open set containing x. Then $A \subseteq (X \setminus U_x)$ and $(X \setminus U_x)$ is a semi$^{\mathcal{G}}$-closed set containing A. Therefore $x \in (X \setminus U_x)$, and this is a contradiction.

Conversely suppose that $U_x \cap A \neq \emptyset$, for every semi$^{\mathcal{G}}$-open set U_x containing x. If possible suppose that $x \notin s_{\mathcal{G}}Cl(A)$, then there exists F subset of X which satisfy

$A \subseteq F$, $X \setminus F$ is semi$^{\mathcal{G}}$-open and $x \notin F$. Therefore $x \in (X \setminus F)$. So $A \cap (X \setminus F) = \emptyset$ for an semi$^{\mathcal{G}}$-open set $X \setminus F$ containing x. It is a contradiction. □

Corollary 2.11. *For a subset K of a grill topological space $(X, \tau, \mathcal{G})$, $s_{\mathcal{G}}Cl(K) = K \cup Int(Cl_{\mathcal{G}}(K))$.*

Proof. Suppose that $x \in Int(Cl_{\mathcal{G}}(K))$ but $x \notin s_{\mathcal{G}}Cl(K)$. Then there exists a semi$^{\mathcal{G}}$-open set U_x containing x, such that $U_x \cap K = \emptyset$ (by Theorem 2.10). Since U_x is semi$^{\mathcal{G}}$-open set, then there exists $N \in \tau_{\mathcal{G}}$ such that $N \subseteq U_x \subseteq Cl(N)$ (by Theorem 2.6). So, $N \cap K = \emptyset$, i.e., $x \notin Cl_{\mathcal{G}}(K)$. Therefore $x \notin Int(Cl_{\mathcal{G}}(K))$, a contradiction. So, $x \in s_{\mathcal{G}}Cl(K)$. Again if $x \in K$ then it is obvious that $x \in s_{\mathcal{G}}Cl(K)$. Finally we get, $Int(Cl_{\mathcal{G}}(K)) \cup K \subseteq s_{\mathcal{G}}Cl(K)$.

For reverse inclusion we shall prove that $Int(Cl_{\mathcal{G}}(K)) \cup K$ is a semi$^{\mathcal{G}}$-closed set containing K. Now $IntCl_{\mathcal{G}}(IntCl_{\mathcal{G}}(K) \cup K)) \subseteq Cl_{\mathcal{G}}(IntCl_{\mathcal{G}}(K) \cup K) \subseteq Cl_{\mathcal{G}}(Cl_{\mathcal{G}}(K)) \cup Cl_{\mathcal{G}}(K) = Cl_{\mathcal{G}}(K)$. This implies that, $IntCl_{\mathcal{G}}(IntCl_{\mathcal{G}}(K) \cup K)) \subseteq IntCl_{\mathcal{G}}(K)$. Therefore $IntCl_{\mathcal{G}}(IntCl_{\mathcal{G}}(K) \cup K)) \subseteq IntCl_{\mathcal{G}}(K) \cup K$. So $IntCl_{\mathcal{G}}(K) \cup K$ is a semi$^{\mathcal{G}}$-closed set containing K and hence $s_{\mathcal{G}}Cl(K) \subseteq IntCl_{\mathcal{G}}(K) \cup K$.

Hence the result. □

Definition 2.12. *Let $(X, \tau, \mathcal{G})$ be a grill topological space and $K \subseteq X$. K is called*

(1) generated semi$^{\mathcal{G}}$- closed ($gs_{\mathcal{G}}$- closed) in $(X, \tau, \mathcal{G})$ if $s_{\mathcal{G}}Cl(K) \subseteq O$ whenever $K \subseteq O$ and O is an open set in $(X, \tau, \mathcal{G})$;

(2) generated semi$^{\mathcal{G}}$- open set ($gs_{\mathcal{G}}$- open) in $(X, \tau, \mathcal{G})$ if $X \setminus K$ is a $gs_{\mathcal{G}}$- closed set in $(X, \tau, \mathcal{G})$.

Theorem 2.13. *For a subset M of a grill topological space $(X, \tau, \mathcal{G})$, M is called $gs_{\mathcal{G}}$- open if and only if $T \subseteq s_{\mathcal{G}}Int(M)$ whenever $T \subseteq M$ and T is a closed set in $(X, \tau, \mathcal{G})$ where $s_{\mathcal{G}}Int(M) = M \cap Cl(Int_{\mathcal{G}}(M))$*

Proof. Suppose M is a $gs_{\mathcal{G}}$- open set in $(X, \tau, \mathcal{G})$. Let $T \subseteq M$ and T be a closed set in $(X, \tau, \mathcal{G})$. It follows that $X \setminus M \subseteq X \setminus T$, where $X \setminus T$ is open set. Since $X \setminus M$ is a $gs_{\mathcal{G}}$- closed, then $s_{\mathcal{G}}Cl(X \setminus M) \subseteq X \setminus T$, where $s_{\mathcal{G}}Cl(X \setminus M) = (X \setminus M) \cup Int(Cl_{\mathcal{G}}(X \setminus M))$. Since $(X \setminus M) \cup Int(Cl_{\mathcal{G}}(X \setminus M)) = (X \setminus M) \cup (X \setminus Cl(Int_{\mathcal{G}}(M))) = X \setminus (M \cap Cl(Int_{\mathcal{G}}(M))) = X \setminus s_{\mathcal{G}}Int(M)$. It follows that $s_{\mathcal{G}}Cl(X \setminus M) = X \setminus s_{\mathcal{G}}Int(M)$. Thus, $T \subseteq X \setminus s_{\mathcal{G}}Cl(X \setminus M) = s_{\mathcal{G}}Int(M)$ and hence $T \subseteq s_{\mathcal{G}}Int(M)$.

The converse part is similar. □

Theorem 2.14. *Let $(X, \tau, \mathcal{G})$ be a grill topological space and $N \subseteq X$. The following properties are equivalent:*

(1) N is an R-$\mathcal{G}$-open set;

(2) N is open and $gs_{\mathcal{G}}$-closed.

Proof. (1) ⇒ (2): Let N be an R-$\mathcal{G}$-open set in $(X, \tau, \mathcal{G})$. Then we have $N = Int(Cl_{\mathcal{G}}(N))$. It follows that N is open and semi$^{\mathcal{G}}$-closed in $(X, \tau, \mathcal{G})$. Thus, $s_{\mathcal{G}}Cl(N) \subseteq K$ whenever $N \subseteq K$ and K is an open set in $(X, \tau, \mathcal{G})$.

(2)⇒ (1): Let N be open and $gs_{\mathcal{G}}$-closed in $(X,\tau,\mathcal{G})$. We have $N \subseteq Int(Cl_{\mathcal{G}}(N))$. Since N is $gs_{\mathcal{G}}$-closed and open, then we have $s_{\mathcal{G}}Cl(N) \subseteq N$. Since $s_{\mathcal{G}}Cl(N) = N \cup Int(Cl_{\mathcal{G}}(N))$, then $s_{\mathcal{G}}Cl(N) = N \cup Int(Cl_{\mathcal{G}}(N)) \subseteq N$. Thus, $Int(Cl_{\mathcal{G}}(N)) \subseteq N$ and $N \subseteq Int(Cl_{\mathcal{G}}(N))$. Hence $N = Int(Cl_{\mathcal{G}}(N))$ and N is an R-$\mathcal{G}$-open set in $(X,\tau,\mathcal{G})$. □

Definition 2.15. *A subset K of a grill topological space $(X,\tau,\mathcal{G})$ is said to be*

(1) an $\mathcal{A}_{\mathcal{G}}$-set if $K = L \cap M$, where L is an open set and $M = Cl(Int_{\mathcal{G}}(M))$;

(2) a locally closed set [11] if $K = L \cap M$ where L is an open set and M is a closed set in $(X,\tau,\mathcal{G})$.

Remark 2.16. *Let $(X,\tau,\mathcal{G})$ be a grill topological space. Any open set and any R-$\mathcal{G}$-closed set in $(X,\tau,\mathcal{G})$ is an $\mathcal{A}_{\mathcal{G}}$-set. The reverse of this implication is not true in general as shown in the following example.*

Example 2.17. *Let $X = \{a,b,c,d\}, \tau = \{X, \emptyset, \{a\}, \{b,c\}, \{a,b,c\}\}$ and $\mathcal{G} = \{\{b\}, \{a,b\}, \{a,b,c\}, \{c,b,d\}, \{b,c\}, \{b,d\}, \{a,b,d\}, X\}$. Then the set $K = \{b,c,d\}$ is an $\mathcal{A}_{\mathcal{G}}$-set but it is not open. The set $L = \{a,b,c\}$ is an $\mathcal{A}_{\mathcal{G}}$-set but it is not R-$\mathcal{G}$-closed.*

Remark 2.18. *Let $(X,\tau,\mathcal{G})$ be a grill topological space. Any $\mathcal{A}_{\mathcal{G}}$-set is a locally closed set in X. The reverse implication is not true in general as shown in the following example.*

Example 2.19. *Let $X = \{a,b,c,d\}, \tau = \{X, \emptyset, \{a\}, \{b,c\}, \{a,b,c\}\}$ and $\mathcal{G} = \{\{b\}, \{a,b\}, \{a,b,c\}, \{c,b,d\}, \{b,c\}, \{b,d\}, \{a,b,d\}, X\}$. Then the set $K = \{d\}$ is locally closed but it is not an $\mathcal{A}_{\mathcal{G}}$-set.*

Theorem 2.20. *Let $(X,\tau,\mathcal{G})$ be a grill topological space, $N \subseteq X$ and $K \subseteq X$. If N is a semi$^{\mathcal{G}}$-open set and K is an open set, then $N \cap K$ is semi$^{\mathcal{G}}$-open.*

Proof. Suppose that N is a semi$^{\mathcal{G}}$-open set and K is an open set in X. It follows by Lemma 2.2 that

$N \cap K \subseteq Cl(Int_{\mathcal{G}}(N)) \cap K \subseteq Cl(int_{\mathcal{G}}(K) \cap K) = Cl(Int_{\mathcal{G}}(N \cap K))$. Thus, $N \cap K \subseteq Cl(Int_{\mathcal{G}}(N \cap K))$ and hence, $N \cap K$ is a semi$^{\mathcal{G}}$-open set in X. □

Lemma 2.21 ([11])**.** *For a subset A of a topological space (X,τ), A is locally closed if and only if $A = U \cap Cl(A)$ for an open set U.*

Definition 2.22. *A subset K of a grill topological space $(X,\tau,\mathcal{G})$ is said to be*

(1) $\beta_{\mathcal{G}}$-open if $K \subseteq Cl(Int_{\mathcal{G}}(Cl(K)))$;

(2) $\beta_{\mathcal{G}}$-closed if $X \setminus K$ is $\beta_{\mathcal{G}}$-open.

Theorem 2.23. *Let $(X,\tau,\mathcal{G})$ be a grill topological space and $K \subseteq X$. The following properties are equivalent:*

(1) K is an $\mathcal{A}_{\mathcal{G}}$-set;

(2) K is semi$^{\mathcal{G}}$-open and locally closed;

(3) K is a $\beta_{\mathcal{G}}$-open set and a locally closed set.

Proof. (1)⇒ (2): Suppose that K is an $\mathcal{A}_{\mathcal{G}}$-set in X. It follows that $K = L \cap M$ where L is an open set and $M = Cl(Int_{\mathcal{G}}(M))$. Then K is locally closed. Since M is a semi$^{\mathcal{G}}$-open set, then by Theorem 2.20, K is a semi$^{\mathcal{G}}$-open set in X.

(2)⇒ (3): It follows from the fact that any semi$^{\mathcal{G}}$-open set is $\beta_{\mathcal{G}}$-open.

(3)⇒ (1): Let K be a $\beta_{\mathcal{G}}$-open set and a locally closed set in X. We have $K \subseteq Cl(Int_{\mathcal{G}}(Cl(K)))$. Since K is a locally closed set in X, then there exists an open set L such that $K = L \cap Cl(K)$. It follows that

$K = L \cap Cl(K) \subseteq L \cap Cl(Int_{\mathcal{G}}(Cl(K))) \subseteq L \cap Cl(K) = K$ and then $K = L \cap Cl(Int_{\mathcal{G}}(Cl(K)))$. We take $M = Cl(Int_{\mathcal{G}}(Cl(K)))$. Then $Cl(Int_{\mathcal{G}}(M)) = M$. Thus, K is an $\mathcal{A}_{\mathcal{G}}$-set in X. □

Theorem 2.24. *Let $(X, \tau, \mathcal{G})$ be a grill topological space. If every subset of $(X, \tau, \mathcal{G})$ is an $\mathcal{A}_{\mathcal{G}}$-set, then $(X, \tau, \mathcal{G})$ is a discrete grill topological space with respect to $\tau_{\mathcal{G}}$.*

Proof. Suppose that every subset of $(X, \tau, \mathcal{G})$ is an $\mathcal{A}_{\mathcal{G}}$-set. It follows from Theorem 2.23 that $\{x\}$ is semi$^{\mathcal{G}}$-open and locally closed for any $x \in X$. We have $\{x\} \subseteq Cl(Int_{\mathcal{G}}(\{x\}))$. Thus we have $Int_{\mathcal{G}}(\{x\}) = \{x\}$. Hence $(X, \tau, \mathcal{G})$ is a discrete grill topological space with respect to $\tau_{\mathcal{G}}$. □

3 Decompositions of $\mathcal{A}_{\mathcal{G}}$-continuous functions

Definition 3.1. *A function $f : (X, \tau, \mathcal{G}) \to (Y, \sigma)$ is said to be*

(1) $\mathcal{A}_{\mathcal{G}}$-continuous if $f^{-1}(T)$ is an $\mathcal{A}_{\mathcal{G}}$-set in X for each open set T in Y;

(2) LC-continuous [11] if $f^{-1}(T)$ is a locally closed set in X for each open set T in Y.

Remark 3.2. *For a function $f : (X, \tau, \mathcal{G}) \to (Y, \sigma)$, the following diagram holds. The reverses of these implications are not true in general as shown in the following example.*

$$\text{Continuous} \Longrightarrow \mathcal{A}_{\mathcal{G}}\text{-continuous} \Longrightarrow \text{LC-continuous.}$$

Example 3.3. *Let $X = \{a, b, c, d\}$, $\tau = \{X, \emptyset, \{a\}, \{b, c\}, \{a, b, c\}\}$ and $\mathcal{G} = \{\{b\}, \{a, b\}, \{a, b, c\}, \{c, b, d\}, \{b, c\}, \{b, d\}, \{a, b, d\}, X\}$. The function $f : (X, \tau, \mathcal{G}) \to (X, \tau)$, defined by $f(a) = a$, $f(b) = b$, $f(c) = b$, $f(d) = c$ is $\mathcal{A}_{\mathcal{G}}$-continuous but it is not continuous. The function $g : (X, \tau, \mathcal{G}) \to (Y, \tau)$, defined by $g(a) = b$, $g(b) = c$, $g(c) = c$, $g(d) = a$ is LC-continuous but it is not $\mathcal{A}_{\mathcal{G}}$-continuous.*

Definition 3.4. *A function $f : (X, \tau, \mathcal{G}) \to (Y, \sigma)$ is said to be*

(1) semi-$\mathcal{G}$-continuous if $f^{-1}(T)$ is a semi$^{\mathcal{G}}$-open in X for each open set T in Y;

(2) $\beta_{\mathcal{G}}$-continuous if $f^{-1}(T)$ is a $\beta_{\mathcal{G}}$-open set in X for each open set in Y.

Theorem 3.5. *The following properties are equivalent for a function $f : (X, \tau, \mathcal{G}) \to (Y, \sigma)$:*

(1) f is $\mathcal{A}_{\mathcal{G}}$-continuous;
(2) f is semi$^{\mathcal{G}}$-continuous and LC-continuous;
(3) f is $\beta_{\mathcal{G}}$-continuous and LC-continuous.

Proof. It follows from Theorem 2.23. □

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